Numerical simulation on a novel cascade thermoacoustic prime mover

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A novel cascade thermoacoustic prime mover is numerically studied based on linear thermoacoustic theory. With the structure of system and the external conditions such as heat input and room temperature given, the fundamental resonant frequency and the distributions of all concerned variables are gained. From the simulation, it is proved that the key components of the machine—the regenerator and stack can work under the traveling-wave mode and standing-wave mode respectively when there is an imaginary load, so the efficiency is improved. Some particular characteristics of this new engine are described.

INTRODUCTION

It is well known that heat and acoustic energy can be converted each other by thermoacoustic process, and a thermoacoustic engine is a device to achieve the conversion. Different modes of machine do not have the same conversion efficiency, for example, a standing-wave engine (SWE) has lower efficiency because of an intrinsically irreversible thermodynamic cycle; and a traveling-wave engine (TWE) has 50% higher efficiency due to excellent thermal contact in regenerator. But it is inevitable that the Gedeon streaming would come into being in the traveling-wave loop, which results in a decreased efficiency. Another point of disadvantage is that there exits a huge thermal stress in the loop. Recently a cascade thermoacoustic engine was invented, whose geometry, only straight-line topology, is simple to build like the standing-wave engine, and achieved efficiency up to 20%[1]. In this paper we will make a numerical analysis for this type of machine. The calculation is based on linear thermoacoustic theory [2], which is a powerful tool to understand the thermoacoustic effect under low amplitude.

MODEL AND NUMERICAL METHOD

In an ideal half-wavelength standing-wave tube, the phase differences between oscillating pressure and volume velocity are always 90°. However, for a lossy one, there is a usual transition zone, where the traveling-wave component becomes dominant. If a regenerator is placed at this special location of a standing-wave engine, the acoustic power coming from a standing-wave stage will be amplified. The total efficiency of the cascade machine could be likely increased, since the traveling-wave Stirling engine is more efficient than the primary stage. However, not any in-phase zone adapt to be inserted a regenerator,

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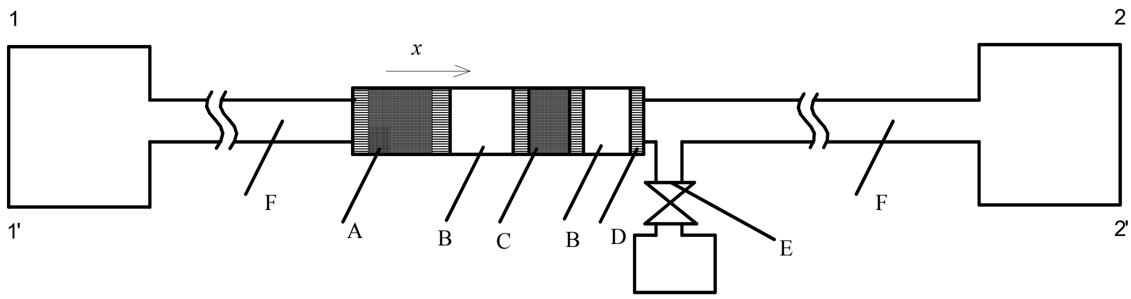


Figure 1   Schematics of a cascade thermoacoustic engine with 2-open ends

or large viscous losses could be produced. In fact, only the one with high acoustic impedance would be a suitable choice. So a standing-wave pipe attached to two infinitely large reservoirs is selected as a model. For this pipe, its fundamental frequency is expected. Figure1 is the schematic of our model cascade engine, which comprises of (A)standing-wave stage(first engine), (B)thermal buffer tube*(TBT)* (C)traveling-wave stage(second engine), (D)ambient heat exchanger, (E)dissipation load and (F)resonator. The two reservoirs simulate open boundary condition.

Using complex notation [3], we have the following universal expressions for any components,

$$\begin{cases} \dfrac{d}{dx}\begin{bmatrix} \tilde{p} \\ \tilde{U} \end{bmatrix} = \overrightarrow{A_F} \cdot \begin{bmatrix} \tilde{p} \\ \tilde{U} \end{bmatrix} & (1) \\ \dfrac{d}{dx}\begin{bmatrix} T_0 \\ E_x \end{bmatrix} = \overrightarrow{A_T} \cdot \begin{bmatrix} T_0 \\ E_x \end{bmatrix} + \vec{B} \ , & (2) \end{cases}$$

where $\tilde{p}, \tilde{U}$ are the oscillating pressure and volume velocity, T_0, E_x are the mean temperature and second-order total energy flux, which is caused by the hydrodynamic transportation of enthalpy and heat conduction of working medium. We impose the boundary condition of（i）volumetric velocity $\tilde{U} = 0$ (ii) energy flux $E_x = 0$ at the two ends (namely Section11’ and 22’ in Figure 1). We can obtain the following solution by simple integral

$$\begin{cases} \begin{bmatrix} \tilde{p}(x) \\ \tilde{U}(x) \end{bmatrix} = \vec{F}(x, x_0) \begin{bmatrix} \tilde{p}(x_0) \\ \tilde{U}(x_0) \end{bmatrix} & (3) \\ \begin{bmatrix} T_0(x) \\ E_x(x) \end{bmatrix} = \vec{G}(x, x_0) \begin{bmatrix} T_0(x_0) \\ E_x(x_0) \end{bmatrix} + \vec{S}(x, x_0), & (4) \end{cases}$$

where $\vec{F}$ $(x,\ x_0)$ is a 2×2 flow transmission matrix(FTM), $\vec{G}$ (x,x_0) is a 2×2 thermal transfer matrix(TTM) and $\vec{S}$ $(x,\ x_0)$ is a 2×1 thermal response vector (TRV). According to Eqs(3),(4), the acoustical variables $\tilde{p}, \tilde{U}$ and thermal variables T_0, E_x could be respectively regarded as a 4-port network element with 2-input and 2-output. Once the FTM, TTM and TRV of all components are obtained, the whole system is decided, including frequency, oscillating variables and temperature distribution. However, there are couplings between the two matrixes and the response vector, the concerned parameters can only be obtained analytically by iteration. Before starting a simulation, we must provide initial values for oscillating frequency and the distribution of temperature. Then the physical property and the frequency-dependent FTM can be calculated. By solving the acoustical equation, a new frequency is obtained. Then, the distribution of pressure and velocity could be obtained quickly according to Eq.(3). The next step is to solve the temperature distribution by calculating the TTM, TRV with boundary condition of energy equation. Generally, the gained temperature is not same as one given before, so iteration has to be used to repeat above steps until temperature discrepancy between two iterations is fallen within the required accuracy.

COMPUTATION RESULTS AND ANALYSIS

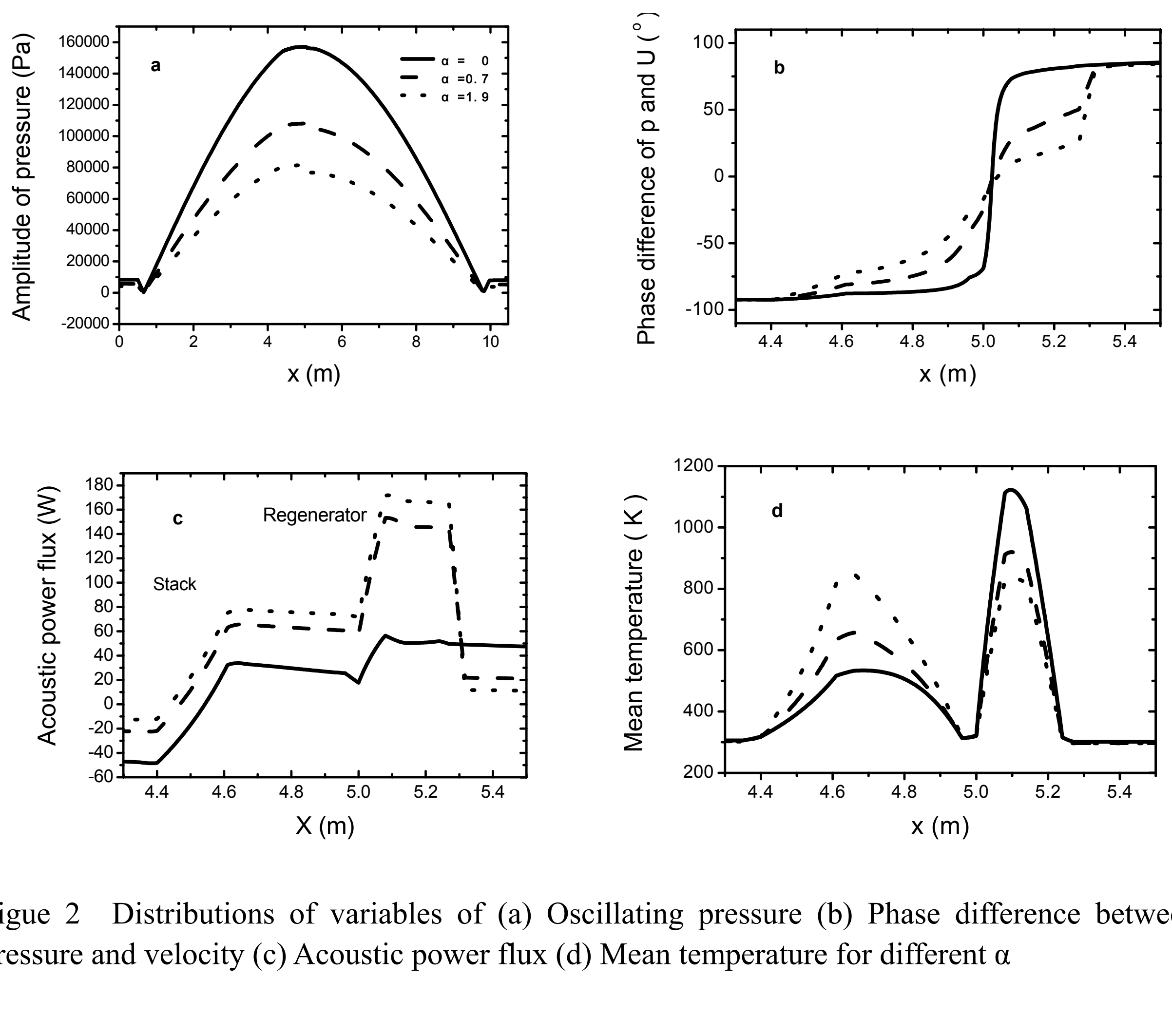
According to the aforementioned analysis, we design a cascade apparatus to match the tested traveling-wave thermoacoustic refrigerator, which works appropriately at a frequency of about 50 Hz. The main parameters are given in Table 1. The heat exchangers have the structure of many parallel channels formed by copper fins with a thickness of 1.0 mm and 1.0 mm gap. The stack and regenerator are made from stainless steel screens of 20 mesh and 120 mesh respectively. The other components are straight stainless tubes. The working fluid is helium gas and the flow is laminar.

Table 1 Main geometry parameters of model

	Reservoir (2)	Resonator (2)	Ambient Heat Exchanger (3)	Stack	Regenerator	Hot heat Exchanger (2)	TBT(2)
Length (m)	0.50/0.50	3.85 / 4.70	0.04/0.04/0.04	0.21	0.08	0.05/0.05	0.30/0.10
Diameter (m)	0.35	0.031	0.05	0.05	0.05	0.05	0.05

In order to examine the performance of cascade engine, we add an imaginary dissipation load R_{load} in the output of the ambient heat exchanger. By adjusting a velocity factor α, $U_{load} = |\tilde{U}_{load}| = \alpha U_{Resonator}$, we can gradually shift the load, where U the magnitude of $\tilde{U}$. $\alpha = 0$ means there is no load and the load resistance is infinity. As the factor α rising, the load resistance will decrease monotonically to zero. In this case, the produced mechanical energy is completely consumed in load.

Figure 2 is the calculated distribution of several interested variables for three loads($\alpha = 0,0.7,1.9$) at mean pressure 2.0 MPa and the total heating power 800 W (the standing-wave stage and traveling-wave stage share the quantity averagely). The pressure antinode is roughly located at the centre of the apparatus



Figue 2 Distributions of variables of (a) Oscillating pressure (b) Phase difference between pressure and velocity (c) Acoustic power flux (d) Mean temperature for different α

(see Figure 2a), where simultaneously is the node of velocity. At the entrances of the two reservoirs, pressure amplitude decrease almost to zero (node) and then rebound to a small value which is nearly stable in reservoir. The oscillation will be weaker while improving the factor α, because more power is used up by the load. Figure 2b is the distribution of phase difference between pressure($\tilde{p}$) and velocity($\tilde{U}$), which expressly indicates the influence of load to the cascade system. When $\alpha = 0$, the traveling-wave region becomes short abruptly vertical and a majority part of the regenerator is already out of in-phase zone, where the phase difference changes from -71.1° to 70.8°. It can be imagined the working status of second engine would be bad. As α rising, the curve is more evenly and the in-phase zone becomes wider. For example $\alpha = 1.9$, the pressure lags velocity by 18.2° at the cold end of regenerator, and leads velocity by 10.3° at hot end, which means the regenerator is working under the traveling-wave mode mainly. It will be very beneficial for the traveling-wave stage to work normally with gas parcels experiencing Stirling cycle and to produce more acoustic work with high efficiency (see Figure 2c). Figure 2d shows that the fist and second engine have different demand for temperature of heat source despite the same heat power. When no acoustic power is delivered, the standing-wave stage needs a lower one and the traveling-wave stage needs a high one. With more acoustic power consumed by the load, the temperature gradient of the stack will increase and that of regenerator will decrease, which means that there should be a limit to the load, or the hot heat exchanger would be burned out. Figure 3 shows the relations of the thermal efficiency and the factor α, where η_1, η_2, η_3 are respectively that of the first engine stage, the second stage and the entire engine. We find η_1 varies slightly; η_2 increases quickly first and transcends η_1, but keeps stable in succession; η_3 is similar to that of η_2. It indicates that, the final delivered power is limited and controlled by the inherent performance of the system.

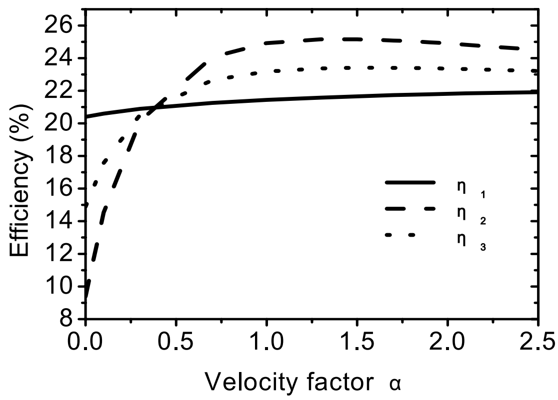


Figure 3 Efficiency versus velocity factor α

CONCLUSIONS

The performance and characteristics of a cascade thermoacoustic engine have been simulated successfully using linear thermoacoustic theory. The calculated results indicate that the cascade engine can operate with a higher thermal efficiency than a single standing-wave stage. It the meantime, because there are different development trend of heat source temperature for the two engine stages, the load has a great influence on the stability of the cascade system. Much efforts on the engine needs to be made in future.

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