A thermodynamic optimization of counterflow recuperative-type heat exchangers

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The counterflow recuperative-type heat exchangers have been used widely in cryogenic systems. Thermodynamic optimization of a steady heat transfer process with constant specific-heat fluids in a counterflow recuperative-type heat exchanger is conducted in this paper. The analytical results show that the irreversibility of a heat transfer process is minimized when the log-mean heat transfer temperature difference equals to the difference of mean thermodynamic temperatures of two fluids. These optimum relations can be conveniently used for optimization design of heat exchangers in cryogenic systems, such as natural gas separation systems, gas liquefaction systems, gas refrigerators, and so on.

INTRODUCTION

The counterflow recuperative-type heat exchangers have been used widely in cryogenic systems, such as natural gas separation systems, gas liquefaction systems, gas refrigerators, and so on. The efficiency of these heat exchangers often directly affects the performance of the cryogenic systems. The irreversibility of these heat exchangers can be minimized by thermodynamic optimization of them. The thermodynamic optimization of a heat exchanger could be defined as the minimization of irreversibility of heat transfer processes through modifying the thermodynamic properties of fluids when specifying heat exchanger structure and total heat transfer rate. From the results of researches [1-3], we can see that the COP of refrigeration systems could be improved at specified size of heat exchangers. But the condition of perfect configuration of heat transfer has been generally accepted as that the heat transfer in the heat exchanger should be carried out with a constant heat transfer driving force, namely, the local temperature difference between heat transfer fluids along the heat transfer surface should be constant [1-4].

In fact, the constant local temperature difference between heat transfer fluids along the heat transfer surface does not definitely lead to minimizing total entropy generation rate in the heat transfer process. The total entropy generation rate and the heat transfer temperature difference in a heat transfer process are independent of each other. Therefore, it leaves room for thermodynamic optimization.

The aim of this paper is to conduct a thermodynamic optimization of a steady heat transfer process which often happens in counterflow recuperative-type heat exchanger with constant specific-heat fluids. The results of this research will be helpful for further research on the heat transfer process with variable specific-heat fluids.

HEAT EXCHANGER MODEL

Consider a counterflow recuperative heat exchanger shown schematically in Figure 1. Following assumptions are employed:
(a) Both of the fluids flow steadily through the heat exchanger at a constant pressure. Their heat capacity rates are kept constant along the heat transfer surface.
(b) The overall heat transfer coefficient is constant throughout the heat exchanger.
(c) Heat losses to the surroundings and longitudinal heat conduction are all neglected.
(d) All frictional pressure drops of both fluids are negligible.

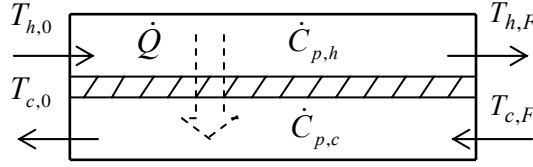(e) The changes in kinetic energy and potential energy of fluids within the heat exchanger are negligible.

Figure 1   Sketch of heat exchanger model

Thus the heat exchanger model could be written as:

Heat transfer equation

$$\dot{Q} = KF\Delta t_{lm} \tag{1}$$

where $\dot{Q}$ is the total heat transfer rate in the exchanger, K is the overall heat transfer coefficient, F is the total heat transfer area and Δt_{lm} is the log-mean temperature difference across the heat exchanger, defined as

$$\Delta t_{lm} = \frac{T_{h,0} - T_{c,0} - T_{h,F} + T_{c,F}}{\ln \dfrac{T_{h,0} - T_{c,0}}{T_{h,F} - T_{c,F}}} \tag{2}$$

where $T_{h,0}$ and $T_{h,F}$ are the absolute temperatures of hot fluid at the hot and cold ends of the exchanger, $T_{c,0}$ and $T_{c,F}$ are the absolute temperatures of cold fluid at the hot and cold ends of the exchanger.

Heat balance equation

$$\dot{Q} = \dot{C}_{p,h}(T_{h,0} - T_{h,F}) = \dot{C}_{p,c}(T_{c,0} - T_{c,F}) \tag{3}$$

where $\dot{C}_{p,h}$ and $\dot{C}_{p,c}$ are the heat capacity rates of hot and cold fluids at constant pressure.

Entropy balance equation

$$\dot{S}_g = \left(\frac{1}{T_{c,m}} - \frac{1}{T_{h,m}} \right) \dot{Q} \tag{4}$$

where $\dot{S}_g$ is the total entropy generation rate in the exchanger. $T_{h,m}$ and $T_{c,m}$ are mean thermodynamic temperatures of hot and cold fluids in the exchanger. Since $\dot{C}_{p,h}$ and $\dot{C}_{p,c}$ are uniform, we have

$$T_{h,m} = \frac{T_{h,0} - T_{h,F}}{\ln \dfrac{T_{h,0}}{T_{h,F}}} \quad \text{and} \quad T_{c,m} = \frac{T_{c,0} - T_{c,F}}{\ln \dfrac{T_{c,0}}{T_{c,F}}} \tag{5}$$

OPTIMIZATION

Based on the above heat exchanger model, following Lagrange function is constructed:

$$L = \dot{S}_g + \lambda\left(\dot{Q} - KF\Delta t_{lm}\right) = \left(\frac{\ln\alpha_c}{T_{c,0} - T_{c,F}} - \frac{\ln\alpha_h}{T_{h,0} - T_{h,F}}\right)\dot{Q} + \lambda\left(\dot{Q} - KF\frac{T_{h,0} - T_{c,0} - T_{h,F} + T_{c,F}}{\ln\beta}\right) \tag{6}$$

where λ is the Lagrange multiplier, and

$$\alpha_h = \frac{T_{h,0}}{T_{h,F}}, \quad \alpha_c = \frac{T_{c,0}}{T_{c,F}}, \quad \beta = \frac{T_{h,0} - T_{c,0}}{T_{h,F} - T_{c,F}} \tag{7}$$

When $\dot{Q}$, F, K are specified, using the necessary conditions for this multi-objective optimization problem, we can get the optimum solution as

$$\frac{T_{c,0}}{T_{c,F}} = \frac{T_{h,0}}{T_{h,F}} \tag{8}$$

DISCUSSION

Other important optimum relations in the heat transfer process could be derived from the above optimum solution of equation (8).

Mean thermodynamic temperature difference ΔT_m
Combining equation (8) with equations (5), we can obtain the optimum mean thermodynamic temperature difference between the hot and cold fluids in the heat exchanger:

$$\Delta T_m = T_{h,m} - T_{c,m} = \Delta t_{lm} \tag{9}$$

Equation (9) shows that $\dot{S}_g$ is minimized when ΔT_m equals to Δt_{lm}. In addition, from heat transfer equation (1), we can see that Δt_{lm} is always constant under the assumptions discussed. Therefore, ΔT_m is minimized while Δt_{lm} keeps constant in the optimization processes. This indicates that ΔT_m is independent of Δt_{lm}. It can be concluded that ΔT_m is an important factor which indicates the value of $\dot{S}_g$ during heat transfer prcesses.

Ratio of heat capacity rate γ
Combining equation (8) with heat balance equation (3), we can express the optimum ratio of heat capacity rate of cold fluid to hot fluid as

$$\gamma = \frac{\dot{C}_{p,c}}{\dot{C}_{p,h}} = \frac{T_{h,0} - T_{h,F}}{T_{c,0} - T_{c,F}} = \frac{T_{h,m}}{T_{c,m}} \tag{10}$$

Substituting equation (9) into (10) results in

$$\gamma = 1 + \frac{\Delta t_{lm}}{T_{c,m}} = \frac{1}{1 - \dfrac{\Delta t_{lm}}{T_{h,m}}} \tag{11}$$

Equation (11) shows that the optimum γ is a function of the ratio of Δt_{lm} to the absolute temperature of hot or cold fluid. This general relation could be conveniently used for thermodynamic optimization of a

heat transfer process.

Assuming that $T_{h,m}$ and $T_{c,m}$ are finite values, more conclusions can be derived:

(a) If $\Delta t_{lm} > 0$, from equation (11), $\gamma > 1$, i.e., the optimum heat capacity rate of cold fluid is greater than that of hot fluid in heat transfer process with finite temperature difference.

(b) If $\Delta t_{lm} = 0$, from equation (11), $\gamma = 1$, i.e., when two fluids undergo a zero temperature difference heat transfer process, the optimum heat capacity rates of both fluids are equal. From these results, we can draw that $\gamma = 1$ is only the optimum condition for a reversible heat transfer process, but not for an irreversible one.

(c) From equation (10), $\dot{C}_{p,c} = \dfrac{T_{h,m}}{T_{c,m}} \dot{C}_{p,h}$. When $\dot{C}_{p,h} \to +\infty$, $\dot{C}_{p,c} \to +\infty$, and vice versa. This accounts that the optimum heat capacity rate of a fluid should be infinite when it exchanges heat with another fluid whose heat capacity rate is infinite.

CONCLUSIONS

For a heat exchanger, the heat transfer area is always finite, which means the mean heat transfer temperature difference is impossible to be zero for a specified total heat transfer rate. Consequently, there must exist a definite difference between the mean thermodynamic temperatures of heat transfer fluids, which results in an existence of certain exergy loss in heat transfer process. When the ratio of heat capacity rate γ conforms to the relation (10), the optimum heat transfer is achieved, so does the minimum of ΔT_m, and ΔT_m equals to $\Delta t_{lm.}$.

The analytical results indicate that the ultimate reason of power saving contributed from thermodynamic optimization is the decrease of the difference of mean thermodynamic temperature between two fluids, even though there is no reduction of the log-mean heat transfer temperature difference between two fluids. In other words, the optimum configuration of heat transfer temperature difference does not need to keep the local temperature difference constant throughout the heat exchanger.

Therefore, it should be emphasized that only the total entropy generation rate should be the fundamental criterion for the thermodynamic optimization. The concepts proposed in this paper can also be used to the optimization of a steady heat transfer process with variable specific-heat fluids.

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