Normalized representation for steady state heat transport in a channel containing He II covering pressure range up to 1.5 MPa

Sato A., Maeda M., Yuyama M. and Kamioka Y. *

TML, NIMS, 3-13 Tsukuba, Ibaraki, 305-0003, Japan
*Taiyo Toyo Sanso Co. Ltd., 3-1-1 Kyobashi, Chuo-ku, Tokyo, 104-0031, Japan

Steady state heat transport along a channel containing He II was measured up to 1.5 MPa. The thermal conductivity functions (TCF) were determined letting exponent *m* in the Gorter-Mellink equation equal to 3.4. The result was represented uniquely by using a normalized TCF. It has a good agreement with the Bon Mardion's heat conductivity function at 0.1 MPa.

INTRODUCTION

The turbulent heat transport in a channel containing He II is described by

$$dT/dx = f(T, P)\, q^{m}\ , \qquad (1)$$

where $f(T, P)$ is a function of He II properties, temperature and pressure, and m is a numerical coefficient. The quantity $f^{-1}(T)$ is called the thermal conductivity function (TCF) . Regarding the m-value, theory indicates that m should be equal to 3 [1]. Van Sciver analyzed the TCF letting m = 3 on the basis of the experiments up to 0.23 MPa [2, 3, 4]. The commercially based data base "HEPAK" also uses exponent m of 3 [5]. On the other hand, Bon Mardion represented his data by using m = 3.4 [6, 7].

It is of interest to know which exponent m is proper for the representation of the heat transport in He II. Therefore, we undertook determining the TCF in a wide range of pressure from saturated pressure to 1.5 MPa. Temperature gradients along the channel containing He II were measured in the temperature range from 1.4 K to 2.1 K [8]. Furthermore, we found that the experimental data deviation from the TCF had a minimum when assuming m was equal to 3.4 [9]. In the process of measurement error estimation, we recognized that the largest error came from the estimation of the inner diameter of the tube. Therefore, we measured the diameter precisely and checked the data again in order to get a universal representation of the thermal conductivity functions.

MEASUREMENT APPARATUS

A test channel was formed in a stainless steel tube with 6 mm or 10 mm inner diameters and 0.5 mm wall thickness. The length of the test section was 19 cm or 10 cm. The inner diameters were measured precisely at several locations along the tube using a three point internal micrometer. The effective cross-sectional areas were estimated considering the size of thermometers and the lead wires, and the thermal shrinkage when cooled at 2 K. The estimated results are shown in Table 1 with the sample tube dimensions.

A test vessel including the channel was immersed in a saturated superfluid helium bath for cooling and was pressurized through a filling tube up to 1.5 MPa [8]. Pressure in the test vessel was measured *in situ* using a piezo-resistive pressure sensor, FPS51B by Fujikura Ltd. [10].

A heater producing heat flux is at the bottom of the channel. The upper end of the channel is open to the subcooled helium bath. Thermometers are located along the channel to measure temperature profiles in the channel. The distance between the thermometers was 20 mm in channels No.1 and

No.2 and 10 mm in the channel No.3. Ruthenium oxide resistors were used as thermometers. The bath temperature is measured using a germanium resistor thermometer. Accuracy of the temperature measurement is within 3 mK. Concerning the detail of the measurement, refer the previous report [8, 9].

Table 1 Estimated cross-sectional area of the channel at 2 K

	Cross-sectional area	Length	Diameter
No. 1	0.225 ± 0.002 cm^2	19 cm	6 mm
No. 2	0.736 ± 0.004 cm^2	19 cm	10 mm
No. 3	0.225 ± 0.002 cm^2	10 cm	6 mm

DATA ANALYSIS

Temperature profiles along the channel were measured in the presence of heat flux at temperatures from 1.4 to 2.1 K. The series of temperature profile data is the same as reported in the previous paper [9]. Heat flux data were checked according to the precise measurement of the cross-sectional area of the channel. The error in determining the temperature gradient mainly depends on the location of the channel where the gradient is determined. Note that the error at the end of the channel is large. Figure 1 shows the measured temperature gradient versus heat flux density for pressurized He II at 0.1 MPa and 1.71 K, compared with the predicted lines by Van Sciver (m = 3). The probable errors are indicated as error bars for the channel No.2. The errors vary from 1.4 to 2 % in the heat flux range from 10^4 to 2×10^4 W/m^2. They are 1.6 - 3.5 % in the higher heat flux region from 2×10^4 to 3×10^4 W/m^2. The straight line indicates the fitted result using the least squares method assuming m is equal to 3.4. The slope of our results suggests that the exponent m should be 3.4.

We checked the exponent m with another method used in the previous work [9]. The data of temperature gradient calculated from the newly estimated heat flux densities were fitted with a TCF by using the least squares method for a certain value of m. The experimental data deviation from the fitted TCF changes depending on m. The exponent m giving the minimum deviation is considered to be the most suitable exponent m. Actually, a series of temperature profile in the same pressure is expressed smoothly in the TCF when it is determined with exponent m giving the minimum deviation. The result at 0.1 MPa is shown in Figure 2, which indicates the exponent m is approximately 3.4 considering the estimation error. Almost the same results were found at the other pressures.

Based on the above analyses, the TCF was determined by using the exponent m of 3.4.

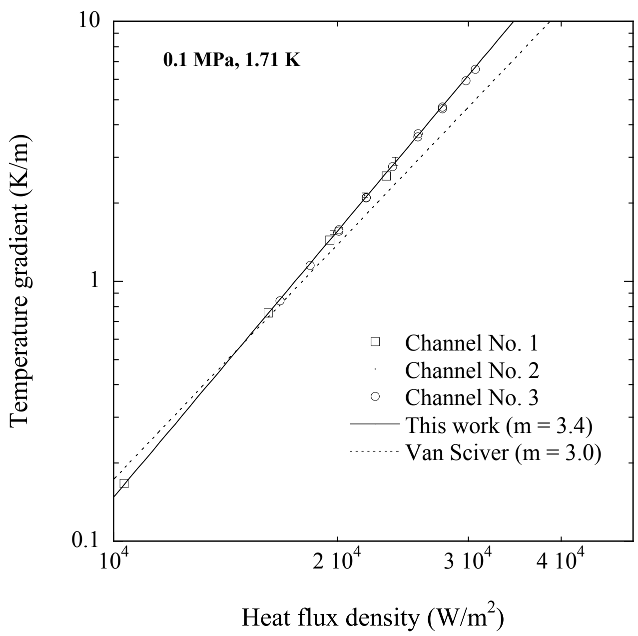

Figure 1 Temperature gradient versus heat flux in pressurized He II at 0.1 MPa and 1.71 K, compared with the predicted line by Van Sciver (m = 3).

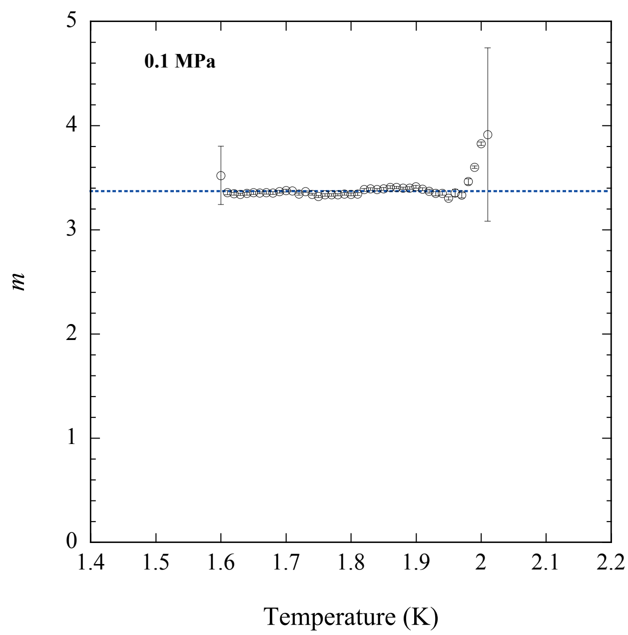

Figure 2 Temperature dependence of the exponent m with the minimum deviation of the experimental data at 0.1 MPa.

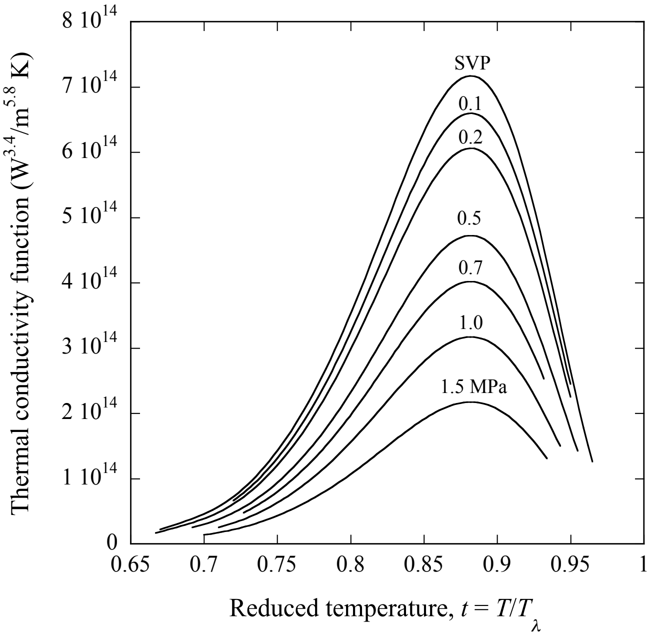

Figure 3 Reduced Temperature dependence of the thermal conductivity function at various pressures.

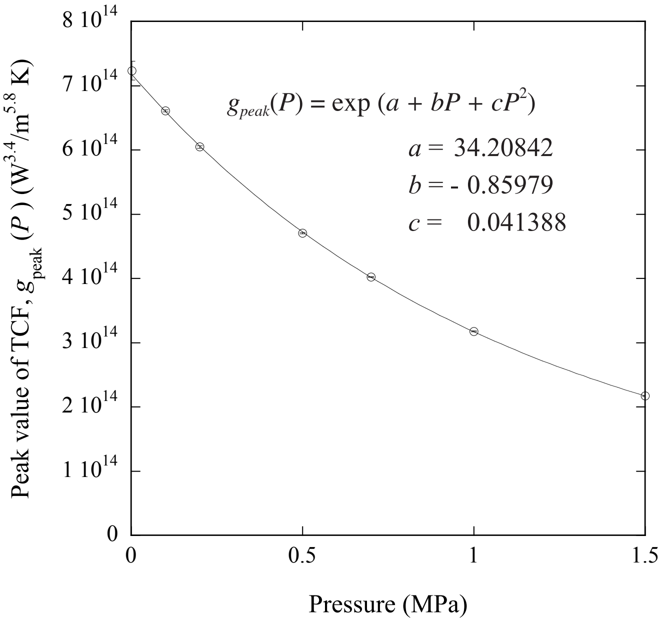

Figure 4 Pressure dependence of the peak value of the thermal conductivity function.

RENORMALIZATION OF HEAT CONDUCTIVITY FUNCTION

Plotted as smooth lines in Figure 3 is the thermal conductivity functions as they depend on reduced temperature, $t = T/T_\lambda$. The function $f^{-1}(T, P)$ is rewritten as a function of reduced temperature t as

$$f^{-1}(T, P) = g(t, P) \ . \tag{2}$$

Peak values occur at $t = 0.882$ which does not depend on pressure. Figure 4 shows the pressure dependence of the peak values. This function is fitted to the equation

$$g_{peak}(P) = \exp(a + bP + cP^2) \ . \tag{3}$$

The functions $g(t, P)$ in equation (2) are normalized at various pressures by the peak value g_{peak} as

$$h(t, P) = g(t, P) \, / \, g_{peak}(P) \ . \tag{4}$$

The normalized thermal conductivity functon, $h(t)$, is shown in Figure 5. All data appear to be expressed as a unique function which does not depend on pressure. Therefore, $h(t, P)$ is expressed as $h(t)$. This function is fitted to the equation

$$h(t) = 1 + (t - 0.882)^2 \sum_{n=0}^{9} \{a_n (t-1)^n\} \ , \tag{5}$$

where

$a_0 = -(0.118)^{-2}$, $a_1 = 1.2172617 \times 10^3$,
$a_2 = -1.4992321 \times 10^4$, $a_3 = -3.9491398 \times 10^5$,
$a_4 = -2.9716249 \times 10^6$, $a_5 = -1.2716045 \times 10^7$,
$a_6 = -3.8519949 \times 10^7$, $a_7 = -8.6644230 \times 10^7$,
$a_8 = -1.2501488 \times 10^8$, $a_9 = -8.1273591 \times 10^7$.

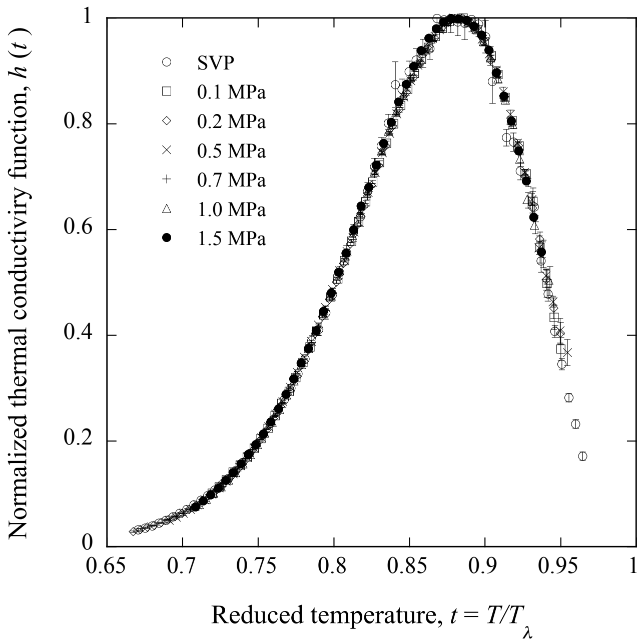

Figure 5 Normalized thermal conductivity functions at various pressures. Peak value appears at $t = 0.882$.

As a result, the heat transport characteristics are represented uniquely by using the normalized TCF as

$$dT/dx = q^{3.4} / g_{peak}(P) \cdot h(t) \qquad (6)$$

Figure 6 shows the thermal conductivity function at 0.1 MPa compared with the Bon Mardion's result . The line calculated from the normalized TCF, $h(t)$, explains the Bon Mardion's data very well.

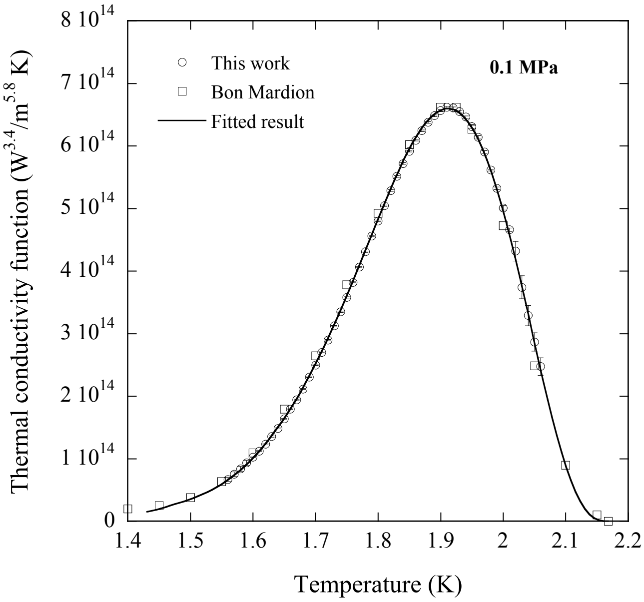

Figure 6 Comparison of the thermal conductivity function at 0.1 MPa with Bon Maridon's results. The straight line is calculated from the normalized TCF, $h(t)$.

SUMMARY

Steady state heat transport through He II in wide channels was investigated up to 1.5 MPa, The exponent in the Gorter-Mellink equation is determined to be 3.4 from the data analysis. The thermal conductivity functions were determined by using m = 3.4. The results in a wide range of pressure from saturated pressure to 1.5 MPa were represented uniquely by using the normalized TCF. Agreement with the Bon Maridon's result is quite good. This normalized equation will be used practically for the design of superfluid equipments. The physical meaning of m = 3.4 needs to be investigated in a future.

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