Study on the optimal characteristic dimension of regenerator in a thermoacoustic engine

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The purpose of this paper is to study the effect of the hydraulic radius of stacked-screen regenerators on the performance of a thermoacoustic engine. Experimental results indicate an optimal ratio of the hydraulic radius versus thermal penetration depth r_h/δ_k which leads to the lowest onset temperature difference ΔT_{onset} in the tested engine. Further more, a simple approximate method is developed to predict this optimal ratio in the regenerator in the case of a traveling-wave phase. The calculated results agree with the experimental results well.

INTRODUCTION

Regenerators are the hearts of traveling-wave thermoacoustic systems. In order to maintain good heat contact between the working gas and solid heat capacity across cross-sectional area, regenerator hydraulic radius should be smaller than thermal penetration depth. However, the further quantitative research on the regenerators in traveling-wave systems is still lacking.

In this paper, according to the network method demonstrated by Backhaus and Swift [1], a thermoacoutic Stirling engine is constructed and tested. A series of experiments have been performed to study the impact of the regenerator hydraulic radius on the onset temperature difference. The relationships between the regenerator hydraulic radius and onset temperature differences are measured and discussed. For each regenerator, the experimental results show an optimal r_h/δ_k, which leads to the lowest onset temperature difference in the tested engine, although the working gas, mean pressure, or regenerator hydraulic radius varies. On the basis of a simple approximate method developed in this paper, the calculated results agree with the experimental results well.

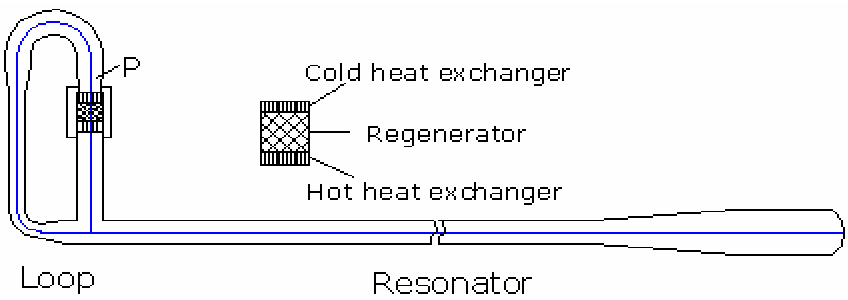

Figure 1 Schematic illustration of the tested engine.

EXPERIMENTAL APPARATUS

The tested engine is shown in Figure 1 schematically. The mean lengths of the looped tube and resonator are about 0.708 and 1.138 m respectively. One piezoelectric pressure sensor is placed above the cold heat exchanger (shown as P in Figure 1) to measure the acoustic pressure. Four thermo-couples are placed along the regenerator to measure the temperatures. The matrix of the regenerator is a pile of stainless-steel screen. The details of the tested engine can be found in our previous paper [2].

EXPERIMENTAL RESULTS

In the following experiments, the input electric power of the heater is fixed at 208 W. The onset points are measured when the mean pressure is changed from 0.1 to 2.6 MPa (absolute pressure) with a step of 0.1 MPa. For the example of 120-mesh regenerator, when the working gas is nitrogen (or helium), the operating frequency is 76 Hz (or 211 Hz). The measured relationships between ΔT_{onset} and p_m are shown in Figure 2 (a). In this figure, there is an optimal mean pressure about 0.4 MPa (or 1.1 MPa) leading to the lowest ΔT_{onset} when the working gas is nitrogen (or helium). Each measured onset point in Figure 2 (a) gives one dimensionless value of r_h/δ_k. So the curves in Figure 2 (a) can be transformed into the ones shown in Figure 2 (b). As shown in this figure, the measured optimal r_h/δ_k is about 0.24 (or 0.20) for nitrogen (or helium).

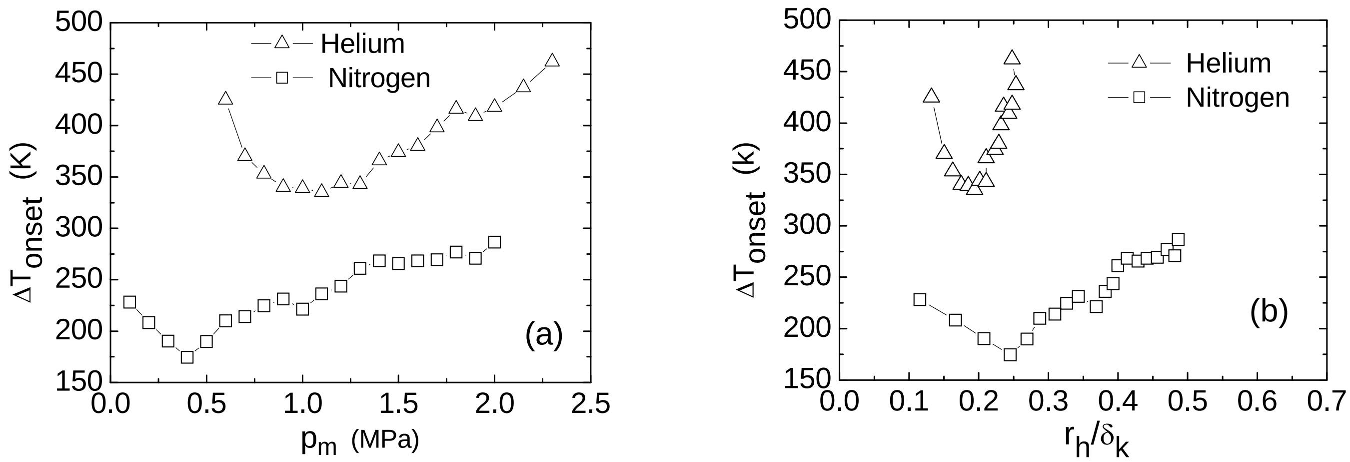

Figure 2 For 120-mesh regenerator, the relationships between ΔT_{onset} and p_m (a). The relationships between ΔT_{onset} and r_h/δ_k (b). The lines are guide for eyes.

By the same means, for nitrogen, the optimal p_m and r_h/δ_k of the tested engine are measured and shown in Table 1 when the mesh number of the stainless-steel screen in the regenerator varies. From Table 1, it can be realized that there is a measured optimal r_h/δ_k about 0.17 - 0.25 for the tested engine.

On the other hand, the mean pressure is fixed at 0.6 MPa. But, the hydraulic radius of the regenerator is varied by changing the mesh number of the stacked screen. The measured relationship between onset temperature difference and hydraulic radius is shown in Figure 3. It is indicated that 200-mesh regenerator is optimal at this mean pressure. The corresponding r_h/δ_k is about 0.20. Therefore, in this engine, there is an optimal $r_h/\delta_k \approx (0.17 \sim 0.25)$ which leads to the lowest ΔT_{onset}. Note that, this optimal r_h/δ_k is obtained by changing the mean pressure when the hydraulic radius is fixed, and vice versa.

Table 1 The measured optimal p_m and r_h/δ_k when r_h varies

Mesh	80	120	150	200	250	300
r_h (μm)	71.1	47.2	37.7	27.2	22.3	18.2
$p_{m,opt}$ (MPa)	0.26	0.4	0.6	0.6	0.8	1.6
$(r_h/\delta_k)_{opt}$	0.25	0.24	0.22	0.20	0.17	0.21

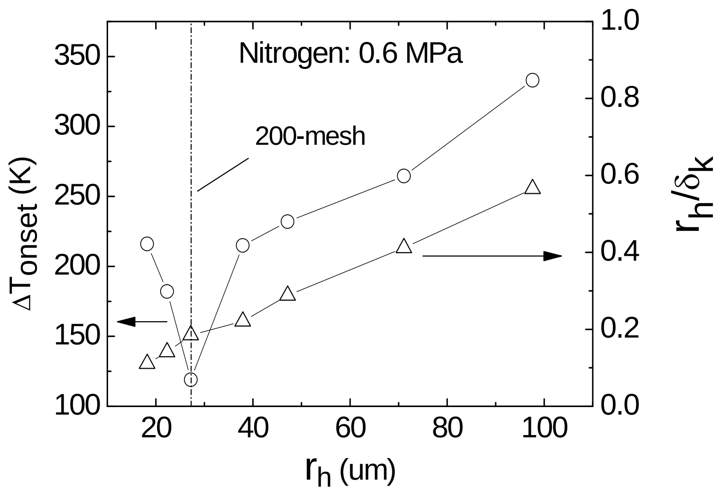

Figure 3 ΔT_{onset} (circles) and r_h/δ_k (triangles) changes as the regenerator hydraulic radius increases.

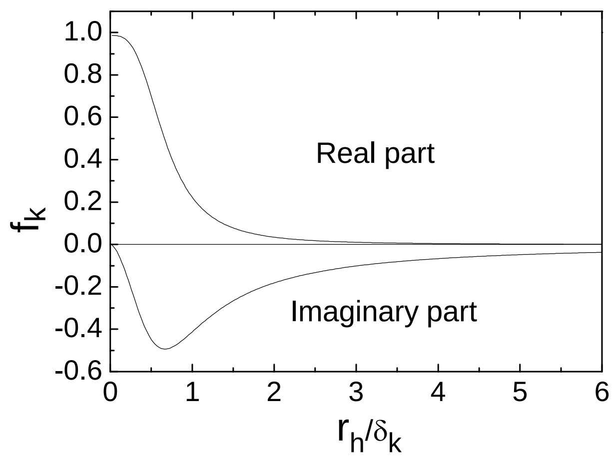

Figure 4 Spatial-average function f_k for a stacked-screen generator.

CALCULATION AND DISCUSSION

According to the harmonic analysis of regenerators by Backhaus and Swift [3], we obtain the equivalent spatial-average function f_k for stacked-screen regenerators (shown in Figure 4). Note that, this equivalent function is only valid for small r_h/δ_k. It helps to understand the above optimal r_h/δ_k, which is a compromise between the viscous and thermal-relaxation effect in the regenerator. On the other hand, in regenerator, the time-averaged acoustic power $d\dot{E}$ produced in a length dx is defined by

$$\frac{d\dot{E}_2}{dx} = \frac{1}{2}\mathrm{Re}\left[\tilde{U}_1\frac{dp_1}{dx} + \tilde{p}_1\frac{dU_1}{dx}\right], \tag{1}$$

with p_1: oscillating pressure, U_1: volume velocity. Letting the right hand side of equation (1) to zero, one can obtain an approximate relation for critical temperature difference [3]. For stacked-screen regenerators, the details of dp_1/dx and du_1/dx can be obtained in Ref. [3]. Substituting them into equation (1) and letting its right hand side to zero yield the following relation

$$\beta_m \frac{dT_m}{dx}\mathrm{Re}\left[\left(1-\frac{\varepsilon_s+\left(g_c-g_v\right)\varepsilon_h}{1+\varepsilon_s+\left(g_c+e^{2i\theta_T}g_v\right)\varepsilon_h}\right)\tilde{p}_1U_1\right]-\frac{\mu_m}{\phi A r_h^2}\left[\frac{c_1(\phi)}{8}+\frac{c_2(\phi)Re_1}{3\pi}\right]\left|U_1\right|^2$$
$$-\frac{\omega\phi A}{p_m}\mathrm{Im}\left[1-\beta_m^2T_m^2\frac{\gamma_m-1}{\gamma_m}\frac{\varepsilon_s+\left(g_c+e^{2i\theta_p}g_v\right)\varepsilon_h}{1+\varepsilon_s+\left(g_c+e^{2i\theta_T}g_v\right)\varepsilon_h}\right]\left|p_1\right|^2=0 \quad . \qquad (2)$$

According to the research by Swift [1], it can be assumed that $\left|p_1/U_1\right|\approx\left(15\sim30\right)\rho_m c/\phi A$ in the regenerator. Furthermore, a traveling-wave phase ($\mathrm{Im}\left[\tilde{p}_1U_1\right]=0$) is presumed. Then, we obtain the optimal $r_h/\delta_k\approx(0.18\sim0.26)$ for the tested system. The calculated results agree with the measured ones well.

CONCLUSION

In the tested engine, there is an optimal r_h/δ_k leading to the lowest ΔT_{onset}, which is a compromise between the viscous and thermal-relaxation effect in the regenerator. The experimental results show that the optimal r_h/δ_k is about 0.17 to 0.25, although the working gas, mean pressure, or regenerator hydraulic radius varies. Furthermore, according to an approximate method developed in this paper, the calculated result is about 0.18 to 0.26, which agrees with the experimental ones well.

ACKNOWLEDGEMENT

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