Comparison of the performance of regenerators to counterflow heat exchangers

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Irreversible processes in regenerators and heat exchangers limit the performance of cryocoolers. In our research we study the possibility to avoid regenerators in pulse-tube refrigerators (PTR's) by using two identical PTR's operating in opposite phase. The two regenerators are replaced by one counterflow heat exchanger. In this contribution we treat the performances of regenerators and heat exchangers from a fundamental point of view. The losses in the two systems are calculated from the entropy production due to the various irreversible processes. The expressions are brought in special forms which make comparison relatively easy.

INTRODUCTION

Irreversible processes in regenerators and heat exchangers limit the performance of cryocoolers. In our research we study the possibility to avoid regenerators in pulse-tube refrigerators (PTR's) by using two identical PTR's operating in opposite phase. The two regenerators are replaced by one counterflow heat exchanger [1]. The performances of heat exchangers and regenerators can be compared by calculating the entropy-production rates by the four different irreversible processes: axial thermal conduction in the gas, axial thermal conduction in the material, flow resistance and heat exchange between the gas and the material.

The regenerator is supposed to be filled with spherical particles with diameter d_h. The free flow area of the regenerator is $A_\mathrm{g} = (1-f)\,A$ where f is the filling factor and A is the area of the cross section. The heat exchanger is supposed to consist of N parallel tubes with diameter d_1 for the high- and N tubes for the low-pressure side. The total cross section for the gas flow is $A_\mathrm{g} = 2N\pi d_1^2/4$. The wall thickness of one tube at pressure p is $\delta_\mathrm{w} = d_1 p/p_\mathrm{c}$ with p_c the breaking stress of the material. As $\delta_\mathrm{w} \ll d_1$ we will disregard the difference between the total area A and the area of the gas flow A_g. So in the case of the heat exchanger $A \approx A_\mathrm{g}$. It will turn out that, in the optimum situation, the tube diameter and the corresponding Reynolds numbers, will be very small so we only consider laminar flow. We will further assume ideal-gas conditions. In order to avoid unnecessary complications we will treat the various contributions to lowest order in the temperature differences. The basic expressions for the entropy-production rates are derived from a paper by De Waele et al. [2] and the thesis from Steijaert [3].

LOSSES

Heat conduction

The entropy-production rate per unit length due to heat conduction in the gas in the axial direction can be written as

$$\frac{\mathrm{d}\dot{S}_\mathrm{cg}}{\mathrm{d}l} = A_\mathrm{g}\frac{N_\mathrm{u}\kappa_\mathrm{g}}{T^2}\left(\frac{\mathrm{d}T}{\mathrm{d}l}\right)^2. \tag{1}$$

Here κ_g is the coefficient of thermal conductivity of the gas, T is the temperature, and l the length co-ordinate. The parameter N_u represents the Nusselt number which is different for the regenerator and the heat exchanger.

The entropy production due to axial heat conduction through the material can be given as

$$\frac{\mathrm{d}\dot{S}_\mathrm{cm}}{\mathrm{d}l} = A_\mathrm{s}\frac{\kappa_\mathrm{s}}{T^2}\left(\frac{\mathrm{d}T}{\mathrm{d}l}\right)^2 \tag{2}$$

with κ_s the thermal conductivity of the solid material. For the regenerator $A_\mathrm{s} = fA$ and $\kappa_\mathrm{s} = C_\mathrm{k}\kappa_\mathrm{m}$ with C_k a factor taking into account the bad thermal contact between the grains and κ_m the thermal conductivity of the material. For the heat exchanger $A_\mathrm{s} = 2N\pi\delta_\mathrm{w}d_1 = 4Ap/p_\mathrm{c}$ and $\kappa_\mathrm{s} = \kappa_\mathrm{m}$.

Flow resistance

The entropy production due to flow resistance can be written as

$$\frac{\mathrm{d}\dot{S}_\mathrm{f}}{\mathrm{d}l} = \eta\frac{z}{AT}\overset{*}{n}{}^2 V_\mathrm{m}^2 \tag{3}$$

with η the viscosity of the gas, V_m the molar volume, $\overset{*}{n}{}^2$ the mean square molar flow, and z a geometrical factor. For the regenerator $z = 1600/d_\mathrm{h}^2$ and for the heat exchanger $z = 128/d_1^2$.

Heat exchange

The entropy production due to the heat exchange between gas and material can be written as

$$\frac{\mathrm{d}\dot{S}_\mathrm{e}}{\mathrm{d}l} = g_\mathrm{e}\frac{C_\mathrm{p}^2\overset{*}{n}{}^2}{A\kappa_\mathrm{g}N_\mathrm{u}}\frac{1}{T^2}\left(\frac{\mathrm{d}T}{\mathrm{d}l}\right)^2 \tag{4}$$

with C_p the molar heat capacity of the gas and g_e a geometrical factor which is $g_\mathrm{e} = d_\mathrm{h}^2/12f$ for the regenerator and $g_\mathrm{e} = d_1^2$ for the heat exchanger.

OPTIMIZATION

The optimal regenerator and heat exchanger can be found by minimizing the entropy productions. If we are looking for the optimum grain size d_h we can write the total entropy production for the regenerator per unit volume by adding up the four contributions and dividing by A. The result is

$$\sigma_\mathrm{r} = (1-f)\,N_\mathrm{ur}\frac{\kappa_\mathrm{g}}{T^2}\left(\frac{\mathrm{d}T}{\mathrm{d}l}\right)^2 + 0.16f\frac{\kappa_\mathrm{m}}{T^2}\left(\frac{\mathrm{d}T}{\mathrm{d}l}\right)^2 + \eta\frac{1600}{Td_\mathrm{h}^2}j^2V_\mathrm{m}^2 + \frac{d_\mathrm{h}^2}{12N_\mathrm{ur}f}\frac{C_\mathrm{p}^2j^2}{\kappa_\mathrm{g}}\frac{1}{T^2}\left(\frac{\mathrm{d}T}{\mathrm{d}l}\right)^2 \tag{5}$$

with $j = \overset{*}{n}/A$ the molar flux. For the heat exchanger

$$\sigma_\mathrm{e} = N_\mathrm{ue}\frac{\kappa_\mathrm{g}}{T^2}\left(\frac{\mathrm{d}T}{\mathrm{d}l}\right)^2 + \frac{4p}{p_\mathrm{c}}\frac{\kappa_\mathrm{m}}{T^2}\left(\frac{\mathrm{d}T}{\mathrm{d}l}\right)^2 + \eta\frac{128}{Td_1^2}j^2V_\mathrm{m}^2 + \frac{d_1^2}{N_\mathrm{ue}}\frac{C_\mathrm{p}^2j^2}{\kappa_\mathrm{g}}\frac{1}{T^2}\left(\frac{\mathrm{d}T}{\mathrm{d}l}\right)^2. \tag{6}$$

The last two terms in Eqs.(5) and (6) depend on the grain size d_h and the tube diameter d_1 respectively. The coefficient of thermal conductivity and the viscosity of the gas are approximated by the relations $\kappa_\mathrm{g} = \kappa_0\sqrt{T}$ and $\eta = \eta_0\sqrt{T}$ with κ_0 and η_0 contants. The optimum values for the diameters are given by

$$d_\mathrm{h0}^4 = 3072\frac{fN_\mathrm{ur}\eta_0\kappa_0T^4}{p^2\left(\frac{\mathrm{d}T}{\mathrm{d}l}\right)^2} \tag{7}$$

$$d_{10}^4 = \frac{512}{25}\frac{N_\mathrm{ue}\eta_0\kappa_0T^4}{p^2\left(\frac{\mathrm{d}T}{\mathrm{d}l}\right)^2} \tag{8}$$

respectively. They result in minimum contributions of the flow and the heat exchange to the entropy-production rates of

$$\sigma^2_{\mathrm{rfe0}} = \frac{10000}{3}\frac{1}{N_{\mathrm{ur}}f}\frac{\eta_0}{\kappa_0}\frac{R^4 j^4}{Tp^2}\left(\frac{\mathrm{d}T}{\mathrm{d}l}\right)^2 \qquad (9)$$

$$\sigma^2_{\mathrm{efe0}} = 3200\frac{1}{N_{\mathrm{ue}}}\frac{\eta_0}{\kappa_0}\frac{R^4 j^4}{Tp^2}\left(\frac{\mathrm{d}T}{\mathrm{d}l}\right)^2. \qquad (10)$$

Note that these terms vary as T^{-1} while the heat conduction terms vary as T^{-2}, so the relative importance of the heat conduction terms increases with lower temperatures. The ratio of the optimum diameters is given by

$$\frac{d^4_{\mathrm{h0}}}{d^4_{10}} = 150\frac{fN_{\mathrm{ur}}}{N_{\mathrm{ue}}} \qquad (11)$$

and the ratio of the minimum dissipation rates is

$$\frac{\sigma^2_{\mathrm{rfe0}}}{\sigma^2_{\mathrm{efe0}}} = \frac{25}{24}\frac{N_{\mathrm{ue}}}{N_{\mathrm{ur}}f} \qquad (12)$$

so, within the limitations of the validity of our calculations, the ratios are only determined by the filling factor and the ratio between the Nusselt numbers and independent of the many possible other parameters.

NUMERICAL VALUES

The gas properties are detremined by the values for helium $\kappa_0 = 0.008$ W/K$^{3/2}$m, $\eta_0 = 1.05$ μPas/K$^{1/2}$, and $C_{\mathrm{p}} = 20.8$ J/molK. For stainless steel $\kappa_{\mathrm{m}} = 15$ W/Km and $p_{\mathrm{c}} = 250$ MPa. The Nusselt number is for the regenerator $N_{\mathrm{ur}} = 10$ and for the heat exchanger $N_{\mathrm{ue}} = 4.36$. In our calculations we use somewhat arbitrarily $p = 1.75$ MPa, $j = 100$ mol/sm^2, $f = 0.5$, $C_{\mathrm{k}} = 0.16$, $T = 200$ K, $\mathrm{d}T/\mathrm{d}l = 200$ K/m. With these values we can calculate the various contributions to the entropy production. For the regenerator

$$\sigma_{\mathrm{r}} = \left[0.56 + 1.2 + 1.07\frac{\mathrm{mm}^2}{d^2_{\mathrm{h}}} + 0.64\frac{d^2_{\mathrm{h}}}{\mathrm{mm}^2}\right]\frac{\mathrm{W}}{\mathrm{Km}^3}. \qquad (13)$$

The diameter dependence of the two last terms is similar to the one given in Fig.1. In the optimum situation $d_{\mathrm{h0}} = 1.14$ mm and $\sigma_{\mathrm{rf0}} = \sigma_{\mathrm{re0}} = 0.83$ W/Km^3. The total entropy-production rate density in the optimum situation is $\sigma_{\mathrm{r0}} = 3.43$ W/Km^3.

For the heat exchanger we get the relation

$$\sigma_{\mathrm{e}} = \left[0.49 + 0.42 + 0.086\frac{\mathrm{mm}^2}{d^2_1} + 8.7\frac{d^2_1}{\mathrm{mm}^2}\right]\frac{\mathrm{W}}{\mathrm{Km}^3}. \qquad (14)$$

The most interesting are the diameter dependent terms. The entropy-production rates of the two last terms together with the total is given in Fig.1 as a function of the tube diameter. In the optimum situation the tube diameter $d_{10} = 0.314$ mm and $\sigma_{\mathrm{ef0}} = \sigma_{\mathrm{ee0}} = 0.87$ W/Km^3. The total is $\sigma_{\mathrm{e0}} = 2.65$ W/km^3.

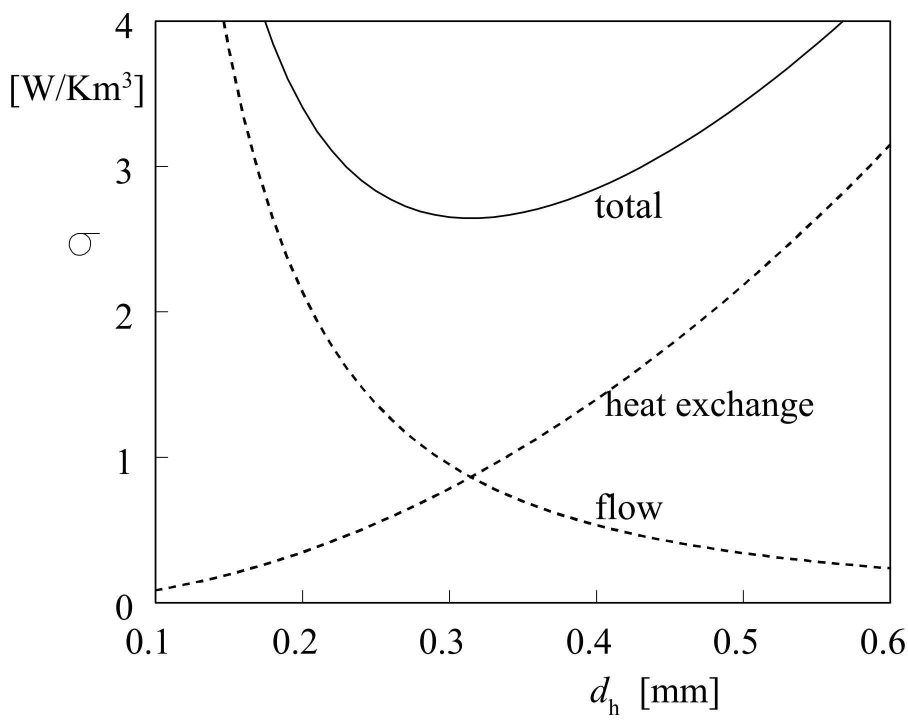

Figure 1: Entropy-production rate densities for a heat exchanger due to the flow resistance and the heat exchange between the gas and the matrix. The total shows a minimum which corresponds to the optimum performance of the regenerator.

DISCUSSION AND CONCLUSIONS

The entropy production in an optimized heat exchanger is less than in an optimized regenerator, so a PTR with a heat exchanger should work better. At temperatures much lower than 200 K the contributions from the flow resistance and the heat exchange become significantly less than the contributions due to the heat conduction in the axial directions. Therefore, one has some freedom to choose diameters which are more convenient than the small values found in the optimization procedure.

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