New perspective on thermodynamic cycles of oscillating flow regenerators

Luo E.[1], Dai W.[1], Radebaugh R.[2]

1. Refrigeration and Cryogenic Division, Technical Institute of Physics and Chemistry, Chinese Academy of Sciences, P.O.Box 2711, Beijing 100080, CHINA
2. Cryogenic Technology Group, Physical and Chemical Properties Division, National Institute of Standards and Technology, Mail Stop 305, Boulder, CO80305, USA

Regenerative machines usually operate in oscillating flow and the regenerators for the machines are the most important thermodynamic elements. However, the real function of the cyclic regenerator has not been understood thoroughly. This paper tries to reveal a unique working mechanism for a regenerator; that is, the regenerator functions as a recuperator but also finishes a complete thermodynamic cycle.

INTRODUCTION

The operations of regenerative machines rely on the compression and expansion of working gas. Classical thermodynamics thinks that a regenerator used in the regenerative machines has been taken just as a recuperative heat exchanger used in a recuperative cycle. This is not true, however, even for an ideal thermodynamic cycle, i.e., without any loss mechanism but having some 'dead' volume for storing working gas.

The thermoacoustic theory has been recently developed to explain working mechanisms of various thermoacoustic machines including traditional regenerative machines like Stirling machines. From the viewpoint of the thermoacoustic theory, the regenerator is an active thermodynamic element, which can reversibly consume some mechanical power and then pump the heat from the cold side to hot side of the regenerator. This viewpoint distinctly noticed that the regenerator consists of the compression, expansion, heat rejection and heat absorption processes. However, perhaps due to the sophisticated thermoacoustic phenomena and due to the Eulerian viewpoint of the thermoacoustic theory, the intuitional explanation has not been well developed yet so that some aspect of the regenerator is ignored such as the recuperative function. This paper tried to reveal unique function of a cyclic regenerator in terms of Lagrangian viewpoint.

PHYSICAL MODEL AND THERMODYNAMIC ANALYSIS

A regenerator is usually a kind of porous solid matrix filled with working gas. Fig.1 schematically shows a typical regenerator in which presents three typical gas parcels. For simplicity, the following assumptions are made: (1) a perfect heat transfer between working gas and solid medium; (2)an inviscous, ideal gas. We first consider a general case that the phase difference between pressure and velocity oscillations is θ ($-\pi/2<\theta<\pi/2$) . Having the velocity phase as a base, there are the following expressions for the velocity and pressure.

$$u(t)=u_d \sin\omega t \tag{1}$$

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$$p(t) = p_0 + p_d \sin(\omega t + \theta) \tag{2}$$

Moreover, the displacement of the gas parcel as the function of time can be expressed by Eq.(3), which is schematically shown in Figure 2, and the time-dependent temperature of the gas parcel can be given by Eq.(4).

$$X(t) = \int u dt = -\frac{u}{\omega}\cos\omega t = -X_d \cos\omega t \tag{3}$$

$$T(t) = T(X(t)) = T_0 + \frac{dT_0}{dx}X(t) = T_0 + \frac{dT_0}{dx}(-\frac{u_d}{\omega}\cos\omega t) = T_0 + T_d \cos\omega t \tag{4}$$

Once the time-dependent temperature and pressure of the gas parcel are known, it is also readily to have the time-dependent volume and entropy for the gas parcel.

$$v(t) = \frac{RT_0}{p_0}[1 + \frac{T_d}{T_0}\cos\omega t - \frac{p_d}{p_0}\sin(\omega t + \theta)] \tag{5}$$

$$s(t) = C_p \ln\frac{T}{T_{ref}} - R\ln\frac{p}{p_{ref}} = C_p \ln\frac{(T_0 + T_d\cos\omega t)}{T_{ref}} - R\ln\frac{[p_0 + p_d\sin(\omega t + \theta)]}{p_{ref}} \tag{6}$$

According to Eq.(2) and Eq.(5), we can give the p-v diagram of the gas parcel. Similarly, the T-s diagram of the gas parcel can be given by Eq.(4) and Eq.(6).

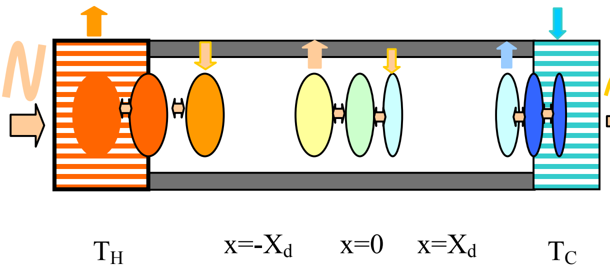

Figure 1. A cyclic regenerator

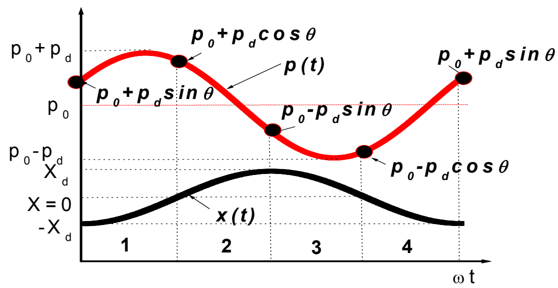

Figure 2. Pressure and displacement

Here we evenly divide a complete thermodynamic cycle into four processes by time: (1) $0 < \omega t < \pi/2$, when the gas parcel moves from its left peak position to its counterbalance position; (2) $\pi/2 < \omega t < \pi$, when the gas parcel moves from its counterbalance position to its right peak; (3) $\pi < \omega t < 3\pi/2$, when the gas parcel returns from its right peak position back to its counterbalance position; (4) $3\pi/2 < \omega t < 2\pi$, when the gas parcel continues back to its left peak position. So far, the gas parcel finishes a complete thermodynamic cycle. The following section gives the conversion and transportation of thermal energy and acoustical power happening in the four processes.

In terms of thermodynamics, the gas parcel does the p-v power within the time range of t_1 to t_2 can be calculated by the following equation.

$$\begin{aligned} w(t) = \int_{t_1}^{t_2} pdv = \omega RT_0\{(-t\frac{1}{2}\frac{p_d}{p_0}\frac{T_d}{T_0}\cos\theta) + [\frac{1}{\omega}\frac{T_d}{T_0}\cos\omega t - \frac{1}{\omega}\frac{p_d}{p_0}\sin(\omega t + \theta)] \\ + [\frac{1}{4\omega}\frac{p_d}{p_0}\frac{T_d}{T_0}\sin(2\omega t + \theta) + \frac{1}{4\omega}(\frac{p_d}{p_0})^2\cos(2\omega t + 2\theta)]\}\Big|_{t_1}^{t_2} \end{aligned} \tag{7}$$

Four these four thermodynamic processes (process 1: $0 < \omega t < \frac{\pi}{2}$, process 2: $\pi/2 < \omega t < \pi$, process 3: $\pi < \omega t < 3\pi/2$ and process 4: $3\pi/2 < \omega t < 2\pi$), the p-v power, the change of internal energy and the absorption heat for the gas parcel are given by Eq.(8) to Eq.(19), respectively.

$$\Delta w_1 = RT_0\{(-\frac{\pi}{4}\frac{p_d}{p_0}\frac{T_d}{T_0}\cos\theta) + [-\frac{T_d}{T_0} - \frac{p_d}{p_0}(\cos\theta - \sin\theta)] - [\frac{1}{2}(\frac{p_d}{p_0})^2\cos 2\theta + \frac{1}{2}\frac{p_d}{p_0}\frac{T_d}{T_0}\sin\theta]\} \tag{8}$$

$$\Delta u_1 = -C_v T_d \tag{9}$$

$$\Delta q_1 = \Delta u_1 + \Delta w_1$$
$$= -C_p T_d + RT_0[(-\frac{\pi}{4}\frac{p_d}{p_0}\frac{T_d}{T_0}\cos\theta) + [-\frac{p_d}{p_0}(\cos\theta - \sin\theta)] - [\frac{1}{2}(\frac{p_d}{p_0})^2\cos 2\theta + \frac{1}{2}\frac{p_d}{p_0}\frac{T_d}{T_0}\sin\theta]\} \tag{10}$$

$$\Delta w_2 = RT_0\{(-\frac{\pi}{4}\frac{p_d}{p_0}\frac{T_d}{T_0}\cos\theta) + [-\frac{T_d}{T_0} + \frac{p_d}{p_0}(\sin\theta + \cos\theta)] + [\frac{1}{2}(\frac{p_d}{p_0})^2\cos 2\theta + \frac{1}{2}\frac{p_d}{p_0}\frac{T_d}{T_0}\sin\theta]\} \tag{11}$$

$$\Delta u_2 = -C_v T_d \tag{12}$$

$$\Delta q_2 = -C_p T_d + RT_0\{(-\frac{\pi}{4}\frac{p_d}{p_0}\frac{T_d}{T_0}\cos\theta) + [\frac{p_d}{p_0}(\sin\theta + \cos\theta)] + [\frac{1}{2}(\frac{p_d}{p_0})^2\cos 2\theta + \frac{1}{2}\frac{p_d}{p_0}\frac{T_d}{T_0}\sin\theta]\} \tag{13}$$

$$\Delta w_3 = RT_0\{(-\frac{\pi}{4}\frac{p_d}{p_0}\frac{T_d}{T_0}\cos\theta) + [(\frac{T_d}{T_0} + \frac{p_d}{p_0}(\cos\theta - \sin\theta)] - [\frac{1}{2}(\frac{p_d}{p_0})^2\cos 2\theta + \frac{1}{2}\frac{p_d}{p_0}\frac{T_d}{T_0}\sin\theta]\} \tag{14}$$

$$\Delta u_3 = C_v T_d \tag{15}$$

$$\Delta q_3 = C_p T_d + RT_0\{(-\frac{\pi}{4}\frac{p_d}{p_0}\frac{T_d}{T_0}\cos\theta) + [\frac{p_d}{p_0}(\cos\theta - \sin\theta)] - [\frac{1}{2}(\frac{p_d}{p_0})^2\cos 2\theta + \frac{1}{2}\frac{p_d}{p_0}\frac{T_d}{T_0}\sin\theta]\} \tag{16}$$

$$\Delta w_4 = RT_0\{(-\frac{\pi}{4}\frac{p_d}{p_0}\frac{T_d}{T_0}\cos\theta) + [(\frac{T_d}{T_0} - \frac{p_d}{p_0}(\cos\theta + \sin\theta)] + [\frac{1}{2}(\frac{p_d}{p_0})^2\cos 2\theta + \frac{1}{2}\frac{p_d}{p_0}\frac{T_d}{T_0}\sin\theta]\} \tag{17}$$

$$\Delta u_4 = C_v T_d \tag{18}$$

$$\Delta q_4 = C_p T_d + RT_0\{(-\frac{\pi}{4}\frac{p_d}{p_0}\frac{T_d}{T_0}\cos\theta) + [-\frac{p_d}{p_0}(\cos\theta + \sin\theta)] + [\frac{1}{2}(\frac{p_d}{p_0})^2\cos 2\theta + \frac{1}{2}\frac{p_d}{p_0}\frac{T_d}{T_0}\sin\theta]\} \tag{19}$$

During the complete thermodynamic cycle, the net p-v power of the gas parcel is given by Eq.(20) and the net heat exchange in the right-half and left-half zones can be given by Eqs.(21) and (22).

$$\sum \Delta w = \Delta w_1 + \Delta w_2 + \Delta w_3 + \Delta w_4 = -\pi RT_0 \frac{p_d}{p_0}\frac{T_d}{T_0}\cos\theta \tag{20}$$

$$\Delta q_c = q_2 + q_3 = RT_0[-\frac{\pi}{2}\frac{p_d}{p_0}\frac{T_d}{T_0}\cos\theta + 2\frac{p_d}{p_0}\cos\theta)] = 2RT_0\frac{p_d}{p_0}\cos\theta(1 - \frac{\pi}{4}\frac{T_d}{T_0}) \tag{21}$$

$$\Delta q_h = q_1 + q_4 = RT_0(-\frac{\pi}{2}\frac{p_d}{p_0}\frac{T_d}{T_0}\cos\theta - 2\frac{p_d}{p_0}\cos\theta) = -2RT_0\frac{p_d}{p_0}\cos\theta(1 + \frac{\pi}{4}\frac{T_d}{T_0}) \tag{22}$$

Furthermore, there is the following correlation for the three parameters.

$$|\Delta q_h| = |\Delta q_c| + |\Delta w| \tag{23}$$

Comprehensively analyzing Eqs.(20) to(23), it is straightforward to achieve such a conclusion that the gas parcel undergoes a complete thermodynamic cycle in which includes compression, heat rejection, expansion and heat absorption; in addition, the gas parcel reversibly consumes a net p-v power $|\Delta w|$, reject a net heat of $|\Delta q_h|$ in hot zone and absorbs a net heat of $|\Delta q_c|$. Actually, Figure 3 and Figure 4 show the p-v and T-s diagrams of the gas parcel, respectively. In a similar way, the p-v and T-s diagrams of all other gas parcels can be given, too. If combining all the diagrams, we can give the p-v and T-s diagram locus of all gas parcels, which is shown in Figure 5 and Figure 6. Two isobaric lines for the locus imply that the thermodynamic cycle is not Stirling cycle. This point is quite different from traditional thermodynamics.

Now let us look at the recuperative process. The net reject heat from the gas to the regenerator solid is given by Eq.(24) when the gas parcel moves from the left-peak position (hot end) to the right-peak position (cold end), and the net absorption heat from the regenerator is given by Eq.(25).

$$\Delta q_{hot-cold} = -2C_p T_d + RT_0\frac{\pi}{2}\frac{p_d}{p_0}\frac{T_d}{T_0}\cos\theta + 2\frac{p_d}{p_0}\sin\theta \tag{24}$$

$$\Delta q_{cold-hot} = 2C_p T_d - RT_0\frac{\pi}{2}\frac{p_d}{p_0}\frac{T_d}{T_0}\cos\theta - 2\frac{p_d}{p_0}\sin\theta \tag{25}$$

$$\Delta q_{hot-cold} + \Delta q_{cold-hot} = 0 \tag{26}$$

Moreover, the net recuperative heat is $\Delta q_{hot-cold} + \Delta q_{cold-hot}$, exactly equal to 0! This is just the traditional function of the regenerator.

The analysis above is for a general case. Here we discuss some extreme cases: one is pure a standing-wave mode and the other is a pure traveling-wave mode in terms of thermoacoustics. For a pure standing-wave mode, one can readily see the net p-v power, the net absorption heat in cold side and the net rejection heat in hot side, all three, are zero, when setting $\theta = 90^\circ$. Thus, pure standing-wave mode does not work. For a pure traveling-wave mode it is readily to see the three parameters are maximized, when setting $\theta = 0^\circ$.

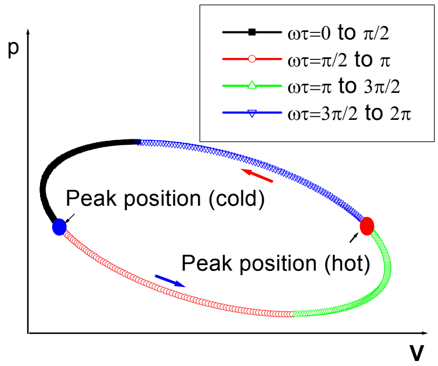

Figure 3. p-v diagram

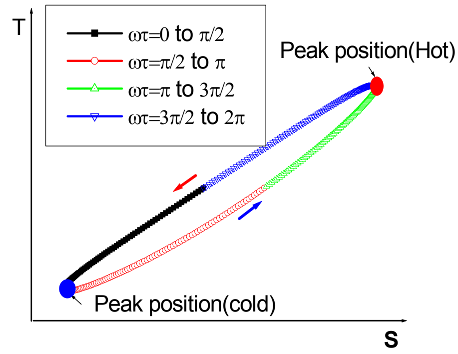

Figure 4. T-s diagram

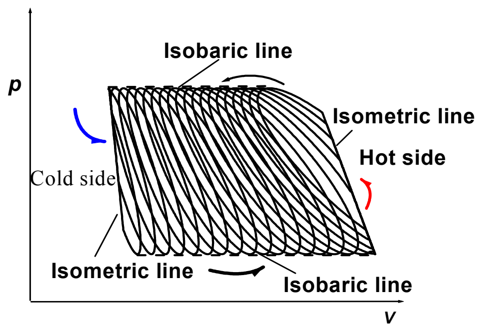

Figure 5. p-v Locus of all gas parcels

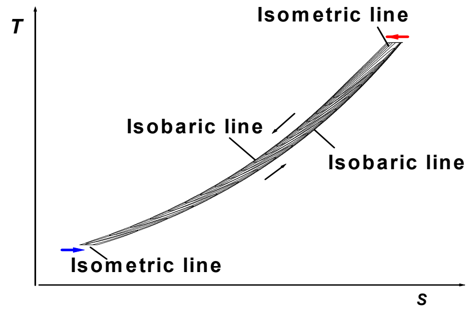

Figure 6. T-s locus of all gas parcels

CONCLUSION

Based on Lagrangian viewpoint, we analyzed the fundamental operation of an ideal regenerator. The analysis shows that the regenerator is active element in which the gas parcel undergoes a complete thermodynamic cycle. The gas parcel does consume a net mechanical power and then does pump heat from cold place to hot place. In the meantime, the regenerator accompanies by a recuperative process. Thus, the regenerator involves compression, heat rejection, expansion, heat absorption and traditional recuperative processes. These processes are parasite together. The regenerator with oscillating flow is definitely not a pure recuperative heat exchanger and it is more like a complete heap pump, which is accompanied by the recuperative function. One of these two functions cannot be separated independently; otherwise the regenerator does not work.

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