Discussion on resonant frequency in thermoacoustic systems

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Besides two traditional methods of calculating resonant frequency, a new method referred to as Standing Wave Minimal Antinode Method is proposed, which makes it possible to consider the influence of resistance, since zero antinode is no long a necessity as in an ideal standing wave system. Numerical simulation will be made on three types of tubes for their resonant frequencies with three methods, focusing on the influences of impedance composition on the resonant frequency. Analysis and comparison will also be made to these three methods.

INTRODUCTION

Analysis and accurate measurement of resonant frequency is vital for improving the performance of a thermoacoustic system, especially a thermoacoustic refrigerator, for the impedance mismatching will lead to a remarkable reduction of the loudspeaker's electroacoustic efficiency. Impedance Matching Method (IMM) and Acoustic Pressure Maximum Method (APMM) are two traditional calculation methods. In this paper, we will propose a new method referred to as Standing Wave Minimal Antinode Method(SWMAM). With this new method, it is possible to consider the influence of resistance, since zero antinode is no long a necessity as in an ideal standing wave system.

Based on three different methods, numerical simulation will be made on three types of tubes for their resonant frequencies, focusing on the influences of impedance composition on the resonant frequency. Results under different conditions will be used to discuss the equality and difference among these methods. We will also observe the influence of impedance at the tube end on the frequency response, with which the importance of radiation impedance may be evaluated.

TYPICAL STRUCTURES AND CALCULATION METHODS

Before describing the calculation methods, we first introduce three typical tubes in thermoacoustic machines, as shown in Figure 1, where (a) represents for the general case with arbitrary ends(with end impedances Z_{A0} and Z_{Al}), (b) for the case with one rigid end and one compliance volume at the other end(1/4 wave length structure, such as Hofler-type thermoacoustic refrigerator), (c) for the case with rigid ends(1/2 wave length structure, such as standing wave thermoacoustic prime mover). In the following analysis, the sound wave propagating inside the tube will be supposed to be plane wave.

Two traditional methods are applied to calculate the resonant frequency. APMM defines the resonant frequency with which the system's acoustic pressure reaches as high as possible. As shown in Figure 1(a),

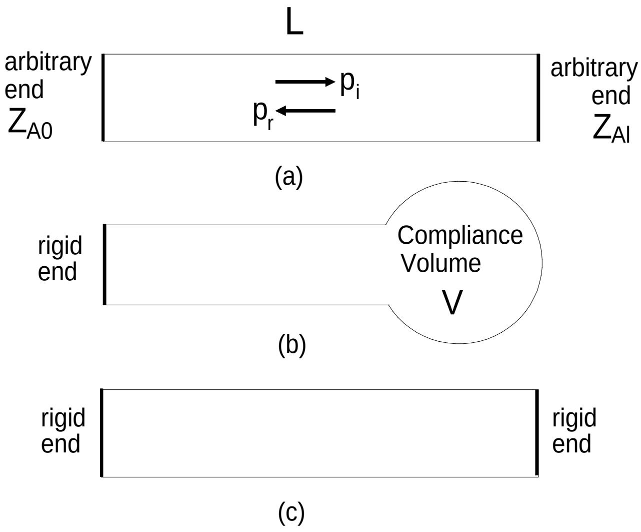


Figure 1 Typical thermoacoustic resonant tubes. (a) arbitrary ends, (b) rigid end + compliance, (c) rigid ends

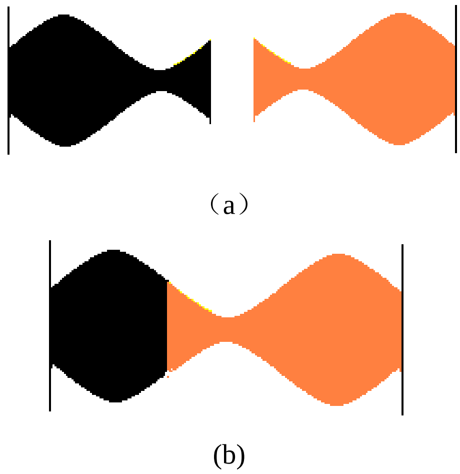


Figure 2 Schematic of standing wave minimal antinode method(SWMAM)

p_{in} is the sound wave entering the tube from left to right, n means the wave after $(n-1)$ times reflection, e.g., p_{i1} represents for the original input, p_{i2} represents for the wave after one reflection at the right end and then also at the left end to re-enter the tube. Similarly, p_{rn} represents for the sound wave propagating from right to left. Except for the ideal cases, the reflection attenuation α and phase shifts at both ends σ_i and σ_r (σ=0 for rigid end) should be considered, based on the acoustic impedance at both ends. The sound pressure can be written as,

$$
\begin{aligned}
p_{in} &= \alpha^{2(n-1)} p e^{j\{\varpi t - k[2(n-1)L + x] + (n-1)\sigma_r \pi + (n-1)\sigma_l \pi\}} \\
p_{rn} &= \alpha^{2n-1} p e^{j\{\varpi t + x - k[2(n-1)L + n\sigma_r \pi + (n-1)\sigma_l \pi\}}
\end{aligned} \tag{1}
$$

After some reflections, the system obtains a balance, and the sound pressure profile versus frequency may be simulated by numerical means, and then the resonant frequency can be determined according to the maximal pressure principle.

We can also use IMM to calculate the resonant frequency of the tube structure shown in Figure 1(a). The tube length is L, and the acoustic impedance at both ends are Z_{A0} and Z_{Al}. According to the fundamental of acoustics, if the impedance transfer equation as follows is satisfied,

$$
Z_{A0} = \frac{\rho_0 c_0}{S} \frac{Z_{Al} + j\rho_0 c_0 \tan kL}{\frac{\rho_0 c_0}{S} + jZ_{Al} \tan kL} \tag{2}
$$

the corresponding frequency can be considered as resonant one, where the reactance equals zero while the resistance is maximal. With the acoustic pressure obtained from Equation (1), we can calculate the acoustic impedance along the tube, and then analyze the profile of the impedance versus frequency.

Besides the above methods, we propose a novel method, Standing Wave Minimal Antinode Method. With the reflection and superposition of sound wave, matching between the frequency and tube length are necessary to form a stable resonant sound field (either standing wave or traveling wave). At any point inside the tube, the sound wave can be regarded as the superposition of two reverse waves, $p = p_i + p_r$, where $p_i = p_{Ai} e^{j(\varpi t - kx)}$ is incident wave, $p_r = p_{Ar} e^{j(\varpi t + kx)}$ is reflection wave. We then have

$$p_{Ar}/p_{Ai} = |r_p| e^{j\sigma\pi} \tag{3}$$

$$\zeta = Z_s/\rho_0 c_0 = Z_A S/\rho_0 c_0 = x_s + jy_s \tag{4}$$

$$|r_p|^2 = \frac{(x_s-1)^2 + y_s^2}{(x_s+1)^2 + y_s^2}, \quad \tan(\sigma\pi) = \frac{2y_s}{x_s^2 + y_s^2 - 1} \tag{5}$$

where r_p is reflection factor, whose module $|r_p|$ and phase angle $\sigma\pi$ depends on the impedances, ζ is impedance ratio, Z_s is impedance of tube end, $\rho_0 c_0$ is air's impedance, *S* is cross-sectional area.

The position *x*, with minimal standing wave, can be determined by σ,

$$x = (1+\sigma)\lambda/4 \tag{6}$$

where λ is wave length of sound wave. If the positions x_1 and x_2 (calculated from both ends) coincide each other, i.e., the sound wave from both ends match inside the tube, we may consider that the system achieves resonant state, when the frequency satisfies $x_1 + x_2 = L + n\lambda/2$, as shown in Figure 2.

SIMULATION RESULTS

In the following simulation, we set the tube length L of 1m, sound velocity c_0 of 340m/s, acoustic impedance of air $\rho_0 c_0$ of 400kg/m^{2}s. In Figure 1(b), the tube length does not include the compliance buffer section. Only the resonant frequency of fundamental is considered as the calculation result.

Figure 3 presents the result of the structure shown in Figure 1(b), with three different methods. Abscissa *x/y* is the ratio between resistance and reactance. The radii of the buffer and the resonant tube are 0.3m and 0.1m, respectively. We can find that the calculation results by various methods agree quite well, which demonstrates the validity of our new proposal.

Figure 4 is for the resonant frequency of a tube with a buffer at one end. The resistance at tube end has obvious effect on resonant frequency, however, it gets less evident when the ratio of real and virtual part of the normalized impedance is larger than 3. This is attributable to the fact that the rise of reactance leads to a decrease of $2y/(x^2+y^2-1)$, which is dominant for σ. However, this effect can not be found in the void buffer for the resistance is not large enough, unless some extra resistance is put inside the buffer.

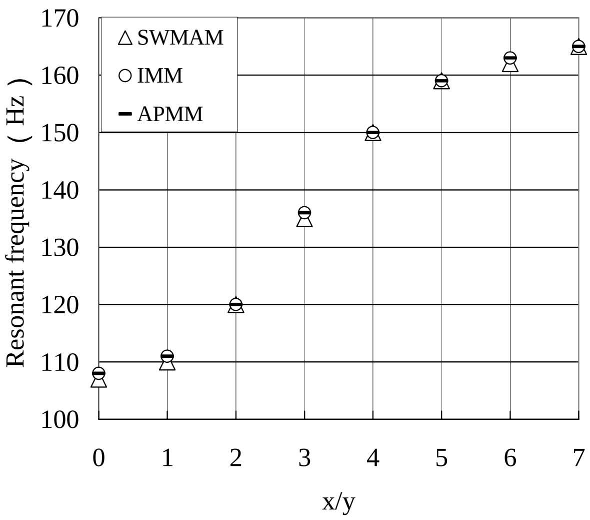


Figure 3 Calculated resonant frequency of the tube with a buffer by three methods

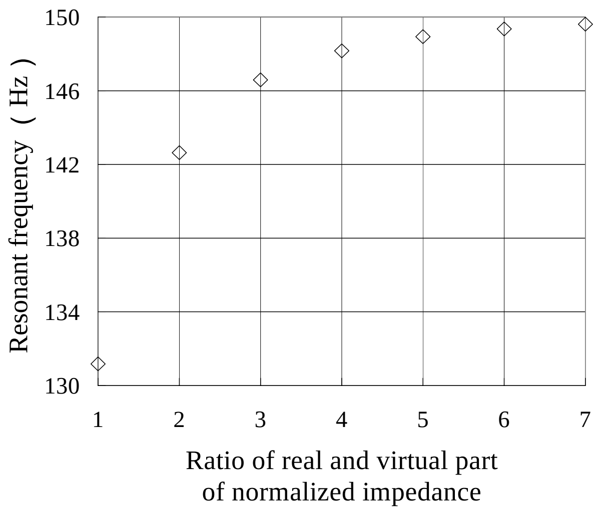
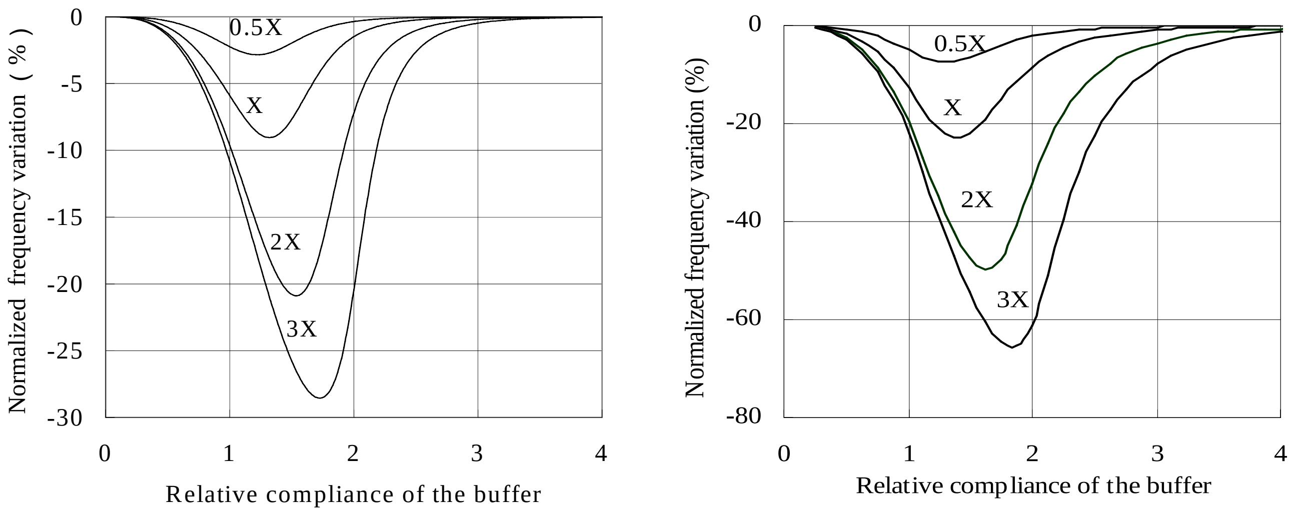


Figure 4 Effect of end impedance on resonant frequency



(a) one rigid end and buffer at the other end (b) two buffers at both end

Figure 5 Normalized resonant frequency variation with relative compliance of the buffer

The influence of the buffer volume on resonant frequency, through relative compliance (the ratio of buffer volume to tube volume), is also analyzed for two cases as shown in Figure 5(a,b), where "X" is the reactance component in the impedance, and "mX" means that the resistance is *m* times of reactance. We find that the buffer volume may greatly affect the resonant frequency at some certain range (say 0.5-3 times of the tube volume for the present calculated case). Also, the resistance component is another obvious factor influencing the resonant frequency.

CONCLUSION

1. The SWMAM is proved valid for calculating resonant frequency, after comparison with two traditional methods. It is convenient for analyzing the influence of impedance on the resonant frequency.
2. The impedance of tube ends has obvious effect on resonant frequency when the resistance is not so large (say the ratio between the resistance and reactance is lower than 3).
3. When the end buffer volume is 0.5-3 times that of tube, the resistance fraction greatly affects the resonant frequency.

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