Analysis of inertance tube in the pulse tube refrigeratior

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Refrigeration effect in pulse tube coolers can be explained in terms of phase shift between oscillations of pressure and mass flow rate at the cold end of the pulse tube. In this paper a linear ordinary differential equation of the second order with variable coefficient has been developed to describe the time-dependent phase shifting effect due to variable frictional coefficient. The comparison between integral average value of the transient parameter on a cycle and the experimental and published data has been conducted. It reveals that different inertance tubes have their own optimal working conditions, and the inertance tube with small internal diameter is more suitable to work under high frequency conditions.

INTRODUCTION

Aiming to improve the performance of pulse tube refrigerator (PTR) further, especially for high frequency operation or large refrigerating capacity cryocooler, the intertance tube has been introduced as a phaser in recent years. Until now there are many different theoretic methods which all focus on the derivation of the impedance of intertance tube based on the thermo electrical analogy. The difference between them is the way to obtain the inductive, capacitive and resistive reactance of interance tube. In general, there are two different ways to derivate the impedance. The first one is that based on the navier-stokes equations of working gas, the nonlinear inertia term has firstly been neglected [1], as a result the gas velocity of the one-dimensional flow in the tube can be derived easily; The other is that based on the analysis of the volume element of gas in the tube, the governing equations have been set up in which the pressure gradient in the inertance tube is taken to be the sum of a resistance term and an inertance term [2]. Then this one dimension equation can be solved. In this paper, a new theoretical model has firstly been developed which reveals the transient oscillatory phase shifting characteristic of inertance tube as well as reservoir. It has been simplified to a linear ordinary differential equation of the second order with variable coefficient. Actually the distributed parameter model has obtained all of relations of physical parameters. Then integrating these equations has got lumped coefficient. In this model the attenuation of pressure amplitude along the longitudinal direction has been involved. Furthermore the fraction coefficient is expressed the function of position, time and geometry size. Finally Comparison has been done to verify this model and some useful characteristics have also been found.

MATHEMATIC MODELS

Mathematical model of intertance tube

Considering a hydrodynamically fully developed reciprocating flow in a pipe, the governing conservation equations of mass and momentum for an incompressible fully developed flow is

$$\begin{aligned} &\frac{\partial u}{\partial x} = 0 \\ &\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) \end{aligned} \qquad (1)$$

with x and r: axial and radial coordinates, u : axial velocity, p: pressure, ρ : density of gas, ν : kinematic viscosity of gas.

It is assumed that the reciprocating flow is driven by a sinusoidal varying pressure gradient in the hot end of the pulse tube. From the published experimental data [3], it is clear that the frequency and phase angle of the pressure wave do not vary,however its amplitude attenuates along the tube. So we assume that the attenuation of pressure amplitude is linear along the tube, the boundary conditions are given by

$$p(x) = a + b \cdot x \,,\ \ x = 0,\ p(0) = p_{hotend} \sin \omega t \ \ , x = l,\ p(l) = p_{reservior} \sin \omega t \tag{2}$$

with p : amplitude of pressure, l :length of inertance tube, ω : frequency of pressure. The coefficients a and b can be got by substituting the boundary conditions into the expression p (x).

An exact solution for the axial velocity profile of a fully developed reciprocating flow in a round tube has been obtained by T.S.Zhao [4]. The velocity distribution (*u*) is given as

$$u = \frac{kD^2}{4\alpha^2 \nu}[B \cos \omega t + (1 - A) \sin \omega t] \tag{3}$$

Then integrating the equation (3), the average velocity (u_m) on the cross section of the tube can be obtained. According to the definition of shearing stress at the wall we get the friction force (τ_w)between the gas and the wall. By integrating u_m and τ_w along the tube, the average friction force is

$$F_1 = \int_0^l \tau_w \cdot 2\pi R dx = \pi R \frac{32 F_w}{A_0} \sin(\phi + \phi_l) \frac{D^4 \sigma^2}{(32 \nu)^2} \int_0^l p^2(x) dx \tag{4}$$

$$\bar{u}_m = \frac{1}{l}\int_0^l \frac{kD^2\sigma}{32\nu} \sin \phi dx = \frac{1}{l} \cdot \frac{D^2\sigma}{32\nu} \sin \phi \int_0^l p(x) dx \tag{5}$$

The relation between F_1 and $\bar{u}_m$ is given as

$$F_1 = \pi R \frac{32 F_w}{A_0} \sin(\phi + \phi_l) \frac{D^2\sigma}{32\nu} \cdot \frac{1}{\sin \phi} \cdot \frac{\int_0^l p^2(x) dx}{\int_0^l p(x) dx} \cdot \bar{u}_m \tag{6}$$

with A , B, F_w and σ : expressions of Bessel function, ϕ : phase difference, D internal diameter of pipe, A_0: internal cross-sectional area of pipe.

Oscillatory flow is a very complex phenomenon. Until now the theory on laminar oscillatory flows has almost been established, with less unknowns in contrast with the theory of transitional and turbulent oscillatory flows [5]. In fact the flow pattern in the inertance tube is turbulent in most cases, because the velocity of the gas in the tube reaches 20-25m/s [3], in this case Re number of working gas is above 10000. So some equations [6] have been adopted to correct the coefficient of friction.

<u>Mathematical model of gas reservoir</u>

The real physical process happening in the gas reservoir is that gas is to be charged and exhausted periodically. From the viewpoint of thermodynamics this process is complex, because it can be influenced by many different thermodynamic and geometry parameters. It is found that the geometry dimension of the gas reservoir is smaller than that of the wavelength of the gas wave, as a result the parameters of the gas in the reservoir can be regarded as a lump parameter. In some degree it acts as the gas spring, the spring force can be imposed on the outlet section of reservoir.

The first law of thermodynamics, perfect gas and mass conservation equation are used to describe the process of gas in the reservoir. Solving those equations, the expression of the pressure imposed on the terminal of the inertance tube by the gas reservoir is given by

$$\mathrm{d}F = S\mathrm{d}p = S\left(\frac{\mathrm{d}Q}{C_v V} + k\frac{T_{in}}{V}\mathrm{d}m_{in}\right) \tag{7}$$

The phase shifter including inertance tube and gas reservoir works together in the vacuum chamber, so it is reasonable to assume that the gases in the reservoir undergo adiabatic process. Meanwhile, due to the frictional heat, the gases in the inertance tube undergoes polytropic process, so the expression of pressure (F_2) is finally given

$$F_2 = \frac{k}{V} T_{hotend} \left(\frac{a + bP_{hotend}}{P_{hotend}}\right)^{\frac{n-1}{n}} \rho A_0 \cdot x \tag{8}$$

with V : volume of reservoir, k: adiabatic coefficient, n: polytropic coefficient.

Mathematical model of phase shifter

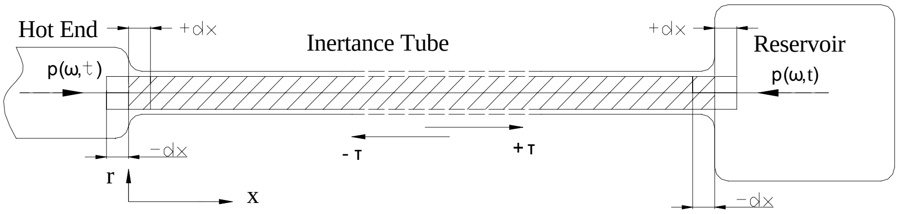

Figure 1 Phase shifter including inertance tube and gas reservior

From published experimental data [3], it is found that the length of the inertance tube has the same dimension as the wavelength of gas parameters. So according to the acoustic theory, it is reasonable to regard the mass in the tube as a lumped mass. A linear ordinary differential equation of the second order with variable coefficient has been derived to describe the physical process in the phase shifter (see Figure 1).

$$\rho A_0 l \frac{d^2 x}{dt^2} + f_1(t, \omega) \frac{dx}{dt} + f_2(t_r, V_r) x = pA_0 \sin \omega t \tag{9}$$

The three terms of at the left-hand of the equation represent the inertia force, friction resistance and the spring force of the gas reservoir respectively. The right-hand term describes the sinusoidal exciting force. We can find that both the friction resistance and the spring force vary with the time and frequency, which is different from the previous theoretical model in which the friction force is regarded as independent of the time.

VERIFICATION

The method of electrical analogy has been proved as an effective way to evaluate the characteristic of a phase shifter. From the viewpoint of it, the resistor of resistance, R, the inductor of inductance, L, and the capacitor of capacitance, C, deduced from the equation (9), are $f_1(t, \omega)/A_0$, ρ l and $1/f_2(t_r, V_r)A_0$ respectively. Those expressions reveal the time-dependent phase shifter effect, which has been ignored previously because more attention has been paid to the average effect in a cycle. In order to verify this model, we firstly calculate the phase shifting characteristic of the IPTR [2], the calculating results agree with the experimental data. Then the further analyses have been conducted to try to get the performance of inertance tubes with different geometry size and under different working conditions. Their dimensions and working condition are shown in Table 1 respectively.

Table 1 Calculation conditions of inertance tube

NO	Dimensions	Working conditions				
	Length×Radius(mm)	Frequency (Hz)	Temperature in hot end (K)	Charge pressure (MPa)	Reservoir (cm^3)	Press ratio
1	1000×0.5	30~70	303	2.5	59	1.17
2	2000×1.0	30~70	303	2.5	59	1.15
3	3000×2.0	30~70	303	2.5	59	1.10

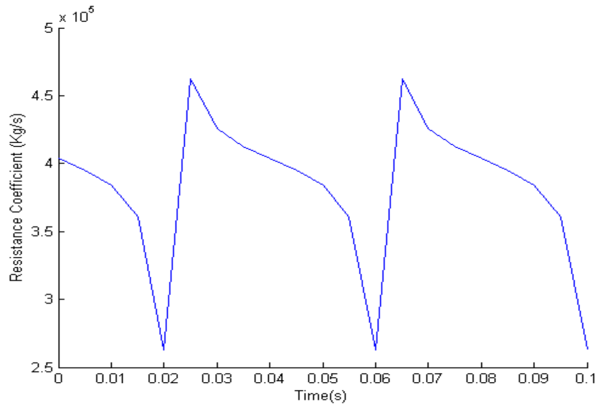

Figure 4 Resistance coefficient as a function of time

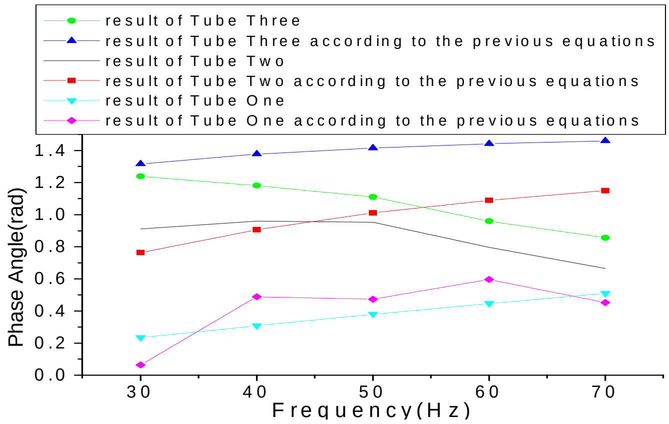

Figure 5 Shifting angle vs frequency for inertance tube with different geometry size

Figure 4 shows the relationship between the coefficient of resistance and the time for the case of tube one (see Table 1) working under 50Hz.We find that the period of resistance coefficient is twice as large as the pressure wave. Meanwhile this coefficient is a strong nonlinear function of the time. It also reveals that there exists an asymmetry in the behavior of the inertance tube in spite of their symmetric structure. In practice, it is undeniable that this effect is very difficult to be observed directly by means of experimental measures because the response time of the testing instruments is always longer than that of the periodical parameters. So by integrating the periodical parameter on a cycle, the average effect of the phase shifting is obtained. In contrast to the common idea about the phase shifting effect [7], according to which the phase shifting angle is directly proportional to the working frequency, there exists an optimal working frequency about a corresponding geometry size (see Figure 5), which is in agreement with some experimental data [2]. In general, it is found that different inertance tubes have their own optimal working conditions, and the inertance with small internal diameter is more suitable to work under high frequency conditions.

CONCLUSION

In this paper a model has been developed to describe the time-dependent phase shifting characteristic of an inertance tube. This model has been simplified to a linear ordinary differential equation of the second order with variable coefficient. In this model the resistance coefficient of the inertance tube is expressed as a function of time, frequency and some other parameters, which are deduced from momentum and mass differential equations of the working gas. Consequently, the phase shifting angle provided by the interance tube also varies with the time. By integrating, the average phase angle on a cycle agrees generally with the published result. It is also found that different inertance tube have their own optimal working conditions, and the inertance with small internal diameter is more suitable to work under high frequency conditions.

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