Stack and frequency's coupling characteristic analysis in thermoacoustic systems

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The performance of a standing-wave thermoacoustic refrigerator system depends on the thermoacoustic stack and frequency modulating phase between the pressure and gas velocity. The quality factor differs for different parameter of stack, so only an effective frequency bandwidth can couple phase to realize heat pumping and excite the resonance. On the contrary, the stack parameter which can realize this goal is an effective bandwidth for a certain frequency too. When frequency or stack parameter is not appropriate to each other, the pumping heat maybe inefficient or vanish.

INTRODUCTION

In the past decades, much attention has been paid on the application of the thermoacoustic technology. Thermoacoustic engines are devices which convert heat energy to acoustic work, and vice versa [1-4]. The phase between oscillating pressure and gas velocity determines voluminous degree of the $p-V$ diagram and the orientation of the cycle for a periodic thermodynamic cycle. Two fundamental conditions must be satisfied in order to pumping heat up the temperature gradient in a standing wave thermoacoustic refrigerator. One is the phase between pressure and velocity can shift from 90°; the other is the resonance can be excited. The phase between pressure and volume is modulated by the crank in a traditional engine. But for a standing wave refrigerator driven by loudspeaker, there is has no crank. The stack and frequency modulate phase between the pressure and gas velocity. Because the particular dynamic resource and particular manner of phase modulated, it is important to determine the frequency and stack for the goal. On the other hand, the stack is the core to realize the conversion between heat energy and acoustic power. The coupling characteristics between stack and frequency are important for the standing wave thermoacoustic refrigerator. When the frequency of the loudspeaker is not appropriate for a certain system, or the stack parameters is not appropriate for a certain frequency, pumping heat up the temperature gradient maybe inefficient.

THE PHASE CONDITION

From the $p-V$ chart, we can find that the phase between oscillating pressure and gas velocity should be $\left(-3\pi/2,-\pi/2\right)$ in order to realize the refrigeration cycle. Acoustics power $\dot{E}_2\left(x\right)$ can be expressed as following [5]

$$E_2(x)=\frac{\omega}{2\pi}\oint \mathrm{Re}\left[p_1(x)e^{i\omega t}\right]\mathrm{Re}\left[U_1(x)e^{i\omega t}\right]dt=\frac{1}{2}\left|p_1\tilde{U}_1\right|\cos\phi_{pU}\,, \qquad (1)$$

with Re: real part, ω: the angular frequency, p_1: oscillating pressure, U_1: oscillating volume flow rate, ϕ_{PU}: phase (p_1, U_1). Using equation (1) for acoustic power $\dot{E}_2$, it is apparent that $\dot{E}_2$ =0 when the phase between pressure and volume flow rate is 90°, i.e., for standing wave phasing. When the phase is departure from standing wave phase, the conversion between heat energy and acoustic power can be caused. This departure is realized by the coupling between stack and acoustic wave.

The momentum equation (including viscosity and arbitrary shape and size of channel) is [6]

$$i\omega\rho_m u_1=-\frac{dp_1}{dx}+\mu\left(\frac{\partial^2 u_1}{\partial y^2}+\frac{\partial^2 u_1}{\partial z^2}\right) \qquad (2)$$

With boundary conditions, $u_1=0$ at the solid surface, the solution is

$$u_1=\frac{i}{\omega\rho_n}\left[1-h_v(y,z)\right]\frac{dp_1}{dx}=\frac{i}{\omega\rho_m}H_v\frac{dp_1}{dx} \qquad (3)$$

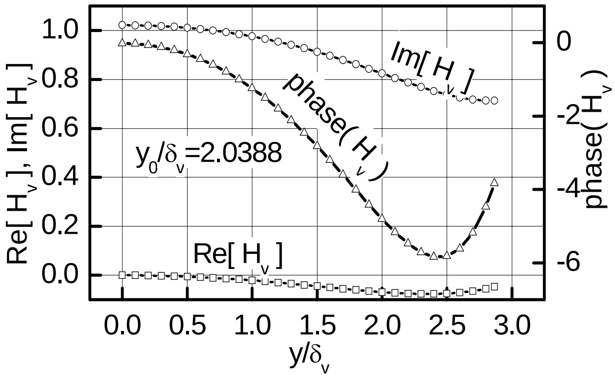

Figure 1 Function H_v and its phase for different layer in Hofler's thermoacoustic refrigerator

The complex function h_v depends on the special channel geometry. There is detailed expression for function h_v in reference [6] for different geometrical channels. Equation (2) indicates that function h_v depends on the hydraulic radius and viscous penetration depth of the system for a given fluid under given thermodynamic conditions. Figure 1 is a plot of H_v and its phase versus the ratio of hydraulic radius r_h and viscous penetration depth δ_v for different fluid layer. According to the calculation model stated in reference [7] (Hofler's thermoacoustic refrigerator, y_0/δ_v =2.0388). The calculated results indicate that the magnitude and phase of the gas velocity was influenced by the hydraulic radius and frequency. Some traveling wave components were added and the capability of energy conversion was possessed.

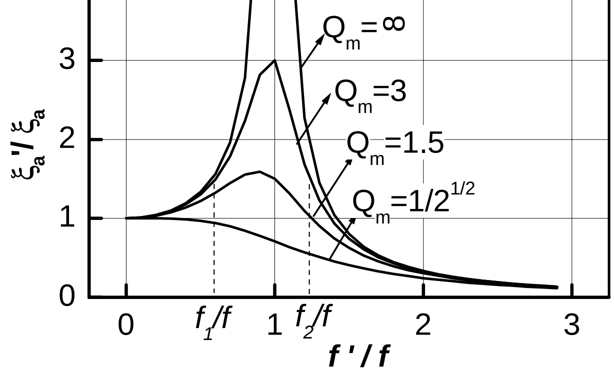

Figure 2 Resonance curve of a simple mechanical system

THE REALIZATION OF RESONANCE

It is a self-sustained oscillation caused by the interaction of the solid with the working fluid at the condition of a gradient temperature existing for thermoacoustic prime mover. The wave equation is

$$M_a \frac{d^2\xi}{dt^2} + R_a \frac{d\xi}{dt} + \frac{1}{C_a}\xi = 0, \tag{4}$$

with M_a, R_a, C_a : acoustic mass, impedance, and compliance respectively. For a thermoacoustic refrigerator driven by a louder-speaker, the louder-speaker provides a periodic driving force to the system. So it is a forced oscillation. The differential equation becomes

$$M_a \frac{d^2\xi}{dt^2} + R_a \frac{d\xi}{dt} + \frac{1}{C_a}\xi = p_a e^{i\omega' t} \tag{5}$$

Where $p = p_a e^{i\omega' t}$ is acoustic pressure externally driving force, ω' is angular frequency of the driving force. The solution of equation (5) is the sum of two parts (a transient term and a steady-state term) which depends on p_a and ω'. After a sufficient time interval the damping term decreases and vanish in the end. There is only the steady-state term is whose angular frequency ω' is that of the driving force in the end.

The displacement amplitude ξ_a' depends not only on magnitude and frequency of the driving force, but also on some inherent parameter such as thermal properties of thermodynamic working substance, viscous and thermal penetration depth. As stated in reference [8], quality factor Q_m was applied to describe the sharpness of resonance of the forced oscillation system. A plot of ξ_a'/ξ_a versus f'/f is shown in Figure 2, where ξ_a' and f' are the amplitude and frequency of forced oscillation respectively, ξ_a and f are inherent amplitude and frequency of free oscillation respectively. Another definition of the sharpness of resonance can be given in term of the frequency bandwidth $f_2 - f_1$, where f_1 and f_2 are the two frequencies above and below resonance f' for which displacement amplitude has drooped to one-half its resonance value. Figure 2 indicates that frequency bandwidth is corresponding to quality factor.

AN EXAMPLE

The standing wave thermoacoustic refrigerator should follow the principle as stated above. When a certain parameter of the system is changed, its quality factor would change, and then the frequency bandwidth would change. We should search frequency bandwidth of the system when we design a standing wave thermoacoustic refrigerator at first; the effective system parameter bandwidth should be found for a certain frequency, otherwise the pumping heat would not realize.

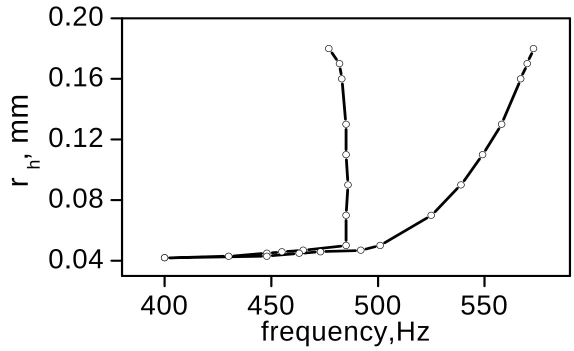

Figure 3 Effective frequency and hydraulic radius bandwidth for Hofler's thermoacoustic refrigerator

The stack is the core of conversion between heat energy and acoustic power, so we have more interest in its characteristics. Different length or hydraulic radius of the stack carries with them corresponding impedance, so their quality factor is different. According to the above principle, the frequency bandwidth of amplitude resonance is different. In the same way, there is an effective stack parameter can realize the goal for a certain frequency.

When we study this kind of problem in a concrete standing wave thermoacoustic refrigerator, same conclusion was obtained. The relation between effective frequency and effective hydraulic radius was shown in Figure 3. The calculation model was Hofler's thermoacoustic refrigerator [8]. The results indicate that the hydraulic radius and frequency is corresponding. i.e. there is an effective frequency bandwidth for a given r_h; there is an effective hydraulic radius bandwidth for a certain frequency. We should pay attention to this problem when we design a thermoacoustic system.

CONCLUSION

For standing wave thermoacoustic refrigerator, the refrigeration performance depends on the phase departing the phase of standing wave and the resonance being excited which as a result of coupling between stack and frequency. Enough attention should be paid to frequency and stack hydraulic radius bandwidth. Resonance frequency and stack hydraulic radius can couple an appropriate phase to realize refrigeration goal.

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