Optimal vector analysis of the phase shifter in the pulse tube refrigerator

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Based on vector analysis, the effect of the performance of the phase shifter in a pulse tube refrigerator on the refrigeration capacity and compressor work has been studied. Refrigeration capacity, compression work and efficiency have been expressed as the function of phase angle and amplitude of mass rate flowed through the phase shifter. Then this complex function has been illustrated in a phasor diagram with the simple geometry relation. By analyzing this geometry relation, some interesting results have been obtained.

INTRODUCTION

Pulse tube refrigeration is based on a cyclic process such that a gas column with cold and warm end temperature is compressed, displaced towards the warm end, expanded and redisplaced towards the cold end. This process is realized by feeding adequate gas flow to both ends of the tube. The phase angle between pressure and mass flow rate at the cold end of pulse tube refrigerators (PTR) is the key parameter for the performance of PTR. The various types of PTR differ mainly by the performance of the phase shifter, which can by realized by use of passive or active elements [1]. Various authors have developed their own theory to explain the mechanism of the PTR. Among them, the phasor analysis is an effective way [2]. In this analysis thermodynamic quantities are expanded in a Fourier series. Only the direct component and fundamental oscillating terms are considered. All higher frequency terms are ignored. Due to different parameters shown in the same phasor diagram, it can be used to predict the performance of PTR directly and clearly.

In this paper, the effect of the performance of the phase shifter in PTR on the refrigeration capacity and compression work has been studied based on vector analysis. In order to get the exact amount of the optimal complex impedance of phase shifter, the complex functions between refrigeration capacity, compression work and the parameters of components have firstly been simplified to the geometrical relationship shown clearly in the phasor diagram. Then through analyzing this geometry relation, many interesting results have been obtained which will be helpful to the design of PTR or the choice of suitable phase shifter.

PHASE ANALYSIS

As a starting point for analyzing the pulse tube, the following idealizations will be used:

1) In an ideal regenerator there is no dead volume, entropy production and pressure drop is zero;
2) In an ideal pulse tube, entropy production is zero;
3) The compressor and pulse tube are adiabatic, i.e. there is no heat transfer to the walls;

4) All oscillating quantities are of small amplitude;

5) The reservoir is large enough for its pressure and temperature to be considered constant.

Then all quantities can be written in the form

$$x(z,t) - x_0(z) = x_1(z)\cos(\omega t + \theta_x) \tag{1}$$

which can be represented by the real part of a phaser in phase space. Phasors behave as vector. But it is only the real part that is of interest. A simple model of a pulse tube is developed below. This model emphasizes the different components of mass flows and pressure and the relationships between them. The pressure and temperature can firstly be expressed as the form of equation (1). Then the ideal gas law can be applied to the adiabatic process and expanding the pressure term into a Taylor series and keeping only the lowest order term, the mass that must flow into the pulse tube through the regenerator to allow the change in m_p is

$$m_p = -(3\omega/5R)P_1\sin(\omega t)\int_0^L T_0^{-1}dz \tag{2}$$

where R is the gas constant(per unit mass), P_1 is the amplitude of pressure wave,T_0 is the temperature of charged working gas.

In the same way, the mass rate due to the phase shifter (m_0) is:

$$m_0(z,t) = (R + Qi)\,p_1 T(L)\cos(\omega t)/T(z) \tag{3}$$

The mass flow in the hot end of PTR is the sum of the two mass flows. The refrigeration power is just the enthalpy flow in the pulse tube， that is

$$\langle H\rangle = \int_0^\tau (m_p + m_0)dt + \frac{2}{5}\frac{C_p T_0}{\tau p_0}\int_0^\tau p_1\cos(\omega t)(m_p + m_0)dt \tag{4}$$

For the basic pulse tube (BPT), m_0=0 since there is no phase shifter, the first term of right-hand of equation is unequal to zero, but the integral average value on a cycle is equal to zero.

With H: enthalpy, m: mass flow, p: pressure, t: time, ω: frequency, C_p: specific heat. τ: period.

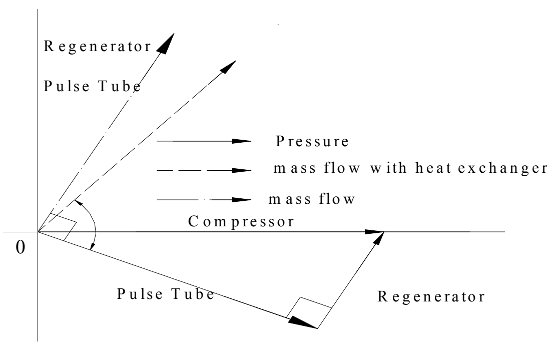

Figure 1 Pressure and mass flow phasor diagram

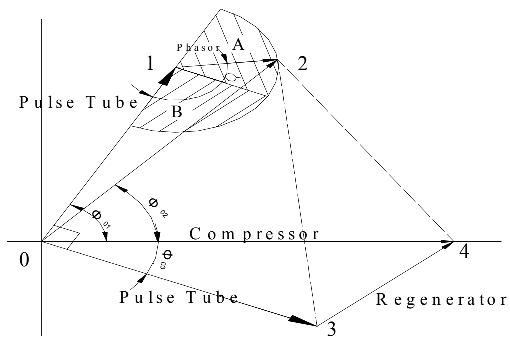

Figure 2 Phasor diagram with phase shifter

Figure 1 shows a phasor diagram for a basic pulse tube. For a basic pulse tube, m_0=0. The mass flow (m_p) is proportional to and shifted 90° from the pressure in the pulse tube. The mass flow is also parallel and proportional to the pressure drop in the regenerator. The latter relationship is a result of the regenerator pressure drop driving the mass flow. The pressure in the pulse tube plus the pressure drop in the regenerator equals the pressure in the compressor. Since the pressure in the pulse tube is 90° from the mass flow there is no refrigeration. In practice, heat transfer between the gas and the pulse tube wall causes a small phase shift resulting in small refrigeration. The compressor pressure times mass flow is proportional to the work done by the compressor. The compressor is doing work, yet there is no refrigeration. all the work is going into the regenerator loss which is proportional to m Δ p.

A phasor diagram for a pulse tube refrigerator with phase shifter is shown in Figure 2. The relationship between the pulse tube pressure and m_p is the same as in the basic pulse tube. In terms of the angle (α), the phase shifter can be divided into two types. When α is less than 90° (Area B in Figure 2), the first type of phase shifter can just provide resistance and capacitive impedance, such as orifice, throttle; when α is bigger than 90° (Area A in Figure 2), the inductive reactance can be provided by the second phase shifer i.e. inertance tube.

VECTOR ANALYSIS

From the phasor diagram (see Figure 2), according to the physical parameters they represent, there exists the following geometrical relationship

$$l_{01} + l_{12} = l_{02}\,,\ l_{03} + l_{34} = l_{04}\,,\ l_{34} \,//\, l_{02} \tag{5}$$

Enthalpy flow through the cold end of the PTR and power consumption of the pulse tube are the directly proportional to $H_{\Delta 203}$, $H_{\Delta 204}$ respectively, which is the area of some corresponding region.

$$H_{\Delta 203} = l_{02} \times l_{03} \times \cos(\phi_{\angle 203}) \quad , \quad H_{\Delta 204} = l_{02} \times l_{04} \times \cos(\phi_{\angle 204}) \tag{6}$$

There exists geometrical relation as follows

$$H_{\Delta 203}{}^{2} + (2S_{\Delta 203})^2 = (l_{02} \times l_{03})^2\,,\quad H_{\Delta 204}{}^{2} + (2S_{\Delta 204})^2 = (l_{02} \times l_{04})^2 \tag{7}$$

With l_{01}, Φ_{01}: magnitude and phase angle of mass rate flowing from refrigerator into the pulse tube, l_{12}, Φ_{12}: magnitude and phase angle of mass rate passing through the phase shifter, l_{02},Φ_{02}: magnitude and phase angle of mass rate passing through the cold end of PTR, l_{03}, Φ_{03}: magnitude and phase angle of pressure in pulse tube, l_{34}: magnitude of pressure in refrigerator, l_{04}: magnitude of pressure provided by compressor. $\Phi_{\angle 012}$: angle formed between vector l_{01} and l_{12}. The others are named in the same manner.

In order to analyze the effect of phase shifter on the performance of PTR, $H_{\Delta 203}$ and $H_{\Delta 204}$ have been expressed as the function of l_{12} and Φ_{12} as follows.

$$H_{\Delta 203} = l_{03} l_{12} \sin(\phi_{\angle 012})\,,\quad H_{\Delta 204} = l_{03}(l_{01} - l_{12})(\tan\phi_{\angle 102} + \tan\phi_{\angle 304})/(1 - \tan\phi_{\angle 102}\tan\phi_{\angle 304}) \tag{8}$$

with $\phi_{\angle 102} = \arctan(l_{12}\sin(\phi_{\angle 012})/(l_{01} - l_{12}\cos(\phi_{\angle 012})))$

According to the definition of the coefficient of performance $\eta = W/H$, it results

$$\eta = l_{12}\sin(\phi_{\angle 102})/((l_{01} - l_{12})\tan(\phi_{\angle 102} + \phi_{\angle 304})) \tag{9}$$

From the Equation (8), (9), it is found that the characteristic of the phase shifter can influence the performance of PTR directly. Unfortunately the globally optimal working condition (l_{12},Φ_{12}) can not be obtained by differentiating the method of multi-variable function. However, Equation (8) still provides much important information on the pulse tube. When l_{12} is equal to zero, it becomes the basic pulse tube refrigeration, which is no refrigeration at ideal conditions. If the angle ($\Phi_{\angle 204}$) keeps constant, the longer the vector (l_{02}) is, the bigger the refrigeration capacity will get. That is the reason why the volume of the reservoir is always adopted as big as possible and the orifice or valve must provide enough resistance. However, we can also find out that with the increase of length of l_{02} the compression work will be increased correspondingly. So it is not the optimal choice to obtain good performance of PTR. When the length of l_{02} is invariable, the refrigeration capacity varies with the angle of $\Phi_{\angle 012}$. If the angle of $\Phi_{\angle 012}$ equals to 90˚, there exists a maximum of refrigeration capacity. According to this conclusion, it is impossible for the first type of phase shifter to get its maximum refrigeration power because the capacitance caused by the reservoir with finite volume cannot be neglected in practical applications. Whereas it is possible for the other kind of phase shifter to shift by 90˚ from the mass flow rate of gas in the pulse tube because the inductance and capacitance can counteract each other. This is the reason why many pulse tube refrigerators using the inertance tube as phase shifter achieve better performance than those using orifice or throttle. It is also found that while the inductance and capacitance are counteracting with each other, in other words while working under the resonant condition in term of electrical analogy, the biggest refrigeration capacity can be achieved. When $\cos(\Phi_{\angle 012})$ equal to l_{12}/l_{01}, the compression work get its minimum value, in which l_{02} will contact the circle with the radius of l_{12}. So we can adjust the phase shifter to this condition for minimal work input.

What is mentioned above can be applied as guidance to the design or adjustment of the phase shifter. If the resistive characteristic as well as the geometry size is known, the optimal design could be obtained. For example [3], under the resonant working condition optimal parameters of the inertance tube can be obtained by iteration from the following two expressions

$$\frac{4\omega^2 \rho l}{\pi d^2} = \frac{RT}{V} \quad , \quad \frac{128\,\mu l}{\pi d^4} = M \tag{10}$$

with V: volume of gas reservoir, d: diameter of tube, l: length of tube, M is the constant.

The first expression of Equation (10) represents the phase shifter working under the resonant condition. In other words, $\Phi_{\angle 012}$ equals to 90˚ in Figure 2. When this expression has been satisfied, the constant, M, in the second expression should be as large as possible to get enough refrigeration capacity. Because the length of l_{12} corresponds to the magnitude of M, equation (8) explains this reason.

CONCLUSION

Effect of the phase shifter on the refrigeration capacity, compression work and efficiency has been studied by means of phase analysis. In the phasor diagram, the geometrical relation between the vectors, which make the analysis clear and simple, express all of these complex functions. By analyzing these relations, it is found that under the resonant condition the refrigeration capacity could get the maximum value. Meanwhile the condition under which the compression work gets its minimum value is also deduced. According to this result, a method to design the phase shifter has subsequently been introduced. In addition, many other interesting results have also been obtained by this easy and direct way.

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