Analytical model for onset temperature difference of thermoacoustic Stirling prime movers

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The onset temperature difference of thermoacoustic Stirling prime movers is a very important parameter for utilizing low quality heat energy. This paper establishes an analytical model to calculate it and finds its dependence on the length and porosity of the regenerator, the working gas, the oscillating frequency and the working pressure.

INTRODUCTION

Thermoacoustic engine is a kind of new machine that can convert heat into mechanical work in the form of an oscillating pressure. They, especially thermoacoustic Stirling prime movers with higher achievable efficiency, have received lots of attention in recent years due to their outstanding advantages of structure simplicity, heat-driven mechanism and environment-friendliness, etc. The onset temperature difference of thermoacoustic Stirling prime movers has crucial influence on the whole device's performance and is a very important parameter for utilizing low quality heat energy, so it is very significant to study the onset temperature difference under different parameters. In this paper we get an analytical expression for it and find out its dependence on various parameters such as the length and porosity of the regenerator, the working gas, the oscillating frequency, the working pressure, and so on.

MODEL

The schematic of the system that we will study in this paper is shown in Figure 1 and the deductions are based on the following assumptions: the temperature distribution along the regenerator is linear; the equation of state of perfect gas is valid; the viscous and thermal dissipations in all components are neglected except in the regenerator; the amplitude of the oscillation is small enough that linear thermoacoustic theory can be used.

According to the theory of literature [1, 2], the onset temperature gradient along the regenerator filled with mesh in thermoacoustic Stirling prime movers is

$$\left(\frac{dT}{dx}\right)_{cr}=\frac{\gamma-1}{a\beta_0}\omega\frac{g_{wk}+\frac{1}{\gamma-1}\left|Y_a\right|^2 g_{w\mu}}{R_e\left[Y_a f_{WT}\right]} \tag{1}$$

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Where, $f_{WT} = (1-\Phi)\Omega/[j\omega\Phi(1-\Phi)+\Omega]$ denotes the factor of power production; $g_{wk} = Re(j^*f_{WT})$ denotes the factor of power dissipation by imperfect heat transfer; $g_{w\mu} = a\mu/\rho\omega$ denotes the factor of power dissipation by flow friction; $Y_a = \rho\alpha U/(\varphi AP)$ denotes the local specific acoustic admittance. All the nomenclature for these physical parameters and mathematical operations used in this paper are listed in Table 1.

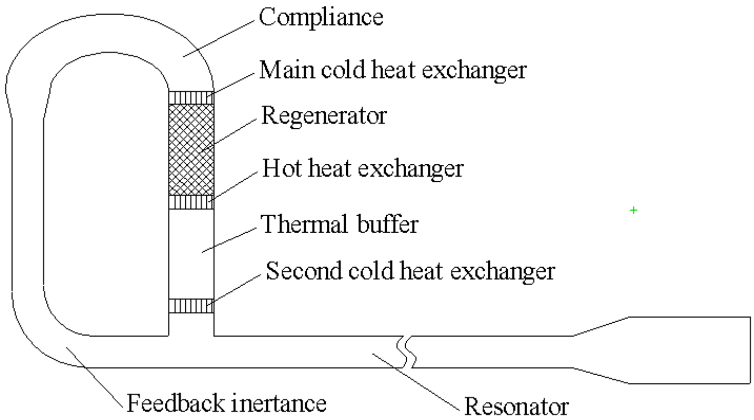

Figure 1 Schematic of a thermoacoustic Stirling prime mover

Obviously, when $dT/dx>(dT/dx)_{cr}$, the local acoustic power flow will be magnified; conversely, when $dT/dx<(dT/dx)_{cr}$, the local acoustic power will be attenuated.

Table 1 Nomenclature

α Adiabatic sound velocity	A Cross-sectional area of regenerator
C_0 Compliance of regenerator	C_1 Compliance of feedback tube
j Imaginary unit	l length of regenerator
L_1 Inertance of feedback tube	L_2 Inertance of thermal buffer tube
P Amplitude of pressure	R Flow resistance of regenerator
R_e Real part of	T Temperature
U Amplitude of volume velocity	
a Flow viscous coefficient	β_0 Thermal expansion coefficient
γ Ratio of isobaric to isochoric specific heat	μ Dynamic viscosity of working gas
ρ Density of working gas	τ Ratio of hot to cold end temperature
φ Volume porosity of regenerator	Φ Heat capacity ratio
ω Angular frequency	Ω Characteristic heat exchanger angular frequency

In the expression for Y_a, the ratio of U to P is unknown, which is the main impediment for us to calculate the onset temperature gradient. Fortunately, literature [3] presents a formula for U/P in the cold end of a regenerator:

$$\frac{U_{cold}}{p_{cold}} = \frac{\omega^2 C_1 L_1 + \dfrac{j\omega C_0 R}{2} g(\tau,b)}{R\dfrac{\tau+1}{2} f(\tau,b) + j\tau\omega L_2 + j\omega L_1} \tag{2}$$

Where, $f(\tau,b)=2*(\tau^{b+2}-1)/[(b+2)*(\tau^2-1)]$; $g(\tau,b)=2*[\tau^{b+2}\ln\tau-(\tau^{b+2}-1)/(b+2)]/[(b+2)*(\tau-1)^2]$, for helium, b=0.68014; τ denotes the ratio of hot-end to cold-end temperatures of the regenerator.

The onset temperature difference can be obtained by integrating the local temperature gradient along the regenerator. However, we already assumed that the temperature along the regenerator is linearly distributed, so we only need to calculate the onset temperature gradient in the cold end of the regenerator. Then we can obtain the onset temperature difference in a simpler way as given by equation (3).

$$\Delta T_{cr} = l \times \left(dT/dx\right)_{cr,cold} \tag{3}$$

NUMERICAL RESULT AND DISCUSSION

The main structure parameters used in this calculation are: the diameters of the regenerator and the thermal buffer tube are 8.75cm, 9cm, respectively, and their lengths are 8cm, 24cm, respectively; the feedback tube consists of two sections, one is 7.8cm in diameter and 56.7cm in length, the other is 10.34cm in diameter, 33.7cm in length.

Figure 2 is the distribution of onset temperature difference versus the length of the regenerator. The triangle, square and diamond lines correspond to different working pressures of 1MPa, 2MPa, 3MPa respectively. The working frequency is 80 Hz; the stainless-steel screen filled in the regenerator is 20-mesh. From this figure one can see that: the onset temperature difference always decreases first and then goes up as the regenerator length increases. It means that there is an optimum length of regenerator. For shorter regenerator, a lower working pressure has a lower onset temperature difference; conversely, for longer regenerator, the higher working pressure has a lower onset temperature difference.

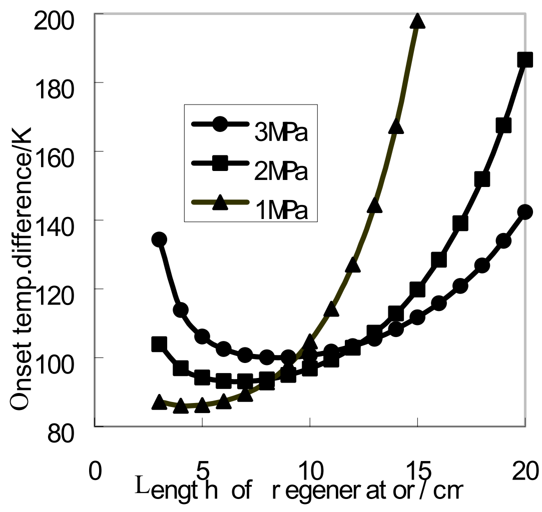

Figure 2 The distribution of onset temperature difference versus the length of the regenerator

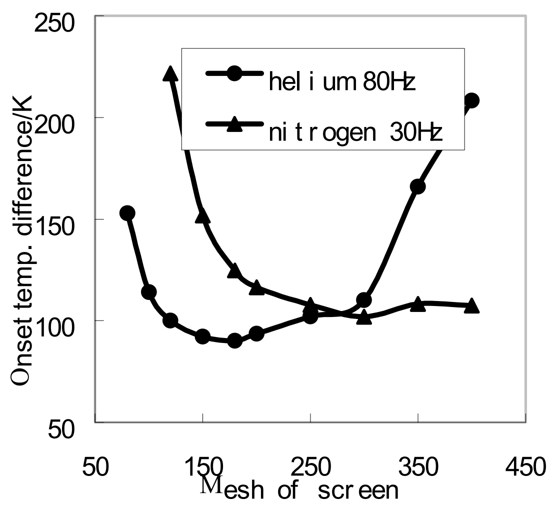

Figure 3 The distributions of onset temperature difference versus the mesh of the screen

Figure 3 is the distribution of onset temperature difference versus the mesh of the screen, and the triangle and circle lines correspond to different working gases, helium and nitrogen. The working pressure is 3MPa and the frequency for helium is 80Hz, nitrogen, 50Hz. In general, with the increase of the mesh, the heat exchange between the screen and the working gas is strengthened, which makes the onset temperature difference lower. Conversely, the viscous dissipation increases and makes the onset temperature difference higher. Thus one can see in the figure the onset temperature difference decreases first and then goes up.

Figure 4 shows the onset temperature difference distribution of helium and nitrogen versus different frequencies. Obviously, the onset temperature difference of nitrogen is higher than that of helium, they all increase monotonically and the nitrogen is more sensitive to the frequency.

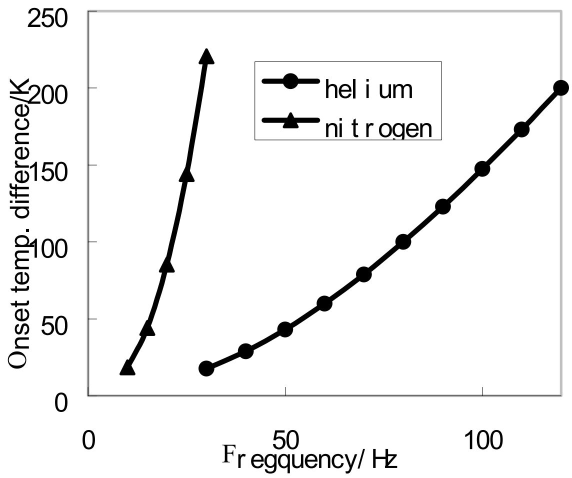

Figure 4 the onset temperature difference distributions versus working frequencies

CONCLUSIONS

From the above calculated results and analyses, some conclusions are drawn as follows:
(1) The onset temperature difference of a thermoacoustic Stirling prime mover deeply depends on the length and porosity of the regenerator, the working gas, the frequency and the pressure (notice: the size of feedback tube, the compliance and the thermal buffer tube is fixed in this study), this analytical model can provide some guidance for designing a low onset-temperature difference engine.
(2) The conclusions are based on some simplified assumptions, in which some factors that may influence the onset temperature difference are not considered in this model, a more accurate model will be developed in future.

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