Numerical simulation of loudspeaker-driven thermoacoustic refrigerator

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The work formulates a numerical simulation model for loudspeaker-driven thermoacoustic refrigerator (TAR) and uses the network method to calculate temperature difference generated across the heat-pump stack. It provides a theoretical basis for improving the TAR performance.

INTRODUCTION

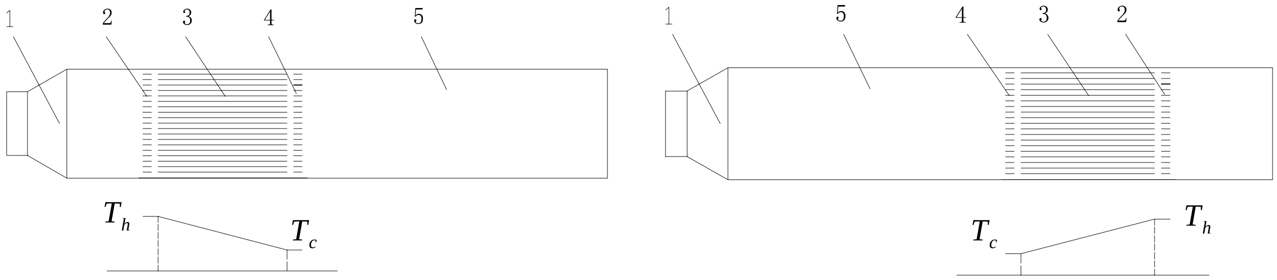
Numerical simulation of loudspeaker-driven thermoacoustic refrigerator (TAR) provides theoretical basis for evaluating and improving TAR performance. In recent years it has received significant attention. However little work has been carried out for accurately quantitative calculation on TAR. So it is very difficult to determine influence factors and propose improvement measures on TAR performance.

Piccolo [1] adopted Swift`s short-stack approximation theory [2] to obtain the expression of temperature difference, and used standard formulas [3,4] and Kirchoff-Rayleigh`s formula [5] for sound absorption in wide tubes to express pressure and velocity of the stationary wave, then to calculate temperature difference. The approximate method is not accurate and cannot be used to compare influence of different stack geometries on temperature difference.

The purpose of this paper is to use the network model to calculate the temperature difference generated across the heat-pump stack and determine the influence factors on temperature difference.

CALCULATION MODEL

A schematic diagram of loudspeaker-driven TAR is shown in Figure 1.



1 Loudspeaker 2 Heat Exchanger 3 Stack 4 Cold Exchanger 5 Resonance Tube

Figure 1 a schematic diagram of loudspeaker-driven

Averaged heat flux generated across heat-pump stack due to hydrodynamics transport is

$$Q_2 = \int T_m \rho_m \overline{s_1 u_1} dA \tag{1}$$

with $s_1 = \dfrac{c_p}{T_m} T_1 - \dfrac{\beta}{\rho_m} P_1$ (2)

and $T_1 = \dfrac{1}{\rho_m c_p}(1 - f_\kappa)P_1 - \dfrac{1}{i\omega}\dfrac{dT_m}{dx}\dfrac{(1-f_\kappa) - \sigma(1-f_\nu)}{(1-f_\nu)(1-\sigma)} u_1$ (3)

where $T_m, \rho_m, c_p, \beta, \sigma, s_1$ are mean temperature, mean density, isobaric specific heat, thermal expansion coefficient, Prandtl number and specific entropy; P_1, u_1, T_1 are first-order acoustic pressure, oscillating velocity and oscillating temperature; ω is angular frequency; i is the imaginary unit; And f_ν, f_κ are respectively dimensionless spatial-average functions of viscous distribution and heat distribution.

Substituting Eq.(2) and Eq.(3) into Eq.(1) and making use of the relation between acoustic pressure and velocity $z_1 = P_1 / u_1$, we can yield the expression of heat flux

$$Q_2 = \frac{A_f}{2}\frac{|P_1|^2}{|z|^2}\left\{ \mathrm{Re}\left[(1-f_\kappa)z\right] - \frac{\rho_m c_p}{\omega}\frac{dT_m}{dx}\mathrm{Im}\left[\frac{(1-f_\kappa)-\sigma(1-f_\nu)}{(1-f_\nu)(1-\sigma)}\right] - \mathrm{Re}(z)\right\} \tag{4}$$

where A_f is fluid channel area of stack; p_1, z_1 are calculated at the mean position of the heat-pump stack in TAR; and dT_m / dx is temperature gradient expected across the stack. The temperature gradient dT_m / dx can be written as

$$dT_m / dx = (T_R - T_L)/\Delta x \tag{5}$$

where T_R, T_L are the temperatures on the right end and left end of the stack. Δx is the stack length.

Heat conduction flux due to temperature gradient can be written as

$$Q_2 = (A_s K_s + A_f K_s)\frac{dT_m}{dx} \tag{6}$$

where A_s is the cross-sectional area of stack material; K_s, K_f are thermal conductivity of stack material and gas.

According to Eq.(4) and Eq.(6), temperature difference generated across the heat-pump stack can be obtained once oscillating pressure, specific acoustic impedance are known.

NUMERICAL SIMULATION METHOD

Network model of TAR is adopted to calculate the temperature difference generated across the stack.

The network transfer equation of TAR can be written as

$$\begin{bmatrix} p_x \\ J_x \end{bmatrix} = R \begin{bmatrix} p_{end} \\ J_{end} \end{bmatrix} \tag{7}$$

with $R = \prod_{i=1}^{i=3} R_i$ (8)

where $R_1 \sim R_3$ are the network transfer matrix of resonance tube, exchangers(hot heat exchanger or cold heat exchanger) and stack respectively. The expressions of $R_1 \sim R_3$ are given in literature [6].

For TAR with rigid end, the boundary condition is $Z_{end} = \infty$ and $P_{end} = DR \cdot P_m$.

where Z_{end} is acoustic impedance on the rigid end of the tube, $Z_{end} = A_f z_{end}$; DR is oscillating pressure ratio.

So the network model of TAR can be adopted to accurately calculate the acoustic pressure, volumetric flow rate and acoustic impedance in arbitrary position of TAR, then to evaluate the temperature difference generated across heat-pump stack.

NUMERICAL SIMULATION RESULTS

Influence of stack position on temperature difference

The system length is 437.5mm. The oscillating frequency is 400Hz. The parallel plate is used to make simulation to compare influence of stack position on temperature difference. The spacing between two plates is 0.8mm and the stack length is 50mm.The calculation results are shown in Figure 2.

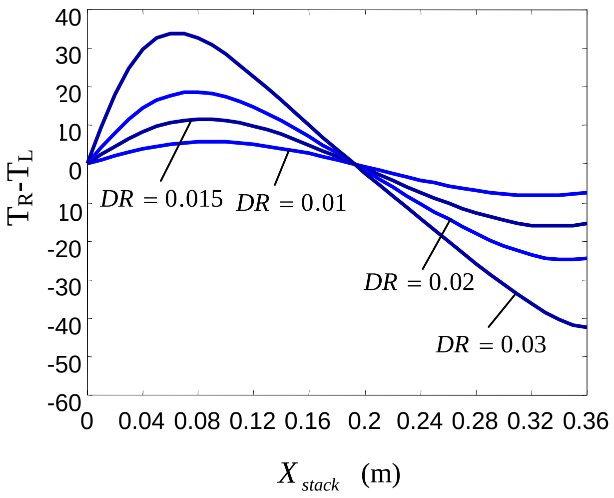


Figure 2 Influence of X_{stack} on temperature difference

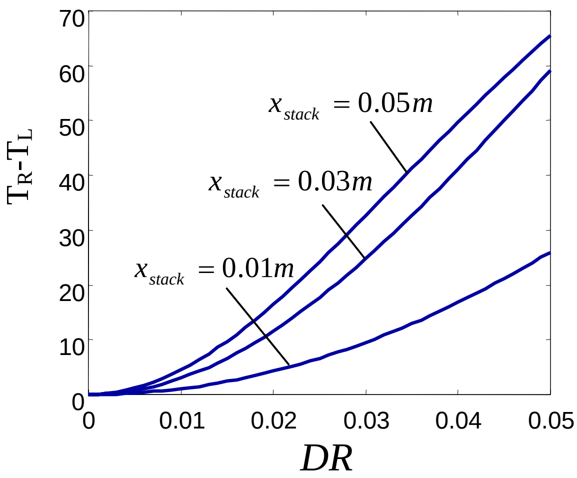


Figure 3 Influence of DR on temperature difference

In Figure 2 X_{stack} is the distance from the rigid end of TAR to the right end of stack. Figure 2 shows that the stack position affects the temperature difference greatly. The hot end of heat-pump stack is on its left end and right end respectively when it is closer to the loudspeaker and the rigid end. And no temperature difference is generated when it is closer to the pressure node.

The temperature difference is the highest when the distance is 50mm and 350mm. The theoretically calculated results are in good agreement with the optimization design of loudspeaker-driven TAR in literature [7].

Influence of DR on temperature difference

Oscillating pressure ratio (DR) is also an important factor to affect the temperature difference. The theoretically calculated results are shown in Figure 3. It can be seen from Figure 3 that the temperature difference is greatly affected by DR and rises with the increase of DR. It also shows that increasing DR is the most effective measure to improve the TAR performance.

Influence of heat-pump stack geometry on temperature difference

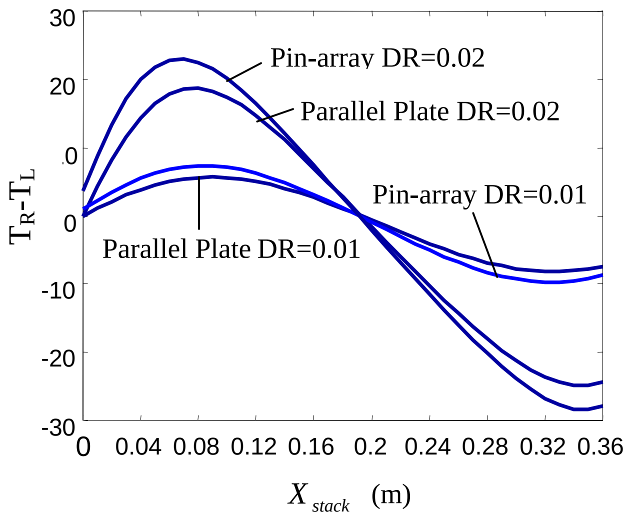


Figure 4 Influence of stack geometry on temperature difference

Parallel plate and pin-array stack are used to make numerical simulation to compare the influence of heat-pump stack geometries on temperature difference.

The steel pins of pin-array stack are made to be rectangular in order to manufacture the special stack feasible. The spacing between pin edges and equivalent radius of pins are optimized to be and 0.6mm and 0.2 mm [8]. The theoretically calculated temperature differences across the heat-pump pin-array stack and heat pump plate stack at DR=0.01,0.02 are shown in Figure 4.

Figure 4 shows that the stack makes some influence on temperature difference and the performance of pin-array stack is superior to that of parallel plate.

RESULTS

The network model can be feasibly used to calculate the temperature differences generated across the heat-pump stack. The theoretically calculated results show that the temperature differences are significantly affected by heat-pump stack position in TAR, different oscillating pressure ratios and different stack geometries. The former two are main factors to affect the temperature difference and the performance of the pin-array stack is superior to that of the parallel plate. The calculated results provide theoretical guidance for designing loudspeaker-driven TAR.

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