Dimensional analysis on multi-component mixed-refrigerants flow characteristics in adiabatic capillary tube

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In multi-component mixed-refrigerants Joule-Thomson cryocooler, the refrigerants flow in the capillary tube is very complicated. Most design methods in the literatures were developed on a theoretical simulation of the two-phase flow through the tube with some simple assumptions such as homogenous two-phase flow and constant friction factor along the tube. Some correlations or methods were valid only for limited refrigerants. In this paper, instead of the complicated mathematical model, a dimensionless correlation is developed to predict the mixed-refrigerants flow characteristics through adiabatic capillary tube. This ease-to-use and fairly accurate method, suitable for hand calculations, can calculate the refrigerants flow rate directly.

INTRODUCTION

Capillary tube is usually used as an expansion and refrigerant flow controlling device in small industry refrigeration systems, air-conditioning systems and household refrigerators because of its simplicity and low cost. It is a long hollow copper with an inside diameter between 0.33 to 4.0 mm and a length from 1 to 6 m. Though a capillary tube itself is very simple, the flow within the tube is quite complicated. The process of refrigerant flow through the capillary tube is a flash process, in which the refrigerant flows at a high speed and changes from liquid to vapor-liquid blend. In practical consideration, main concern is the refrigerants flowing characteristics for matching the system to get the highest performance.

The selection of the proper diameter and length of a capillary tube is important for a given system to run properly. In the most works found in public literatures [1-3], the refrigerants flowing characteristics were analyzed on numerical simulation with an assumption of homogenous two-phase flow. Some other correlations [4-7] were developed based on the Buckingham π-theorem to calculate the flow characteristics, and this ease-to-use and fairly accurate method, suitable for hand calculations, can calculate the refrigerants flow rate directly.

In recent years, there has been a remarkable development of the mixed-refrigerant J-T cryocooler, which cover a large temperature range from below liquid nitrogen temperature (77 K) to 230 K with corresponding mixture refrigerants. There are usually 5 to 7 components in the cryocooler. The mixed-refrigerants flow in the capillary tube is more complicated than pure refrigerants in the refrigeration system. The mixed-refrigerants flow characteristics are very important for the system design. However, the research on mixed-refrigerants flow characteristics is few in the open literatures. In this paper, a dimensionless correlation is developed to predict the mixed-refrigerants flow characteristics through the adiabatic capillary tube based on the experimental data. The correlation can be used directly to give a guide for system design.

DEVELOPMENT OF DIMENSIONLESS CORRELATION

Dimensionless correlation has been developed in order to predict refrigerant mass flow rate in several capillary tubes operating at different conditions. Dimensionless parameters are selected considering the different parameters' effect on flow characteristics based on the Buckingham π-theorem [8].

Dimensionless parameters

The first step of π-theorem was to select parameters that have influence on the mass flow rate through capillary tube.

These parameters are the geometric parameters of the capillary tube (inner diameter d and length L), inlet conditions (inlet pressure P_{in}, degree of subcooling T_{sub}, and quality x), outlet condition (outlet pressure P_{out}), and refrigerant properties (specific volume v, viscosity μ of liquid and gas phase, liquid specific heat C_{Pl}, surface tension σ, and latent heat of vaporization h_{lg}). Therefore, mass flow rate can be represented as function of several parameters shown in Equation (1).

$$m = f_1(d, L, P_{in}, P_{out}, T_{sub}, x, v_l, v_g, C_{Pl}, \mu_l, \mu_g, \sigma, h_{\lg}) \tag{1}$$

$$\pi_0 = f_2(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7, \pi_8) \tag{2}$$

$$\pi_0 = C \cdot \pi_1^{e1} \cdot \pi_2^{e2} \cdot \pi_3^{e3} \cdot \pi_4^{e4} \cdot \pi_5^{e5} \cdot \pi_6^{e6} \cdot \pi_7^{e7} \cdot \pi_8^{e8} \tag{3}$$

Then the parameters d, v_l, μ_l and C_{Pl} are defined as the repeating variables and nine dimensionless π terms presented in Table 1 are obtained. Buckingham π-theorem guarantees that the functional relationship must be the equivalent form shown in Equation (2).

To obtain the function f_2, Equation (2) is rewritten in the form with unknown constants and exponents as shown in Equation (3). Constant C and exponents of the eight π parameters are obtained by applying the least squares method to minimize the errors between mass flow rate calculated from Equation (3) and experimental values.

Table 1 Definitions of dimensionless π parameters

π-terms	Significances	Definitions
π_0	Flow rate	$m/(d \cdot \mu_l)$
π_1	Geometry effect	L/d
π_2	Inlet pressure effect	$(d^2 p_{in})/(v_l \mu_l^2)$
π_3	Outlet pressure effect	$(d^2 p_{out})/(v_l \mu_l^2)$
π_4	Liquid inlet effect	$(d^2 C_{Pf} T_{sub})/(v_l^2 \mu_l^2)$
	Two-phase inlet effect	x
π_5	Density effect	v_g / v_l
π_6	Viscosity effect	$(\mu_l - \mu_g)/\mu_g$
π_7	Vaporization effect	$(d^2 h_{\lg})/(v_l^2 \mu_l^2)$
π_8	Metastable effect	$(d\sigma)/(v_l \mu_l^2)$

Correlations for pure refrigerants

In Table 2, some dimensionless correlations for pure refrigerants in public literatures are listed. The outlet pressure (π_3 term) was not included in all correlations because the choked flow condition was easily reached for typical steady-state applications. Under this condition, the mass flow rate is dependent on the inlet pressure of the capillary tube only, and the pressure at the outlet has no relation with the mass flow rate. Also the metastable effect (π_8 term) was neglected because the influence is little.

In addition, it should be noted that the physical properties were evaluated at the corresponding bubble point temperature at the inlet pressure under the condition that the inlet is in sub-cooled liquid state. Only the Bittle's [4] correlation shown as Equation (5) was developed for two-phase flow inlet condition, in which the physical properties were evaluated at the state of inlet pressure and temperature.

It is important to select proper dimensionless π term to reflect the different parameters' contribution to the flow characteristics in a capillary tube. For sub-cooled liquid inlet condition, only the geometry effect (π_1 term), the inlet pressure effect (π_2 term) and the liquid inlet effect (π_4 term) were included in Equations (6), (7), (8) and (9) for one kind of refrigerant; the density effect (π_5 term), the viscosity effect (π_6 term) and the vaporization effect (π_7 term) were considered in Equations (4) and (10) because the correlations were applicable to three or four kinds of refrigerant. For the two-phase inlet condition, the density effect (π_5 term) was added in Equation (5).

Table 2 Dimensionless correlations for pure refrigerants

Authors	Refrigerants	Inlet state	Correlations
Bittle [4]	R22, R134a, R152a, R410A	Liquid	$\pi_0 = 1.893 \cdot \pi_1^{-0.484} \cdot \pi_2^{1.369} \cdot \pi_4^{0.019} \cdot \pi_5^{0.773} \cdot \pi_6^{0.265} \cdot \pi_7^{-0.824}$ (4)
		Two-phase	$\pi_0 = 836.9 \cdot \pi_1^{-0.740} \cdot \pi_2^{0.417} \cdot \pi_4^{0.981} \cdot \pi_5^{-0.646}$ (5)
Melo [5]	R12	Liquid	$\pi_0 = 0.06135 \cdot \pi_1^{-0.509} \cdot \pi_2^{0.495} \cdot \pi_4^{0.160}$ (6)
	R134a	Liquid	$\pi_0 = 0.125 \cdot \pi_1^{-0.552} \cdot \pi_2^{0.460} \cdot \pi_4^{0.178}$ (7)
	R600a	Liquid	$\pi_0 = 0.719 \cdot \pi_1^{-0.590} \cdot \pi_2^{0.403} \cdot \pi_4^{0.157}$ (8)
Wolf [6]	R134a	Liquid	$\pi_0 = 0.0129 \cdot \pi_1^{-0..387} \cdot \pi_2^{0.492} \cdot \pi_4^{0.187}$ (9)
	R22, R134a, R410A	Liquid	$\pi_0 = 1.892 \cdot \pi_1^{-0..484} \cdot \pi_2^{1.369} \cdot \pi_4^{0.0187} \cdot \pi_5^{0.773} \cdot \pi_6^{0.265} \cdot \pi_7^{-0.824}$ (10)
Wei [7]	R407C	Liquid	$\pi_0 = 108.54 \cdot \pi_1^{-0.4125} \cdot \pi_2^{0.2892} \cdot \pi_4^{0.2377} \cdot \pi_5^{0.2446}$ (11)

Correlation for mixed-refrigerants

In order to obtain the dimensionless correlation for multi-component mixed-refrigerants, a set-up based on the real J-T cryocooler cycle was established [9], and the experimental conditions are listed in Table 3.

Table 3 Experimental conditions in this study

Mixed-refrigerants	N_2+CH_4+C_2H_6+C_3H_8+iC_4H_{10}
Inner diameter of capillary tube (mm)	1.2, 1.7
Length of capillary tube (m)	1, 1.5, 2.0, 3.0, 4.0
Inlet Pressure (bar)	14.9～21.9
Outlet pressure (bar)	2.7～4.4
Temperature range (K)	100～200
Inlet quality	0.1～0.43

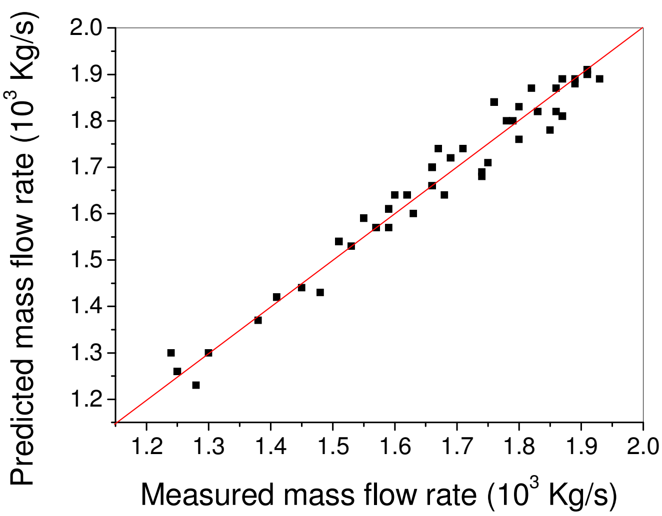

Figure 1 Comparisons of predicted mass flow rate with experimental data

In the multi-component mixed-refrigerants flow characteristics experiments, the state at the inlet of the capillary tube was in two-phase state and the choked flow was not reached at the outlet of the capillary tube. Therefore, the metastable (π_8 term) should be excluded and the outlet pressure effect (π_3 term) should be considered in the correlation. Based on the experimental data, the correlation for mixed-refrigerants flow characteristics shown in Equation (12) was obtained.

It should be noted that the physical properties were evaluated as an average value of the inlet and outlet state in π_0, π_5, π_6 and π_7 terms because the outlet pressure effect was considered in the correlation. The physical properties in π_2 and π_3 terms were evaluated at the corresponding inlet and outlet state.

Figure 1 shows comparisons of the predicted data with measured mass flow rate. Approximately 95.3 % of the data are correlated within a relative deviation of ± 5 %, and the average deviation is 1.80 %.

$$\pi_0 = 2679940.9 \cdot \pi_1^{-0.25744} \cdot \pi_2^{-0.02996} \cdot \pi_3^{0.01504} \cdot \pi_4^{-0.11064} \cdot \pi_5^{-0.44198} \cdot \pi_6^{-0.91566} \cdot \pi_7^{-0.02619} \tag{12}$$

$$m = f_3\left[d^{0.9777}, \left(\frac{\mu_l}{\mu_g} - 1\right)^{-0.91566}, \left(\frac{v_g}{v_l}\right)^{-0.44198}, L^{-0.25744}, \ldots\ldots\ldots \right] \tag{13}$$

Expanding the dimensionless π terms in Equation (12), four parameters that have more great influence on the mass flow rate are obtained as shown in Equation (13). Therefore, for a given multi-component refrigerants mixture, the primary factors that influence the flow characteristics in the adiabatic capillary tube are inner diameter of capillary tube, viscosity ratio and specific volume ratio between the liquid phase and gas phase, and the length of capillary tube.

SUMMARY

1. The dimensional analysis method was developed to predict multi-component mixed-refrigerants flow characteristics in the capillary tube. Some dimensionless parameters that influence mass flow rate were obtained on the basis of the Buckingham π-theorem.
2. A dimensionless correlation for predicting mixed-refrigerants flow characteristics was obtained based on the experimental data. The average deviation of the correlation is 1.80 %. The correlation can be used to the calculation of mass flow rate for a real system design.
3. Different from the correlations in the room-temperature refrigeration system, the pressure at the outlet of the capillary tube should be considered in correlation for mixed-refrigerants because the critical condition was not reached.
4. According to the correlation, the primary factors that influence mixed-refrigerants flow characteristics in adiabatic capillary tube were obtained. The result can give a guide for system design.

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