Lattice gas modeling of thermoacoustic oscillations

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A two-dimensional 9-bit thermal lattice gas model was applied to simulate self-sustained oscillations in a thermoacoustic resonant tube. The time-evolution of pressure oscillation and the change in temperature with time and space were successfully simulated using the present model. The simulated results from our work are qualitatively in good agreement with that measured or simulated from other researchers in earlier literature. The application of a lattice-gas method to thermoacoustics in the present paper has demonstrated that it would be a useful approach for further thermoacoustic research and applications.

INTRODUCTION

The study of thermoacoustic devices is presently attracting considerable interest. Numerical simulations have played important roles in the advance of modern thermoacoustics and in the developments of thermoacoustic devices for a variety of commercial, military and industrial applications. In the present paper, a totally different numerical method — a lattice gas method (LGM), was applied to thermoacoustic research. Compared to other conventional computational methods, the LGM is simple and unconditionally stable in the algorithm. More important feature is that the LGM can offer access, theoretically or numerically, to regimes that cannot be easily reached by other methods. These features of the LGM are attractive to thermoacoustic research.

The LGM has been developed since the1980s. It was originally proposed for the investigation of hydrodynamic problems [1-2] and was later applied thermodynamics problems [3-4] and acoustics [5-6]. At present, lattice-gas automata methods have been applied to a number of classes of problems. These application fields involved to date have hydrodynamic instabilities, flow in porous media, phase transitions, reactive systems, etc. Thermoacoustic oscillations studied in the present paper involve the inter-disciplinary fields of thermodynamics and acoustics. The application of a lattice-gas method to thermoacoustics and the results from our work have demonstrated that it would be a useful approach for further thermoacoustic research and applications.

A TWO-DIMENSIONAL 9-BIT LATTICE GAS MODEL FOR THERMOACOUSTIC ENGINES

A lattice gas model can be viewed as a simple, fully discrete microscopic model, in which the time, space and fluid are all discrete. Therefore, the fluid consists of discrete particles that reside on a finite region of a regular lattice. At each lattice node one particle at most is permitted according with the exclusion principle. These particles move at regular time intervals from one lattice node to another along the lattice links, and collide at a lattice node obeying the mass, the momentum and the energy conservation laws for a thermal lattice gas model. The theoretical and numerical researches [2] on the lattice gas method have manifested that the statistics of large quantity microscopic particle motions can exhibit the physically realistic macroscopic behavior of fluid motion.

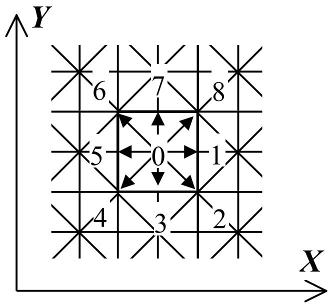

Figure 1 Two-dimensional 9-bit lattice.

We used a 9-velocity thermal model with three speeds $\left(0,1,\sqrt{2}\right)$ on a two-dimensional square lattice in the present study. The two-dimensional square lattice with corresponding velocity vectors is depicted in Fig. 1. All particles have unit mass. The gas particles moving along the axes have speed 1, while particles along the diagonals have speed $\sqrt{2}$, and the particles resting on the lattice have speed 0. The exclusion principle enables a natural Boolean description and encoding of the states of lattice gases. Any possible internal state of an individual automaton can thus be represented by 9 Boolean variables in the present model, which is called 9-bit lattice gas.

In the present model, we used similar two-body collision rules to that stated in Ref. [3]. In order to increase the collision rate and accelerate the rate of approach to equilibrium, thus shorten the computation time, we also considered multi-body collision rules in our model besides these two-body collision rules. This consideration is important for practical computations. These multi-body rules are obtained based on the two-body rules, satisfying the exclusion principle in pre-collision and post-collision. In addition, in order to simulate the thermal boundary condition imposed on the stack and heat exchangers in a thermoacoustic engine, some special collision rules and different probabilities of heat exchange were applied to the stack and heat exchangers in our model [7]. All of these rules used in the present model are reversible and determinate, namely, each input state corresponds to only one output state, or vice versa. All of two-body rules and part of multi-body rules applied in our model are depicted in Fig. 2.

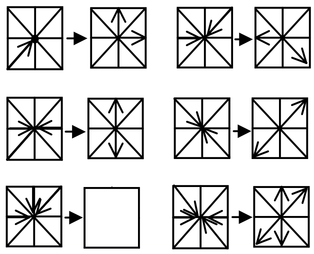

Figure 2 Collision rules

Studies [3] show that a 9-bit lattice-gas model satisfies some macroscopic equations such as the Navier-Stokes equation subject to some particle speed distribution condition. In the low-density approximation, this condition is $d_1/d_2=4,\ d_0/d_1=4$, where $d_0, d_1,$ and d_2 are particle densities corresponding respectively to three speeds 0, 1 and $\sqrt{2}$ in a 9-bit model. This condition for particle speed distribution above was applied in our simulation.

The noise of lattice gas method is its intrinsic nature and can be reduced by a statistic method. The larger the statistical cell, the smaller the residual noise as well as the poorer the effective space resolution. In order to be able to both describe wave characteristic and obtain a macroscopic variable, the size of region for the statistic average should be chosen properly so that it comes from the result of a compromise between the accepted noise level and the desired physical resolution.

SIMULATION RESULTS AND ANALYSIS

The physical model under consideration for lattice gas simulation is a thermoacoustic prime mover as shown in Fig.3. This model engine consists of the working gas air, a medium for thermal rectification called the stack, heat exchangers at each end of the stack, and a 1/2 wavelength of resonator. The thermoacoustic stack in addition to two heat exchangers is generally called thermoacoustic components, which are positioned near the left end of the tube. The temperature distribution along the boundary of tube was prescribed and a temperature gradient was applied to the region of stack, as shown in Fig.3. Thus the stack and the two heat exchangers were imposed a thermal boundary condition under which the heat exchange between local gas particle and the solid boundary would occur. For the regions of the resonant tube outside the thermoacoustic components both thermal and adiabatic boundary conditions were applied with a probability of 0.01 for the thermal boundary condition.

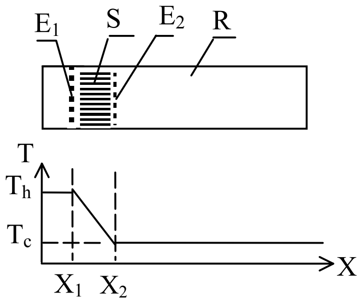

Figure 3 The physical model.
E1—Hot heat exchanger
E2—Cold heat exchanger
S—Stack, R—Resonant tube.

The lattice gas simulation was performed on a rectangular cross-section of a model engine of 2000×100 lattice nodes. The distance of the left end of stack from the left end of tube was 450 in lattice units. The length of stack was 100 in lattice units, and the lengths of two heat exchangers are all 10 in lattice units. The thickness and spacing of plates of stack and heat exchangers are all taken to be 1 lattice unit and 3 lattice units, respectively. The temperature layer in which heat exchanges between the fluid particles and walls would be expected to occur was set at 1 lattice unit. More meaningful conclusions from this model would require more lattice gas units, which will be considered in our next work. The hot temperature of the stack end was fixed at 2.0, while the cold temperature at 1.33, with temperature difference across the stack 0.67. In the initial calculations, the particle density of lattice gas was set at 2.5. The cell size of statistical region was set at 10×10 in lattice gas unit.

The onset behavior of oscillation in a thermoacoustic engine was successfully simulated in our present work. The buildup process of sound pressure at the cold end of tube is demonstrated clearly in Fig. 4. Thermoacoustic oscillation is excited in the resonant tube when the temperature gradient across the stack is greater than some value, and then the amplitude of acoustic pressure increases and finally reaches saturation. The self-excited oscillation will remain stable with the temperature condition along with other parameters constant. These onset oscillation features in a thermoacoustic prime mover were also observed by the experiment in early publication [8] and also simulated in conventional numerical method [9]. By comparing the results from our model with those of the experimental measurement and calculated [8-9], it can be found that the onset behaviors of self-excited thermoacoustic oscillation are qualitatively in good agreement. The pressure wave shape in Fig.4(a) shows the noise nature of the lattice gas method, and the pressure wave curve in Fig.4(b) shows somewhat non-linear feature. The asymmetry of profile curve in the top figure is also evidence of the non-linearity. It could be the consequence of the large temperature gradient across the stack and large probability of heat exchange.

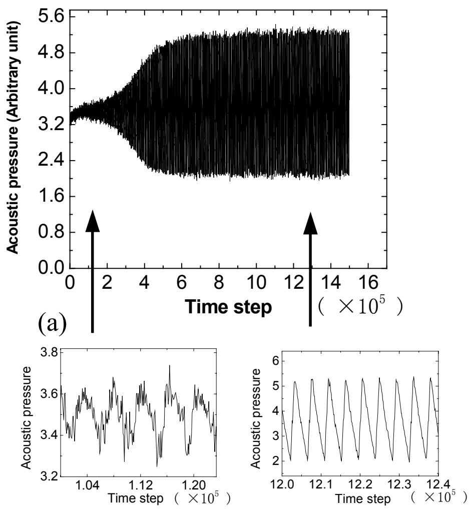

Figure 4 Acoustic oscillations at $\Delta T = 0.67$ applied across stack

We studied numerically as well the distribution of temperature field in the thermoacoustic engine. The contours of temperature in engine are shown in Fig.5, in which the isothermal lines can be observed. The time evolution process of temperature and its propagation process from the hot region towards two cold regions of tube ends can be clearly observed from Fig. 5(a). It can also be seen from Fig. 5 that at the unstable stage of onset oscillation the temperature distribution is inhomogeneous in the hot region, while at the stable-state of oscillation the temperature distribution becomes homogeneous in the hot tube. But, in the stack region the temperature shows the oscillation feature, and the temperature gradient is established and stable. Such a temperature re-distribution is just the consequence of thermoacoustic effect in stack. It can be observed as well from Fig. 5(b) that in the hot region of the tube the temperature is highest in the engine, while at the cold region of the tube (the right hand end) is a slightly higher temperature region. That could be because the dissipation heat of acoustic power in the overall engine is accumulated in the tube end, and/or because the heat is pumped to the right end region of tube by thermal acoustics generated at the left end of tube.

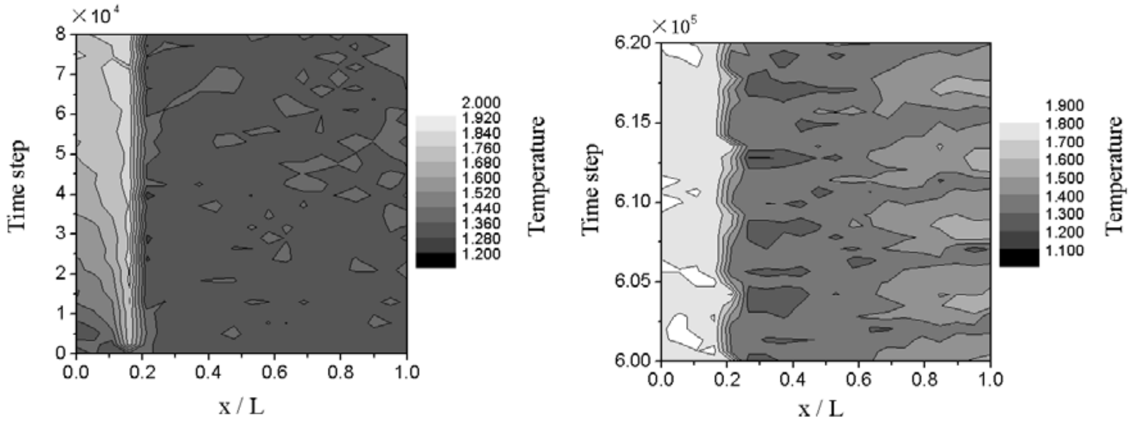

Figure 5 Distribution of temperature field in a thermoacoustic engine. (a) At onset phase of oscillation. (b) At stable-state of oscillation. x denotes position in engine, and L length of the engine.

CONCLUSIONS

In this paper we used a two-dimensional 9-bit thermal lattice gas model to simulate thermoacoustic oscillations. The building process of self-excited oscillation and the onset behavior of an acoustic wave in a thermoacoustic prime mover were successfully simulated. The temperature distribution and temperature time evolution in a thermoacoustic engine were obtained. The results from our model showed qualitatively good agreement with those from other researchers in earlier literature. The present paper work demonstrated that the lattice gas method would be a new powerful approach to investigate some complicated thermoacoustic problems. To reduce model noise and to extend its applications to thermoacoustic refrigerators and other thermoacoustic systems with complex geometric boundaries, an improved lattice gas model and more experimental work would be desirable. Efforts in all these directions are currently underway.

REFERENCES

1. M. J. Biggs and S. J. Humby, Lattice-gas automata methods for engines, Trans Ichem E (1998), 76 162-174
2. J.-P. Rivet, and J. P. Boon, Lattice Gas Hydrodynamics, Cambridge University Press (2001)
3. S. Chen, M. Lee, K. H. Zhao and G. D. Doolean, A lattice gas model with temperature, Physica D (1989), 37 42-59
4. S. P. Das and M. H. Ernst, Thermal transport properties in a square lattice gas, Physica A (1992) 187 191-209
5. Y. Sudo and V. W. Sparrow, Sound Propagation Simulation Using Lattice Gas Methods, AIAA Journal (1995), 33(9) 1582-1589
6. P. Lavallee, Attenuation of sound waves in lattice gases, Physics Letters A (1992), 163 392-396
7. Yu Chen, Xiaoqing Zhang, Xu Liu, A lattice gas model of thermoacoustic engines, J. Acoust. Soc. Am. (2004), (Accepted)
8. Wheatley J., Hofler T., Swift G. W., and A.Migliori, Understanding some simple phenomena in thermoacoustics with application to heat engines, Am. J. Phys. (1985), 53(2) 147-161
9. S. Karpov and A. Prosperetti, A nonlinear model of thermoacoustic devices, J. Acoust. Soc. Am. (2002), 112(4) 1431-1444