Extended power law of nonlinear transport properties of superconducting materials

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The nonlinear transport properties of superconductors near the transition are usually described by the so called power law $E/E_c = (J/J_c)^n$. We report a wide-range resistive transition equation with the form of an "extended power law". This equation fits the experimental data of MgB_2 and high T_c cuprates. The applicability of this equation for the transport properties of normal-metal-sheathed superconducting material is also discussed.

INTRODUCTION

For designing superconducting magnets, fault current limiters, cables and many other devices thorough knowledge of the electromagnetic response near the critical state is necessary. In principle this should be determined by the Maxwell equations combined with a proper materials equation $J(E, T, B)$. At present, power law $E(J)$ characteristics of the form

$$E = E_c[J/J_{c0}(T, B)]^n \tag{1}$$

are often used (see e.g.[1], [2], [3], [4] and references there-in). In this study we show that the $E(J)$ isothermal characteristics for wider range including the crossover to Ohmic-like regimes have the general form of extended power law which well fits experimental data.

EXTENDED POWER LAW

This form can be derived from the Ginzburg-Landau (GL) free energy density[5]

$$f = f_{n0} + \alpha \left|\Psi\right|^2 + \frac{\beta}{2} \left|\Psi\right|^4 + \frac{1}{2m^*} \left| \left(\frac{\hbar}{i}\nabla - \frac{e^*}{c}\mathbf{A} \right) \Psi \right|^2 + \frac{h^2}{8\pi} \tag{2}$$

where f_{n0} is the free energy density of normal state, $\Psi\left(r\right)$ is the complex order parameter, $\mathbf{A}$ is the vector potential. Gorkov found that the GL theory based on Eq.(2) is derivable as a rigorous limiting case of the BCS microscopic theory with $\Psi\left(r\right)$ proportional to the local value of the gap parameter $\Delta\left(r\right)$ and the effective charge e^* in Eq.(2) equal to $2e$ [6].

Working in the London limit, Nelson and co-workers showed that according to the GL free energy Eq.(2) a system of N flux lines with a field H along the z direction in a sample length L can be described with the free energy represented by the trajectories $\{\overrightarrow{r_j}\left(z\right)\}$ of these flux lines[7]. Considering further the pinning potential $V_P\left(\overrightarrow{r}\right)$ arising from inhomogeneities and defects in sample[8, 9], the free energy of such a sample with N flux lines can be expressed as

$$\begin{aligned} F &= \frac{1}{2}\varepsilon_l \sum_{j=1}^{N} \int_0^L \left| \frac{d\,\vec{r_j}\left(z\right)}{dz} \right|^2 dz + \frac{1}{2}\sum_{i\neq j} \int_0^L V\left(r_{ij}\right) dz \\ &+ \sum_{j=1}^{N} \int_0^L V_P\left[\vec{r_j}\left(z\right)\right] dz \end{aligned} \tag{3}$$

Here $V(r_{ij}) = V(|\overrightarrow{r_i} - \overrightarrow{r_j}|) = 2\varepsilon_0 K_0(r_{ij}/\lambda_{ab})$ is the interaction potential between lines with in-plane London penetration depth λ_{ab} and $K_0(x)$ is the modified Bessel function $K_0(x) \approx (\pi/2x)^{1/2} e^{-x}$. ε_l is the linear tension of flux line and $\varepsilon_0 \approx (\Phi_0/4\pi\lambda_{ab})^2$ is the energy scale for the interaction.

Thermally activated flux motion can be considered as the sequence of thermally activated jumps of the vortex segments or vortex bundles between the metastable states generated by disorder. Every elementary jump is viewed as the nuclearation of a vortex loop, and the mean velocity of the vortex system is determined by the nuclearation rate[8, 9]

$$v \propto \exp(-\delta F/kT) \tag{4}$$

here δF is the free energy for the formation of the critical size loop or nucleus which can be found by means of the standard variational procedure from the free energy functional due to the in-plane displacement $\overrightarrow{u}(z)$ of the moving vortex during loop formation

$$F_{loop}[\overrightarrow{u}] = \int dz \left[\frac{1}{2}\varepsilon_l \left| \frac{d\overrightarrow{u}(z)}{dz} \right|^2 + V_P(\overrightarrow{u}(z)) - \overrightarrow{f}_s \cdot \overrightarrow{u} \right] \tag{5}$$

with

$$\overrightarrow{f}_s = \overrightarrow{f}_L + \overrightarrow{f}_\eta = \frac{J_P \Phi_0}{c} \times \overrightarrow{e_z} \qquad\qquad J_P = J - \frac{E}{\rho_f} \tag{6}$$

where $f_L = \overrightarrow{J} \times \overrightarrow{e_z}/c$ is the Lorentz force due to applied current J and f_η is the viscous drag force on vortex, $f_\eta = -\eta v_{vortex}$, with $v_{vortex} = d\overrightarrow{u}/dt$ and viscous drag coefficient $\eta \approx (\Phi_0 B_{c2})/(\rho_n c^2)$ as estimated by Bardeen and Stephen[10].

By a derivation similar to that used Ref.9, one finds the barrier energy

$$\delta F = U(J_p) \approx U_c(\frac{J_c}{J_p})^\mu \tag{7}$$

which implies a current-voltage characteristic of the form

$$E(J) = \rho_f J exp[-\frac{U_c}{kT}(\frac{J_c}{J_p})^\mu] \tag{8}$$

where U_c is a temperature- and field-dependent characteristic pinning energy related to the stiffness coefficient and J_c is a characteristic current density related to U_c ,μ is an numerical exponent.

Considering the real size effect Eq.(8) leads to a general normalized form of the current-voltage characteristic[11] in the form

$$y = x \exp[-\gamma(1 + y - x)^p] \tag{9}$$

where

$$\gamma = \frac{U_c}{kT}\left(\frac{J_c}{J_L}\right), \qquad x = \frac{J}{J_L}, \qquad y = \frac{E(J)}{\rho_f J_L}, \qquad p = \mu$$

with J_L the transport current density corresponding to the case where the critical size of loop formation is equal to the real sample size L. The ρ_f in Eq.(8) is the flux flow resistance of a pinning free mixed state as derived by Bardeen and Stephen.Its relation with the normal resistance is of the form

$$\frac{\rho_f}{\rho_n} = \frac{2\pi a^2 B}{\Phi_0} = \left(\frac{a}{\xi}\right)^2 \frac{B}{B_{c2}} \approx \frac{B}{B_{c2}} \tag{10}$$

In an earlier work[12], this response equation has also been shown in connection with the Anderson-Kim model.Compared with the widely used power law nonlinear response $E \propto J^n$ (see Refs.[1],[2],[3],[4]), equation(9) can also be asymptotically expressed in an extended power law form

$$\frac{y}{x} = a_0 + f(x - y)^{n-1} \tag{11}$$

with n the maximal slope at the inflection point of $lny \sim lnx$ curve so

$$\frac{\partial^2 lny}{\partial^2 lnx}|_{x=x_i} = 0 \quad \text{and} \quad n = \frac{\partial lny}{\partial lnx}|_{x=x_i} \tag{12}$$

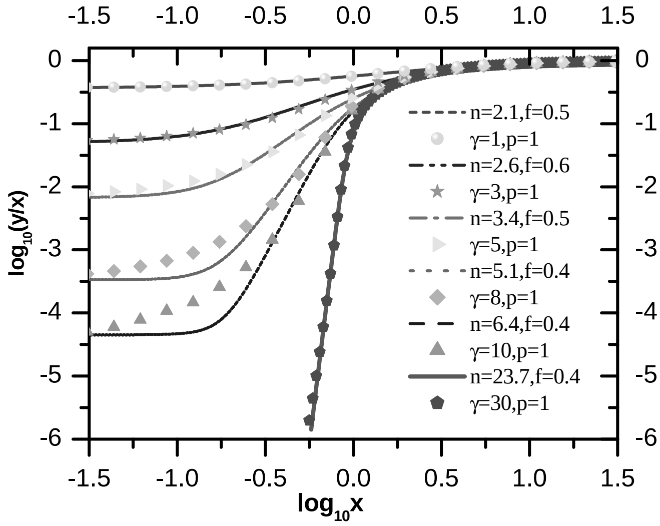


Figure 1: The numerical solutions of Eq.(9)(solid symbols) and Eq.(11)(lines).

and $a_0 = e^{-\gamma}$, f is a numerical factor depending on the parameters γ and p in Eq.(9).

In Fig.1 we show the numerical solutions of Eq.(9) and Eq.(11) for comparison.

COMPARISON WITH EXPERIMENTS

Accounting further the resistive transition between pinning free flux-flow superconducting phase and the normal phase one finds a wide range resistive transition equation of the form

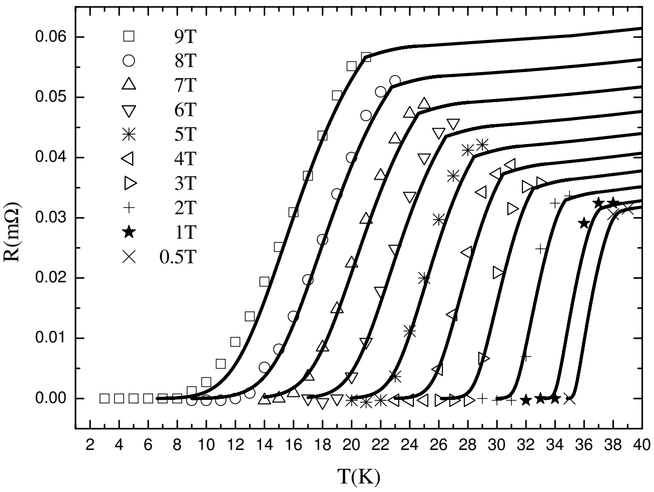
$$\frac{R}{R_n} = exp[-\sum_{i=1}^{2} \gamma_i (1 + y_i - x_i)^{p_i} \theta(T_i - T)] \qquad (13)$$

with x_i, y_i, γ_i, and T_i the normalized current, voltage, symmetry-breaking factor, and critical temperature, respectively, defined as

$$\begin{aligned} & x_1 \equiv \frac{I}{I_d(T,B)},\ x_2 \equiv \frac{I}{I_{c0}(T,B)}, \\ & y_1 \equiv \frac{R}{R_n}[\frac{I}{I_d(T,B)}],\ y_2 \equiv \frac{R}{R_f}[\frac{I}{I_{c0}(T,B)}], \\ & \gamma_1 \equiv \ln\frac{R_n(T,B)}{R_f(T,B)},\ \gamma_2 \equiv \frac{U_c(T,B)}{kT}, \\ & T_1 \equiv T_c(B),\ T_2 \equiv T_m(B), \end{aligned} \qquad (14)$$

where I_d is the depairing current and I_{c0} is the critical current of vortex solid for overcoming the activation energy barrier $U_c(T,B)$. $\theta(x)$ is the Heaviside function and p_i are exponents[13]. R_f is the unpinned flux-flow resistance of the mixed state in type-II superconductors. In Fig.2 we show the comparison of the resistive transition Equation(13) with the experimental data of the temperature dependent resistance of MgB_2 in different applied fields measured by Finnemore et al.[14]. The current density used in their standard four-probe technique is sufficiently low with the value of $0.1 - 0.3A/cm^2$.

It is widely believed that the mechanism of superconductivity in high-T_c cuprates may be essentially different from the familiar s wave type pairing on which conventional BCS theory is based[15]. Kwok et al. studied the width and shape of the resistive transition fo untwined and twinned single crystals of YBCO in fields up to 8T[16]. We compare our resistive Equation(12) with their experimental data of untwinned YBCO crystal in Fig.3.



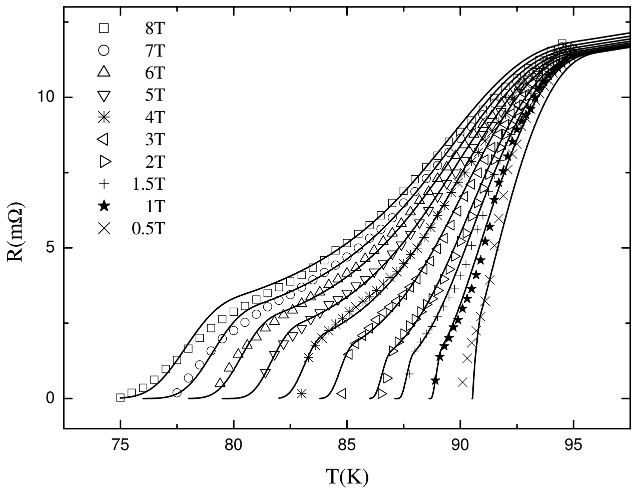


Figure 2: Comparison of Eq.(13) with the experimental resistance data of MgB_2 samples measured by Finnemore et al.(Ref.14). $\Box, \circ, \triangle, \bigtriangledown, *, \triangleleft, \triangleright, +, \star, \times$ denote experimental data of resistive transitions in different applied fields and lines denote the theoretical curves in Eq.(13) with corresponding applied fields. The parameters in Eq.(13) are $(1) U_c(T,B) \propto [T_m(B)-T]^{0.8}(B+5.46)^{-4.3}; (2) I_{c0} \propto T_m(B)-T, I_d \propto [T_c(B)-T]^{1.5}\gamma_1^{0.5}; (3) T_m(B) = T_c(0)[1-(B/21.7)^{0.84}], T_c(B) = T_c(0)(1-B/22.4), T_c(0) = 40.2K; (4) \gamma_1 \equiv \ln[R_n(T,B)/R_f(T,B,J\to 0)] = \ln[B_{c2}(T)/B]$, where $B_{c2}(T) = 0.6[T_c(0)-T]^{0.98}$.

Figure 3: Comparison of Eq.(13) with the experimental resistance data of an untwinned YBCO crystal measured in different applied fields for $H_{\|c}$ by Kwok et al.(Ref.16). $\Box, \circ, \triangle, \bigtriangledown, *, \triangleleft, \triangleright, +, \star, \times$ denote experimental data and lines denote the theoretical curves in Eq.(13) with corresponding applied fields. The parameters in Eq.(13) are $(1) U_c(T,B) \propto [T_m(B)-T]^3 B^{-4.22}; (2) I_{c0} \propto T_m(B)-T, I_d \propto T_c(B)[1-T/T_c(B)]^{2.3}\gamma_1^{0.5}; (3) T_m(B) = T_c(0)[1-(B/1200)^{0.36}], T_c(B) = T_c(0)[1-(B/3.6\times 10^6)^{0.3}], T_c(0) = 97K; (4) \gamma_1 \equiv \ln[R_n(T,B)/R_f(T,B,J\to 0)] = \ln[B_{c2}(T)/B]^m$, where $m = 0.84B^{-0.59} - 0.07$ and $B_{c2}(T) = 0.6[T_c(0)-T]^{3.3}$.

METAL-SHEATHED CONDUCTORS

The material equation Eq.(9) for a homogeneous type-II superconductor can be generalized to describe the characteristics of metal-sheathed HTS conductor. For a uniform long conductor, the voltage at a total current I according to Eq.(9) has the form

$$V(I) = R_{eff} I e^{-U(J_P)/kT} \tag{15}$$

with R_{eff} the effective resistance of the conductor

$$R_{eff} = \frac{L}{a}[\frac{1}{r+1}\rho_f^{-1} + \frac{r}{r+1}\rho_m^{-1}]^{-1} \tag{16}$$

where L, a, ρ_m and r are the length, area of section, resistivity of metal sheath and the ratio: Metal/HTSC of this conductor respectively. Taking the $U(J)$ of the form of Anderson-Kim model one may have the form

$$U(J_P) = \frac{\hbar I_0}{e}(1 + \frac{V}{I_0 R_{eff}} - \frac{I}{I_0}) \tag{17}$$

Measurements of the $I \sim V$ characteristics of individuals bare filaments extracted from very high J_c multifilament Ag-sheathed $(Bi,Pb)_2Sr_2Ca_2Cu_3O_x$ superconducting tapes show that all filaments possessed local characteristics of c-axis transport across intrinsic Josephson junctions although current was always injected to flow along the ab planes[17]. Based upon this important finding and considering the current shunting into the metal sheath we attempt to describe the $I-V$ characteristics of real wires or tapes with a model of series connection and express the voltage on unit length of conductor with transport current I as

$$V(I) = V_{ab}(I) + V_c(I) \tag{18}$$

where $V_{ab}(I)$ is the contribution from the main part of conductor with the characteristic of the ab plane transport

$$V_{ab}(I) = \frac{I(1-l_c)}{a}[\frac{1}{r+1}\rho_f^{-1} + \frac{r}{r+1}\rho_m^{-1}]^{-1} e^{-U(J_P)/kT} \tag{19}$$

and $V_c(I)$ is the contribution from the part with c-axis transport which has the total path length $l_c \ll 1$ in unit length of conductor. In an earlier work it is shown that the $I-V$ characteristics of a resistive shunted Josephson junction can be described with an equation like Eq.(9)[18]. Thus we have

$$V_c(I) = IR_c exp[-\frac{\hbar I_J}{ekT}(1+\frac{V_c(I)}{I_J R_c} - \frac{I}{I_J})] \tag{20}$$

where

$$R_c = \frac{l_c}{a}(\frac{1}{r+1}\rho_c^{-1} + \frac{r}{r+1}\rho_m^{-1})^{-1} \tag{21}$$

is the shunting resistance of the c-axis transport part with total path length l_c and resistivity ρ_c. I_J is the maximum Josephson current in the absence of noise.

Usually, the resistivity of metal $\rho_m \ll \rho_f$, ρ_c and the maximal Josephson current I_J is much smaller than the critical current I_0 of the conductor. Thus, when $I > I_J$, one finds from Eq.(18)-(21) approximate reduced relation for sheathed conductor similar to

$$y' = x' exp[-\gamma'(1+y'-x')^p] \tag{22}$$

where

$$x' \equiv I/I_0' \ , \ y' \equiv V(I)/(R_{eff} I_0') \ , \ \gamma' \equiv \hbar I_0'/ekT \tag{23}$$

with

$$R_{eff} \approx \frac{L}{a}(\frac{r+1}{r})\rho_m \tag{24}$$

and

$$I_0' = I_0[1 - \frac{ekT}{\hbar I_0}\ln(1 + l_c exp[\frac{\hbar I_0}{ekT}])] \tag{25}$$

The analytic relation(22) is useful for studying the operation of superconducting devices electric engineering such as fault current limiter, magnet etc..

SUMMARY

We show an extended power law form of current-voltage characteristics which well fits the wide range experimental data of high-Tc MgB_2 and YBCO materials. This equation combined with the Maxwell equation may provide a useful tool for applied superconductivity design.

ACKNOWLEDGEMENT

This work is supported by the Ministry of Science & Technology of China (NKBRSG-G 1999064602) and the National Natural Science Foundation of China under Grant No. 10174003 and No. 50377040.

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