Thermoacoustic turbulent-flow model for inertance tubes used for pulse tube refrigerators

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Inertance-tube phase shifter can avoid DC-flow of the double-inlet phase shifter besides. The flow inside the inertance tube is usually turbulent, which makes linear network mode not accurately workable for describing performance of the inertance tube. Thus, a thermoacoustic turbulent-flow model is proposed to solve the problems, which can consider both turbulent-flow effect and heat transfer effect.

INTRODUCTION

Efficient regenerative cryocoolers operate on proper phase relations between pressure and velocity. A Stirling refrigerator reaches this proper phase relation by complicated mechanical connection of the cranks of both the compressor and the displacer. Pulse tube refrigerators have various devices to achieve these phase relations. A double-inlet phase shifter is a very powerful phase shifter, which is widely used in the pulse tube refrigerators. However, this shifter can bring about some negative effects and sometimes lead to instabilities of operating temperatures due to a so-called DC-flow occurring in this looped pipe system. An inertance tube is supposed to be another powerful shifter, which has the additional function of eliminating DC flow. For typical operating conditions, the flowing of the inertance tube is turbulent. Thus, this paper tried to develop a thermoacoustic turbulent-flow model for describing the inertance tube, by which can both incorporate turbulent-flow effect and heat transfer effect between gas and wall.

PHYSICAL AND MATHMATICAL MODEL

Figure 1 shows the schematic of the inertance tube phase shifter. It consists of an inertance tube and a reservoir. The reservoir is a short cylinder, but with a large cross-sectional area. Because of this special structure, the reservoir can be modeled as a lumped-parameter compliance. The inertance tube is a long tube with a considerable length, so it has an obvious acoustic behavior. Thus, it is to be modeled here with distributed-parameter models. The operating conditions and the structures for an inertance tube phase shifter are given as follows: (1) the inertance tube has a length of L, a diameter of D and a cross-sectional area of A, and the reservoir has a volume of Vr; (2) the oscillating flow inside the inertance is laminar in an angular frequency of ω; (3) the working gas is helium, having a sound velocity of a_0 and an adiabatic index, γ. In addition, the time-averaged temperature and pressure in the inertance tube are T_0 (~300K) and p_0, respectively. The dynamic viscosity of helium, corresponding to the pressure and temperature, is μ. The wavelength for helium at a frequency of ω is λ.

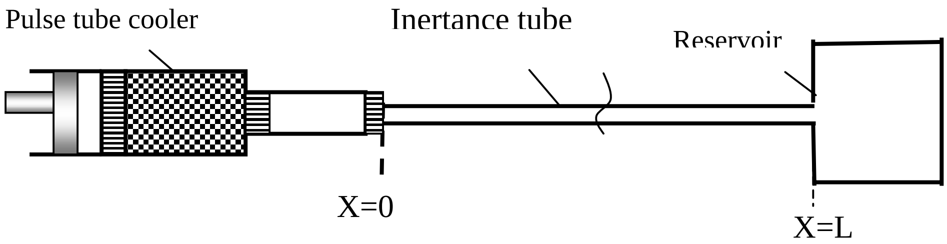

Figure 1 Schematic of Inertance Tube Phase Shifter

In fact, most practical inertance-tube shifters operate in a state of turbulent flow. Therefore, a complete description for such inertance tube shifters may need to be based on some complicated simulations of computational fluid dynamics (CFD) modeling, which is beyond the contents of this paper. In fact, the phase shifting and acoustic power transmission of an inertance tube is still a result of resistive, inertance and compliance whether the flow inside the inertance tube is laminar or turbulent. Thus, Eq.(1) and Eq.(2) are proposed to describe the two cases. The most obvious difference between laminar and turbulent flows is that the turbulent flow makes a significant contribution to viscous flow impedance of an inertance tube. Especially for small-diameter inertance tubes, the viscous component is generally predominant so that the capability of the phase shifter is greatly limited.

$$\frac{d\tilde{p}}{dx} = -(\xi_\mu r_\mu + i\xi_l \omega\, l)\tilde{U} \tag{1}$$

$$\frac{d\tilde{U}}{dx} = -[i\xi_c \omega\, c + \frac{1}{\xi_k r_k}]\tilde{p} \tag{2}$$

where $\tilde{p}$ and $\tilde{U}$ are the acoustical pressure and volumetric velocity in complex number; ξ_μ, ξ_l, ξ_c and ξ_k are the four correction factors for turbulent flow. r_μ, l, c and r_k are the viscous flow resistance, inertia, compliance and thermal-relaxation resistance per unit length for laminar flow, respectively, are given by Eq.(3).

$$r_{\mu,L} = \frac{\omega\rho_0}{A}\frac{\mathrm{Im}[-f_\mu]}{|1-f_\mu|^2},\quad l = \frac{\rho_0}{A}\frac{1-\mathrm{Re}[f_\mu]}{|1-f_\mu|^2}, c = \frac{A}{\gamma p_0}[1+(\gamma-1)f_k], \frac{1}{r_k} = \frac{\gamma}{\gamma-1}\frac{\omega A\,\mathrm{Im}(-f_k)}{p_0} \tag{3}$$

where f_μ and f_k are the viscous and thermal factors which can be found elsewhere[1].

In principle, the empirical expressions for ξ_μ, ξ_l, ξ_c and ξ_k may be approximately obtained based on numerous experimental data or CFD simulations. However, some simplified expressions for the four correction factors need to be obtained based on some reasonable deductions and qualitative understanding. For turbulent flow, the viscous flow resistance $r_{\mu,T}$ can be described as follows [2].

$$r_{\mu,T} = \xi_\mu r_\mu = \frac{64\rho_0 |\tilde{U}|}{3\pi^3 D^5}[f_M - (1-\frac{9\pi}{32})R_e \frac{df_M}{dR_e}] \tag{4}$$

where f_M is the friction factor for steady turbulent flow; R_e is the peak Reynolds number which is defined as $R_e = \frac{\rho_0 |\tilde{U}| D}{A\mu}$. (5)

Qualitatively speaking, under the same operating conditions and the same geometric parameter, the inertia of fluid in turbulent flow may decrease due to the increase of "effective" flowing area. Thus the correction factor for inertia, ξ_b may approach 1, while the correction factors, ξ_c and ξ_k, may increase due to enhanced heat transfer from turbulent flow. For the case of perfect heat transfer, f_k=1 and 1/ f_k =0. As a simplified modeling, a hypothetical, thermal conductivity resulting from this enhanced heat transfer is introduced to make a correction for ξ_c and ξ_k; that is, enhancement of the heat transfer leads to a decreased, effective dimensionless diameter of the inertance tube. The effective dimensionless diameter $\overline{D}_T$ may be approximately evaluated by

$$\overline{D}_T = \overline{D}\sqrt{\frac{Nu_L}{Nu_T}} \tag{6}$$

where $\overline{N}u_L$ and $\overline{N}u_T$ are the Nusselt numbers of laminar and turbulent flows, respectively. Therefore, ξ_{dk} and ξ_k, can be estimated by calculating $f_{k,T}$ with $\overline{D}_T$. Moreover, Eq.(1) and Eq.(2) can be rewritten in the following forms,

$$\frac{d\tilde{p}}{dx} = -(r_{\mu,T} + i\omega l_T)]\tilde{U} \tag{7}$$

$$\frac{d\tilde{U}}{dx} = -(i\omega c_T + \frac{1}{r_{k,T}})\tilde{p} \tag{8}$$

where $r_{\mu,T}$, l_T , c_T and $r_{k,T}$ are the flow resistance, inertia, compliance and thermal-relaxation components of the impedance for the inertance tube with turbulent flow. The above-described analysis shows how to evaluate these components in a simplified way. It should be noted here that Eq.(7) and Eq.(8) are highly nonlinear, coupled equations. Moreover, this is usually a two-boundary-value problem, thus, a numerical iteration for their solutions is required. To solve Eq.(7) and Eq.(8), we used the shooting target method for them and adopt the fourth-order Runge-Kutta method for integration. The above turbulent model was compared with the experimental results and is described in the following section.

EXPERIMENTAL VERIFICATION

An experimental set-up was constructed to measure the performance of the phase shift and acoustical power transmission of various inertance tubes and to verify the applicability of the turbulent models previously developed. The experimental system consists of a linear compressor, an inertance tube and a reservoir. In the experiments, the operating frequency for the compressor is adjustable, ranging from 30 Hz to 90Hz. In addition, the pressure ratio can be controlled and varies from 1.05 to 1.40. Several inertance tubes with different diameters and lengths were tested.

A comparison between the turbulent model and the experimental result obtained from the hot-wire anemometer was made, which is shown in Figures 2 to 5. Figures 2 and 3 show the phase angles for different operating frequencies and pressure ratios, and Figures 4 and 5 show the acoustical powers for different operating frequencies and different pressure ratios. There are still some obvious errors between the turbulent flow model and the experimental data. With regard to the errors for the phase angle, one reason is that the phase angle is only for the fundamental frequency of the oscillating turbulent flow; another reason is that the turbulent model developed here is actually semi-theoretical and semi-empirical model that certainly needs to be improved with further study. The reasons are also applicable to errors for the acoustical powers.

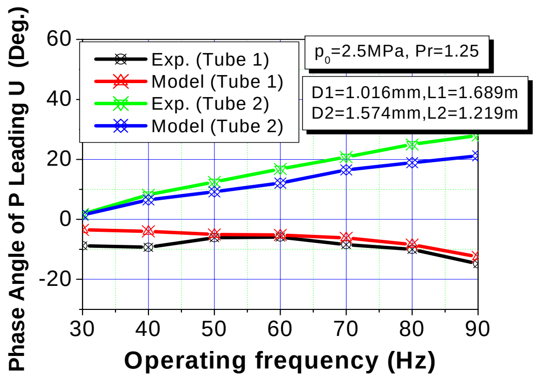

Figure 2 Phase angle

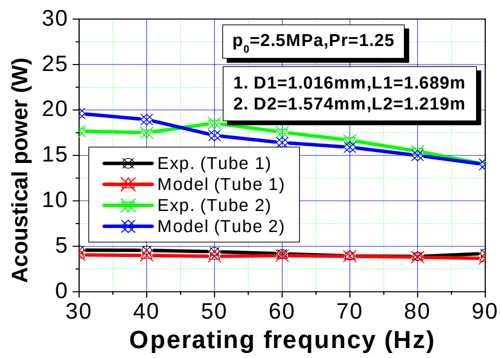

Figure 3 Acoustical power

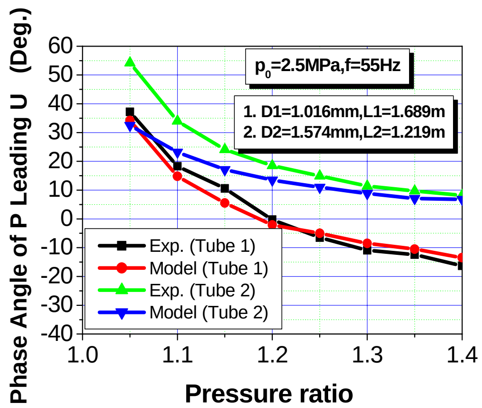

Figure 4 Phase angle

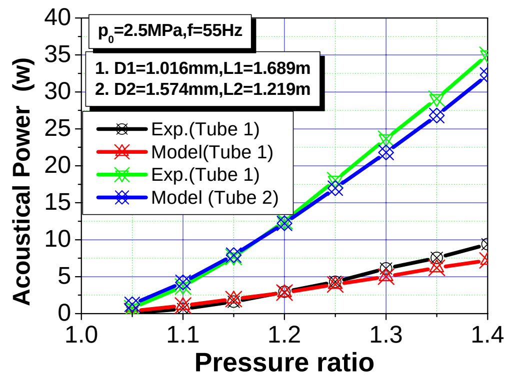

Figure 5 Acoustical power

CONCLUSIONS

The turbulent-flow model in this paper is developed to describe practical inertance tube shifters with acceptable accuracy. In addition, relatively extensive experiments on various inertance tubes for low-capacity pulse tube refrigerators were made and only some of the experiment data are shown in this paper. Comparisons between the simplified turbulent-flow model and the experimental data by hot-wire anemometry were made, which showed a good agreement qualitatively, and also provided an acceptable accuracy quantitatively. Certainly, much more effort is still needed on this topic.

ACKNOWDGEMENT

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