Basic operation principle of a novel cascade thermoacoustic prime mover

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This paper presents an analysis for basic operation principle of the cascade thermoacoustic engine. Based on the analysis, the paper points out that an efficient cascade thermoacoustic system can be realized by using some special features of a half-wavelength, standing-wave system in which there is usually a high impedance zone with a traveling-wave component being predominant, and has no closed-loop topology. Thus, placing a regenerator-based engine in this zone as a second-stage engine can realize a more efficient thermoacoustic process than a single-stage standing-wave engine. In addition, there is no DC-flow problem due to without a closed-loop topology in the cascade system. Finally, the in-line system can avoid imbalanced thermal stress existing in the thermoacoustic Stirling engines, thus simplifying mechanical consideration and improving lifetime.

INTRODUCTION

Thermoacoustic machines are new kinds of environment-friendly devices. Thermoacoustic prime movers can be classified into two categories, standing wave and traveling-wave movers. In principle, the standing-wave prime mover relies on an intrinsically irreversible thermodynamic process, the so-called medium heat transfer process, consequently, its highest efficiency is bounded to about 60% of the ideal Carnot cycle [1]. On the other hand, the thermoacoustic Stirling prime mover can theoretically achieve the efficiency of the ideal Carnot cycle if heat transfer process is perfect [2]. However, almost any real thermodynamic process is accompanied by various irreversibilities. In the thermoacoustic prime movers, the irreversibilities resulting from imperfect heat transfer and viscous friction are two main ones that usually affect each other. In a one-wavelength, pure traveling-wave prime mover, the in-phase relation between the pressure and velocity is indeed beneficial to the conversion of heat and acoustical power, so is the good heat transfer between the gas and solid matrix of the regenerator of the prime mover. However, the velocity amplitude is too large in the pure traveling-wave system and thus produces significant viscous loss.

The thermoacoustic Stirling prime mover that was developed by Los Alamos Laboratory and other laboratories, can provide a predominantly traveling-wave acoustical field yet with small velocity by making using of the special feature of standing-wave filed and designing a lumped-parameter traveling-wave loop [3]. The demonstrated prime mover achieved the efficiency of about 30% from heat to acoustical power, which can be competitive to modern internal combustion engine. However, the thermoacoustic Stirling engine may exist a significant DC-flow loss and also has imbalanced thermal stresses. The DC-flow loss greatly decreases the efficiency of the looped system and the imbalanced

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thermal stress decreases reliability of the system. Recently, a concept of cascaded thermoacoustic systems was proposed [4], but there is little work reported on its basic operation principle. Thus, the paper tried to present an explanation for fundamental mechanism of the cascaded thermoacoustic systems. Firstly, special features of a half-wavelength, standing-wave system were analyzed. Then, by analyzing the production rate of local acoustical power, the operation principle of the cascaded thermoacoustic engine was described.

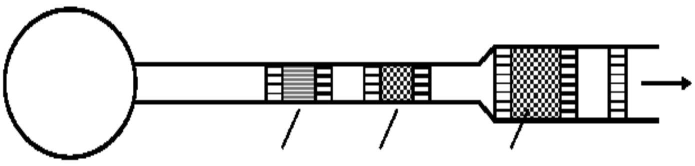

Figure 1. Schematic for one kind of cascade thermoacoustic prime mover [4]
(1. Standing-wave engine 2. First traveling-wave engine 3. Second traveling-wave engine)

MODEL AND ANALYSIS

To understand basic operating principle of the cascade thermoacoustic engine, let us first examine the standing-wave systems shown in Figure 1. Under linear acoustical assumption, the expressions for pressure and velocity distributions $\tilde{p}(x)$ and $\tilde{u}(x)$ can be given in Eq.(1). In addition, Eq.(2) gives all eigenvalue frequencies of the acoustical system.

$$\tilde{p}(x) = i2C_1 \sin(kx);\ \tilde{u}(x) = -\frac{1}{\rho_0 a_0} 2C_1 \cos(kx) \qquad (k = a_0/\omega\text{, wave number}) \tag{1}$$

$$f = \frac{na_0}{2l}, (n = 0,1,2,...) \qquad (a_0\text{, sound velocity}) \tag{2}$$

For fundamental oscillation mode, the expressions of the pressure and velocity distributions are given as follows,

$$\tilde{p}(x) = i2C_1 \sin(\pi x / l);\ \tilde{u}(x) = -\frac{1}{\rho_0 a_0} 2C_1 \cos(\pi x / l) \tag{3}$$

Furthermore, the local acoustical impedance can be obtained as follows,

$$\frac{\tilde{p}(x)}{\tilde{u}(x)} = -i\rho_0 a_0 \sin(\pi x / l) / \cos(\pi x / l) \tag{4}$$

Examining the local acoustical impedance can know the phase angle between the oscillating pressure and velocity. In terms of thermoacoustics, the larger the amplitude of the acoustical impedance is, the less viscous loss the thermoacoustic engine has. Actually, the local production rate of acoustical power is a key parameter to evaluate if the thermoacoustic engine can deliver net acoustical power or not. The local production rate of acoustical power is given by the following equation,

$$\frac{dW_x}{dx} = \frac{1}{2}\mathrm{Re}[\tilde{U}(x)\tilde{p}^*(x) f_{WT}]\beta_0 \frac{dT_0}{dx} - \frac{1}{2}\frac{(\gamma - 1)A}{\rho_0 a_0^2}|\tilde{p}(x)|^2 g_{wK} - \frac{1}{2}\frac{\rho_0}{A}\omega|\tilde{U}(x)|^2 g_{w\mu} \tag{5}$$

It should be noticed that only the first item in Eq.(5) can be positive because the other two items denote power losses by imperfect heat transfer and viscous losses. Obviously, the first item is also strongly determined by various factors such as the temperature gradient and direction, the amplitudes and relative phases of pressure and velocity, and the power-production factor f_{WT}, etc.

Obviously, the acoustical field of the thermoacoustic engine is extremely important for thermoacoustic processes. For a standing-wave acoustical field, it is more appropriate to use a so-called "stack" as solid medium; contrariwise, a traveling-wave acoustical field requires a so-called "regenerator". In figure 1, for the fundamental oscillation mode of the half-wavelength standing-wave system, the left and right zones are dominated by standing-wave components whereas in the middle zone a traveling-wave component predominantly exists for the real system. Thus, if designing a first-stage standing-wave engine in the left zone and a second-stage traveling-wave engine in the middle zone, such a cascade thermoacoustic system should be more efficient than a single-stage standing-wave engine. In addition, adjacent zone to the middle zone has larger pressure and smaller velocity, so the first-stage standing-wave should not be far from the middle zone where the second-stage traveling-wave is located.

Now let us consider what temperature-gradient direction should be imposed on the two-stage engine. Assuming the propagation direction of the wave is from left to right. From Eq.(4), it can be shown that the pressure lags the velocity in the left of the middle location while leading in the right of the middle location. Roughly speaking, in the far left side, the pressure lags the velocity by about 90˚; in the middle zone, the lagging phase varies from about -90˚ to about 90˚; in the far right side, the pressure leads the velocity by 90˚. Thus, there are the following approximate expressions for the pressure distributions in the different zones.

In the far left-side, the pressure distributions of the real cascade thermoacoustic system can be expressed approximately by Eq.(6) to Eq.(8), respectively

$$\widetilde{p}(x) = (a - bi)\widetilde{U}(x), (a, b > 0, and\ b >> a) \tag{6}$$

$$\widetilde{p}(x) = (a - bi)\widetilde{U}(x), (a, b > 0, and\ a \approx b) \tag{7}$$

$$\widetilde{p}(x) = a\widetilde{U}(x), (a > 0, and\ b \approx 0). \tag{8}$$

In addition, the power-production factor f_{WT} is given by the following equation,

$$f_{WT} = c - id, (c, d > 0) \tag{9}$$

For a stack-based solid medium, $c \approx d \approx 0.5$; for a regenerator-based solid medium, $c >> d$, $c \approx 1$, $d \approx 0$.

As mentioned above, it is better to design a stack-based, standing-wave engine in the far left-side zone. Thus, when neglecting the two dissipation items of acoustical power, the production rate of acoustical power can be simplified as follows

$$\sum w'_{s\tan d} = \frac{1}{2}\mathrm{Re}[(a + bi)(c - di)] \,|\widetilde{U}(x)|^2\, \beta_0 \frac{dT_0}{dx} \approx \frac{1}{2} bd \,|\widetilde{U}(x)|^2\, \beta_0 \frac{dT_0}{dx}\ . \tag{10}$$

To make the power-production rate be positive, it is necessary to impose a positive temperature gradient dT_0 / dx. Moreover, substituting the pressure and velocity distributions into Eq.(10) yields the following analytical expression for the power-production rate

$$\sum w = \frac{C_1^2 A \sin(2\pi x / l)}{\rho_0 a_0} \beta_0 \frac{dT_0}{dx}. \tag{11}$$

It can be found from Eq.(11) that the local power-production rate reaches its maximum value if the stack is placed one-fourth distance from the middle point.

By making a similar analysis, we can obtain the following expression for the local power-production rate if a regenerator is placed in the middle zone

$$\sum w'_{travel} = \frac{1}{2}\mathrm{Re}[a(c-di)]\,|U(x)|^2\,\beta_0\frac{dT_0}{dx} \approx \frac{1}{2}ac\,|U(x)|^2\,\beta_0\frac{dT_0}{dx}. \tag{12}$$

To make the power-production rate be positive, it is also necessary to impose a positive temperature gradient dT_0/dx. Because $b<a$ (in the middle point, $\tilde{p}(x)/\tilde{u}(x)$ reaches its maximum) and $c>d$, $\sum w'_{travel} > \sum w'_{s\tan d}$, which implies the second-stage traveling-wave engine is more efficient than the first-stage standing-wave engine. Moreover, the second-stage traveling-wave engine has small viscous loss and no DC-flow loss. Thus, the cascade thermoacoustic system would be more efficient than a single-stage standing-wave thermoacoustic engine. This is just the basic operation principle of the cascade thermoacoustic system.

According to the above analysis, it is also possible to design other topology of efficient cascade thermoacoustic system, which was described in the literature [5].

CONCLUSION

The basic operation principle was presented in this paper. The analysis shows that any efficient thermoacoustic system has these requirements: high impedance, traveling-wave acoustical field, and non-looped flow passage. The high impedance can minimize the viscous loss by velocity. The traveling-wave acoustical field can maximize the conversion between heat and acoustic power. And, the non-looped flow passage can avoid the DC-flow loss. The proposed, cascaded thermoacoustic systems can satisfy these requirements, therefore having the potential for high efficiency. In addition, the cascaded thermoacoustic systems do not exist imbalanced thermal stress; therefore, they are more reliable.

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