Fractal description of thermal contact resistance between rough surfaces at low temperature

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In order to model the heat transfer of cryogenic contact refrigeration system, the fractal recursive thermal contact resistance model is established. In this model real contact surfaces are described based on developed Cantor set fractal theory and the volume conservation of plastically deformed asperities is considered. It is concluded from comparison with the experimental results at low temperature that this model can predict thermal contact resistance well.

INTRODUCTION

Complex scientific instruments such as the space infrared detective facility or miniature thermal contact switch apparatus in satellite are often cooled through bolted or pressed links to their refrigeration systems. When two rough nominally flat surfaces are brought together under load, the discrete real contact points impede the heat flow through contact surfaces and result in the temperature drop and thermal contact resistance (TCR) at the interface. In the past many TCR models based on statistics theory had been proposed to predict this resistance. However, the principal statistical roughness parameters such as roughness height, slope, and curvature in those models are always dependent of length scale and resolution of the instrument. And at the same time, roughness measurements on a variety of surfaces have demonstrated that their structure follows fractal geometry where similar rough images of surfaces appear under repeated magnification [1]. This implies that Fractal geometry may be an effective method to study TCR phenomenon. In this paper the Cantor set fractal theory is used to describe the surface morphology of the interface and the fractal TCR network model is obtained based on the elastic-plastic theory. This model considers the volume conservation of plastically deformed materials and the constriction resistance of asperities. The calculating results agree well with the experimental ones at low temperature.

CANTOR SET FRACTAL THERMAL CONTACT RESISTANCE MODEL

Topographic description of contact surfaces

The self-affined Cantor set fractal method of Warren and Krajcinovic [2,3] is further developed to represent isotropic contact surface. At each step of Cantor set construction of the surface (see Figure 1), the middle section of the initial segments are removed so that the remaining horizontal length of segments at the (i)th generation is 1/fr of the length constructed at (i-1)th generation (fr>1). Similarly, the recess depth at the (i)th generation of the Cantor set surface is 1/fz of the depth at (i-1)th generation (fz>1). It is also shown from Figure 1 that the horizontal length L_i in x or y direction and recess depth h_i in z direction of the (i)th generation are $L_0(1/fr)^i$ and $h_0(1/fz)^i$ respectively.

At (i)th generation, the Cantor set profile contains the $N=s^i$ asperities and the Cantor set surface contains $N=s^{2i}$ segments where s is the number of asperities on a repeating segments. It is noted that Figure 1 is just an example of s=3. And the length of each asperity at the (i)th generation is $\delta_i=L_0(1/sfr)^i$. The height is $z_i=h_0(fz-1)(1/fz)^i$. Further, the gap between adjacent asperities at (i)th generation which is generated on the single asperity at (i-1)th generation is $g_i=L_0(fr-1)/[(s-1)fr^i s^{i-1}]$. Here L_0 corresponds to the profile length, and h_0 is equal to twice of the R.M.S. roughness height.

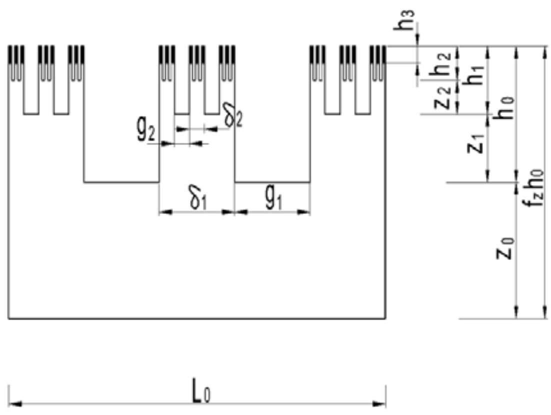


Figure 1 Fractal surface profile of Cantor set (s=3)

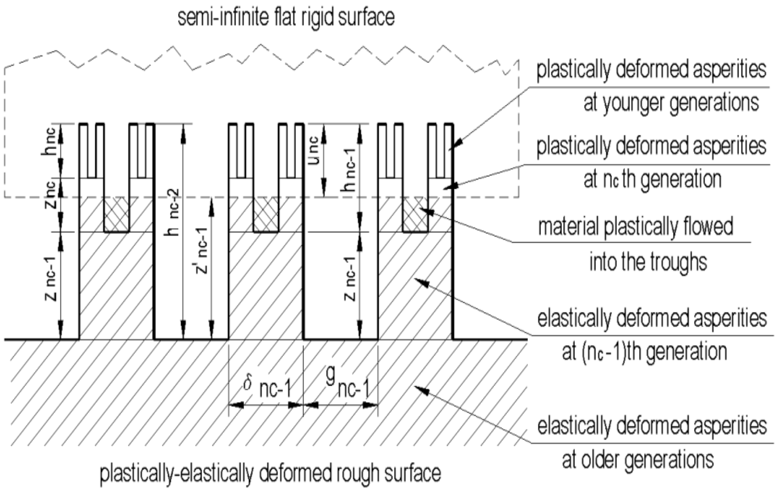


Figure 2 Elastic-plastic contact model of asperities

Deformation of the cantor set surfaces

Here the real contact surfaces are equivalent to that an elastic-plastic solid surface with Cantor set fractal structure is in contact with an idealized smooth rigid semi-infinite surface. Since the Cantor set fractal surface is composed of the small asperities stacking upon the large asperities self-similarly, each asperity behaves as an elastic-plastic axial loaded column.

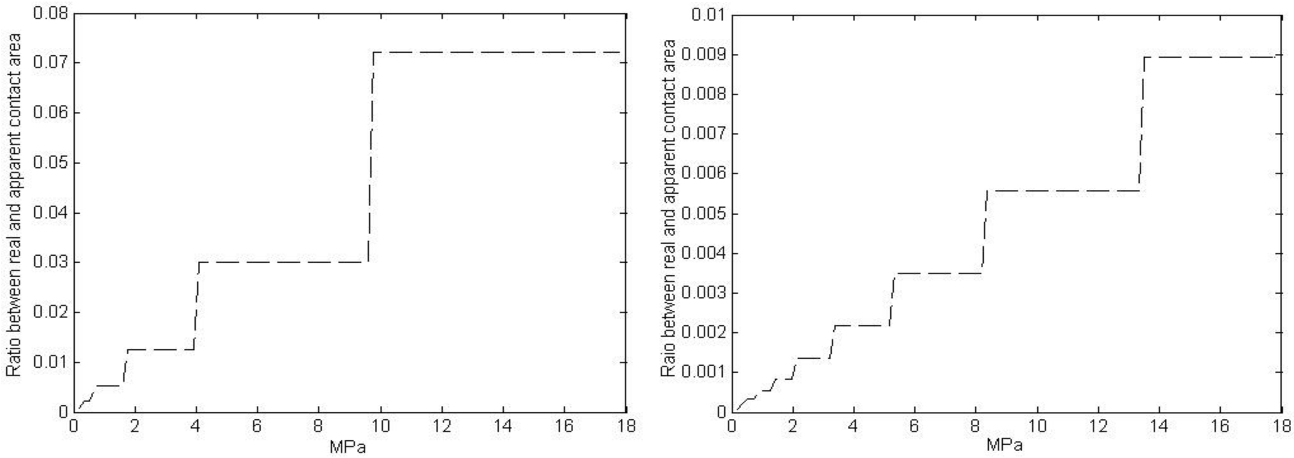
When the normal load P is given, the smooth rigid surface will first be in contact with the youngest generation ($i\rightarrow\infty$). If $P>P_{c,i}$ (critical load of plastic deformation), the ith generation asperities will flow plastically into troughs surrounding the asperities and then the smooth surface will be in contact with (i-1)th generation asperities. Such process will not stop until the (n_c-1)th generation of asperities whose critical load is large than the exerted load. Thus the oldest plastically deformed asperities is the (n_c)th generation (see Figure 2). And the corresponding contact area is

$$A_c=\left(\frac{1}{f_r^2}\right)^{n_c}L_0^2=\frac{P}{\sigma_y} \tag{1}$$

$$n_c=\begin{cases}\zeta,\zeta\in Z^+\\ 1+trun(\zeta),\zeta\notin Z^+\end{cases}\quad;\quad \zeta=\frac{\ln\xi}{\ln(1/f_r^2)},\xi=\frac{P}{L_0^2\sigma_y},(0\le\xi\le1) \tag{2}$$

where Z^+ represents the positive integers, σ_y is the yield stress, *trun*(ζ) truncates ζ to an integer value. Therefore, for the given normal load, the deformation of asperities at the whole generations can be classified into two parts: (a) asperities at $[n_c,\infty]$ generations deform plastically and (b) asperities at $[0, n_c-1]$ generations deform elastically. The contact interface is deforming from plastically to elastically. This is absolutely different from the traditional theory of Greenwood and Williamson [4] in which deformation of the identical asperities transits from elasticity to plasticity.

Figure 3 shows the change of the ratio between real and apparent contact area under increasing load. From it, you can see that how small the real contact area is compared with apparent ones. Since Equation (2) includes the truncation function, the calculating results of real contact area are discrete and increase with load like stairs. In addition, because aluminum is softer than stainless steel, the ratio of aluminum is much larger than stainless steel.



(a) Aluminum s=12,D=2.56,fr=1.55,fz=1.506 (b) Stainless Steel s=14, D=2.767, fr=1.266, fz=1.219

Figure 3 Ratio between numerical results of real and apparent contact area of aluminum and stainless steel

Thermal contact resistance model

It is assumed that the plastically deformed asperities at [n_c,∞] generations will fill in adjacent troughs and increase the height of the (n_c-1)th generation by h_{nc-1}-u_{nc} (see Figure 2). Hence, the increased volumes of asperities at (n_c-1)th generation is equal to the plastically flowed volume of ones at [n_c,∞] generations:

$$\delta_{nc-1}^{2}\cdot s^{2(nc-1)}\cdot(h_{nc-1}-u_{nc})=\sum_{i=nc}^{\infty}V_i \tag{3}$$

$$l_i=\begin{cases}h_0 f_z^{(1-nc)}(f_z-\alpha\beta)(1-\xi\varepsilon f_r^{2(nc-1)}), i=n_c-1\\ h_0 f_z^{-i}(f_z-1)(1-\xi\varepsilon f_r^{2i}), i=0,1,2,...,n_c-2\end{cases};\alpha=1-\frac{1}{f_r^2};\beta=\sum_{i=0}^{\infty}\left(\frac{1}{f_r^2 f_z}\right)^i;\varepsilon=\frac{\sigma_y}{E} \tag{4}$$

Considering the volume conservation of plastic deformation and elastic deformation of asperities, the actual height of asperities at [0,n_c-1] generation under the given load is derived to equation (4).

According to Fourier law and CMY model [5], heat conduction resistance $R_{a,i}$ and constriction resistance $R_{c,i}$ of single asperity at (i)th generation in [0,n_c-1] can be obtained. Because of recursive construction of Cantor set fractal contact surfaces, the total thermal contact resistance $R_{t,0}$ is the recursive serial and parallel resistance of the constriction resistance and heat conduction resistance.

$$R_{a,i}=\frac{l_i}{k\cdot\delta_i^2};\quad R_{c,i}=\frac{\sqrt{\pi}\left(\delta_i+g_i\right)^2}{4k\delta_i}\cdot f\left(\frac{\delta_i}{\delta_i+g_i}\right);\quad R_{t,0}=\frac{\sum_{i=0}^{nc-1}s^{2i}(R_{a,(nc-1-i)}+R_{c,(nc-1-i)})}{s^{2\cdot nc}} \tag{5}$$

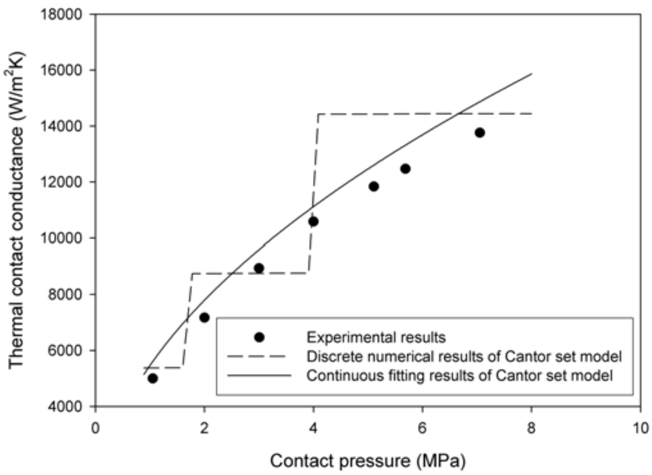
COMPARISON AND DISCUSSION

In order to check the Cantor set fractal TCR model, the experiment was conducted at the interface of Al5052 and stainless steel 304 respectively. The temperature of experiments was 155K and the vacuity was less than 1.5 Pa. The normal contact pressure is from 1.0307 to 6.9675 MPa. The surface morphology of the specimens was measured by STRA-1 stylus profilometer. The physical properties of materials were measured in the experiment, which is listed in Table1.

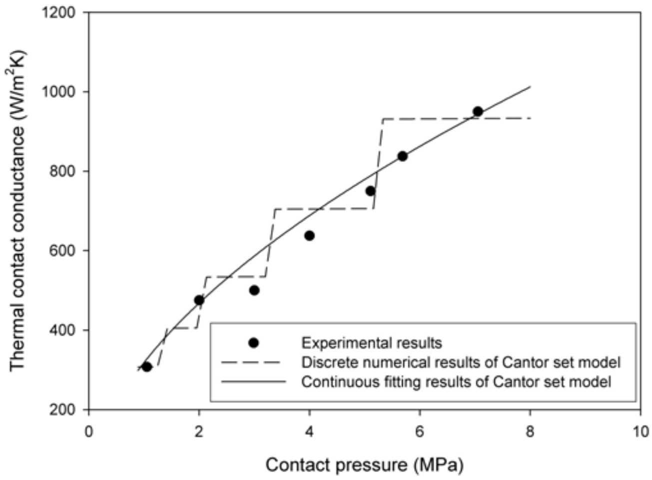
The experimental results are illustrated in Figure 4 in the form of Thermal contact conductance (reciprocal of TCR). Because of the discrete real contact area, the calculating results are discrete, too. The fitting curves of staircase numerical results are used to compare and predict the real conductance. The equations of fitting curves are $y=5470x^{0.5121}$ for aluminum and $y=318.5x^{0.5562}$ for stainless steel respectively. From Figure 4, it is shown that the fitting curve of Cantor set fractal TCR model can predict the experiment results well.

Table 1 Experimental parameters of Aluminum and Stainless steel samples

Material	Thermal conductivity (W/m^2K)	Elastic modulus (GPa)	Yielding strength (MPa)	R.M.S height (μm)	h_0 (μm)	Fractal dimension D
Al-5052	107	73.3	450	3.2692	6.5384	1.71
SS-304	10.7	201	2400	2.1328	4.2656	1.85



(a) Aluminum s=12, D=2.56, fr=1.55, fz=1.506



(b) Stainless Steel s=14, D=2.767, fr=1.266, fz=1.219

Figure 4 Comparison between experiment and numerical results

CONCLUSION

In this paper Cantor set fractal theory is used to describe rough surface topography. Considering the volume conservation of plastically deformed asperities, the fractal recursive TCR model is established. It is concluded from the model that the asperities of the contact interface are deforming from plastically to elastically under normal load. The simulated results are in good agreement with the experimental results at low temperature. This model provides a new way to study the phenomenon of TCR.

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