Routing, Flow, and Capacity Design in Communication and Computer Networks

Chapter 13: Resilient Design: Single-/Multi-Layer Fair Networks

Slides by
Yong Liu1, Deep Medhi2, and Michał Pióro3
1Polytechnic University, New York, USA
2University of Missouri-Kansas City, USA
3Warsaw University of Technology, Poland & Lund University, Sweden
October 2007

Outline

- Note: Knowledge of Chapter-8 assumed
- Resilient Design of Single Layer Proportionally Fair (PF) Networks
- Design of two-layer PF Networks
  - Reconfiguration in Layer-1
  - Reconfiguration in Layer-2
  - Reconfiguration in both Layer-1 and Layer-2
**LMX: DR/PF/PR+UR**

**Link-Path Formulation**

**Path Restoration with Unrestricted Flow Reconfiguration**

**indices**
- \( d' = 1, 2, \ldots, D \) demands
- \( p = 1, 2, \ldots, P_d \) candidate paths for demand \( d \)
- \( e = 1, 2, \ldots, E \) links
- \( s = 1, 2, \ldots, S \) situations (normal state is not distinguished)

**constants**
- \( \delta_{ed} \) = 1 if link \( e \) belongs to the fixed path of demand \( d \), 0, otherwise
- \( \tau_{ds} \) revenue from demand \( d \) in situation \( s \)
- \( \xi_e \) unit cost of link \( e \)
- \( \alpha_{ed} \) fractional availability coefficient of link \( e \) in situation \( s \) (0 ≤ \( \alpha_{ed} \) ≤ 1)
- \( B \) assumed budget

**variables**
- \( y_e \) capacity of link \( e \)
- \( x_{dp} \) flow allocated to path \( p \) of demand \( d \) in situation \( s \)
- \( X_{ds} \) total flow allocated to demand \( d \) in situation \( s \)
- \( X = (X_s; X_{1s}, \ldots, X_{Ss}) \), \( X_s = (X_{ds}; d = 1, 2, \ldots, D) \) for \( s = 1, 2, \ldots, S \)
- \( R_s \) logarithmic revenue in situation \( s \), \( R = (R_1, R_2, \ldots, R_S) \)
- \( \bar{R} \) vector \( \bar{R} = (\underline{R}_1, \underline{R}_2, \ldots, \underline{R}_S) \) of the revenues sorted in the non-decreasing order

**objective**

maximize lexicographically \( \bar{R} \) \hspace{1cm} (13.1.1a)

**constraints**

\[
\sum_{e \in E} y_e \leq B \hspace{1cm} (13.1.1b)
\]

\[
\sum_{d \in D} \sum_{p \in P_d} \delta_{ed} x_{dp} \leq \alpha_{ed} y_e, \hspace{1cm} e = 1, 2, \ldots, E \hspace{1cm} (13.1.1c)
\]

\[
X_{ds} = \sum_{p \in P_d} x_{dp}, \hspace{1cm} d = 1, 2, \ldots, D, \hspace{1cm} s = 1, 2, \ldots, S \hspace{1cm} (13.1.1d)
\]

\[
R_s \leq \sum_{d \in D} \sum_{s \in S} \log X_{ds}, \hspace{1cm} s = 1, 2, \ldots, S \hspace{1cm} (13.1.1e)
\]

\[
\tau - R_s \leq 0, \hspace{1cm} s = 1, 2, \ldots, S. \hspace{1cm} (13.1.1f)
\]

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**CXP: DR/PF/PR+UR {FS}**

**Link-Path Formulation**

**First Step of DR/PF/PR+UR**

**additional variable**
- \( t \) auxiliary continuous variable (with unlimited sign)

**objective**

maximize \( t \) \hspace{1cm} (13.1.2a)

**constraints**

\[
\sum_{e \in E} y_e \leq B \hspace{1cm} (13.1.2b)
\]

\[
\sum_{d \in D} \sum_{p \in P_d} \delta_{ed} x_{dp} \leq \alpha_{ed} y_e, \hspace{1cm} e = 1, 2, \ldots, E \hspace{1cm} (13.1.2c)
\]

\[
X_{ds} = \sum_{p \in P_d} x_{dp}, \hspace{1cm} d = 1, 2, \ldots, D, \hspace{1cm} s = 1, 2, \ldots, S \hspace{1cm} (13.1.2d)
\]

\[
R_s \leq \sum_{d \in D} \sum_{s \in S} \log X_{ds}, \hspace{1cm} s = 1, 2, \ldots, S \hspace{1cm} (13.1.2e)
\]

\[
\tau - R_s \leq 0, \hspace{1cm} s = 1, 2, \ldots, S. \hspace{1cm} (13.1.2f)
\]
Algorithm 13.1 [cont'd]

Step 2: Solve the following CXP to improve the currently best revenues:

\[
\begin{align*}
\text{maximize} & \quad t, \\
\text{subject to} & \quad \sum_{d} \delta_{e}y_{de} \leq B, \quad (13.1.4a) \\
& \quad \sum_{e} \sum_{p} \delta_{de} x_{de} \leq \alpha_{e}y_{de}, \quad e = 1, 2, \ldots, E, \quad (13.1.4b) \\
& \quad X_{ds} = \sum_{p} \delta_{ds} x_{de}, \quad d = 1, 2, \ldots, D, \quad (13.1.4c) \\
& \quad t_{s} \leq \sum_{d} \delta_{ds} \log X_{ds}, \quad s \in Z_{0}, t_{s} - \text{constants}, \quad (13.1.4d) \\
& \quad t \leq \sum_{d} \delta_{ds} \log X_{ds}, \quad s \in Z_{1}. \quad (13.1.4e)
\end{align*}
\]

Step 3: Let \((t^{*}, x^{*}, X^{*}, R^{*}, y^{*})\) be the optimal solution of CXP (13.1.4). Put \(t_{s} : = t^{*}\) for each \(s \in Z_{1}\) and go to Step 1. 

\{Note that it may happen that the optimal solution of (13.1.4) will not increase current \(t^{*}\) because there may be more blocking situations besides the one detected in Step 1.\}
Figures for Example 13.1: Irregular and regular sets of blocking situation

Figure 13.1 Two-Link Network

Figure 13.2 Three-Link Network

Linear Approx of (13.1.2)

**LP: DR/PF/PR+UR {FS-LA}**

**Linear Approximation of DR/PF/PR+UR {FS}**

**Additional variables**

\( z_{ds} \) approximation of \( \log X_{ds} \) (continuous)

**Objective**

\[ \text{maximize } t \]  \hspace{1cm} (13.1.23a)

**Constraints**

\[ \sum_{e} z_{ele} \leq B \]  \hspace{1cm} (13.1.23b)

\[ \sum_{e} \sum_{d} \alpha_{ed} z_{dpe} \leq \alpha_{ed} z_{ele} \]  \hspace{1cm} (13.1.23c)

\[ X_{ds} = \sum_{d} x_{dps} \]  \hspace{1cm} (13.1.23d)

\[ z_{ds} \leq g_k X_{ds} + f_k \]  \hspace{1cm} (13.1.23e)

\[ t \leq \sum_{s} z_{des} \]  \hspace{1cm} (13.1.23f)
Design of Resilient Two-layer PF Networks: Reconfiguration in Layer-1

Indices
- \( d = 1, 2, \ldots, D \) demands
- \( p = 1, 2, \ldots, P_d \) candidate paths of layer 2 for demand \( d \)
- \( e = 1, 2, \ldots, E \) links of layer 2
- \( q = 1, 2, \ldots, Q_e \) candidate paths of layer 1 for link \( e \)
- \( g = 1, 2, \ldots, G \) links of layer 1
- \( s = 1, 2, \ldots, S \) situations (normal state is not distinguished)

Constants
- \( r_d \) revenue from demand \( d \)
- \( h_{\ell}, H_{\ell} \) lower and upper bound, respectively, for total flow of demand \( d \)
- \( \gamma_{p,q} = 1 \) if link \( e \) belongs to path \( p \) realizing demand \( d \), 0 otherwise
- \( \zeta_{g,s} \) unit cost of link \( g \)
- \( \alpha_{g,s} \) fractional availability coefficient of link \( g \) in situation \( s \) \((0 \leq \alpha_{g,s} \leq 1)\)
- \( B \) assumed budget

Variables
- \( x_d \) flow allocated to path \( p \) of demand \( d \)
- \( X_d \) total flow allocated to demand \( d \), \( X = (X_1, X_2, \ldots, X_D) \)
- \( y_e \) capacity of link \( e \)
- \( u_{g,s} \) flow allocated to path \( q \) of link \( e \) in situation \( s \)
- \( u_g \) capacity of link \( g \)

Objective

\[
\text{maximize } F = \sum_d r_d \log X_d \quad (13.3.1a)
\]

Constraints

\[
\sum_g b_g u_g \leq B \quad (13.3.1b)
\]

\[
\sum_p x_{dp} = X_d, \quad d = 1, 2, \ldots, D \quad (13.3.1c)
\]

\[
h_d \leq X_d \leq H_d, \quad d = 1, 2, \ldots, D \quad (13.3.1d)
\]

\[
\sum_s \sum_q \gamma_{p,q} z_{e,s} = y_e, \quad e = 1, 2, \ldots, E \quad s = 1, 2, \ldots, S \quad (13.3.1e)
\]

\[
\sum_e \sum_g \gamma_{p,q} z_{e,s} \leq \alpha_{g,s} u_g, \quad g = 1, 2, \ldots, G \quad s = 1, 2, \ldots, S \quad (13.3.1g)
\]
Design of Resilient Two-layer PF Networks: Reconfiguration in Layer-2

**LXM: DR/2L/NPR(1)-PRU(2)**

**Link-Path Formulation**

**Flow Reconfiguration Only in Layer 2**

**Additional Constants**

- $\gamma_{ds}$: revenue from demand $d$ in situation $s$
- $h_{ds}$, $H_{ds}$: lower and upper bound, respectively, for total flow of demand $d$ in situation $s$
- $\alpha_{gs}$: binary availability coefficient of link $g$ in situation $s$ ($\alpha_{gs} \in \{0, 1\}$)
- $\theta_{eq}$: binary availability coefficient of path $q$ realizing link $e$ in situation $s$

**Variables**

- $x_{dps}$: flow allocated to path $p$ of demand $d$ in situation $s$
- $X_{ds}$: total flow allocated to demand $d$ in situation $s$, $X = (X_1, X_2, \ldots, X_S)$
- $y_{gs}$: capacity of link $g$ in situation $s$
- $z_{eq}$: flow allocated to path $q$ of link $e$
- $u_g$: capacity of link $g$
- $R_s$: logarithmic revenue in situation $s$, $R = (R_1, R_2, \ldots, R_S)$
- $R$: vector $R = (R_1, R_2, \ldots, R_S)$ of the revenues sorted in the non-decreasing order

**Objective**

maximize lexicographically $R$ \hspace{1cm} (13.3.2a)

**Constraints**

- $\sum q_{dps} \leq B$, $d = 1, 2, \ldots, D$, $s = 1, 2, \ldots, S$ \hspace{1cm} (13.3.2b)
- $\sum x_{dps} = X_{ds}$, $d = 1, 2, \ldots, D$, $s = 1, 2, \ldots, S$ \hspace{1cm} (13.3.2c)
- $h_{ds} \leq X_{ds} \leq H_{ds}$, $d = 1, 2, \ldots, D$, $s = 1, 2, \ldots, S$ \hspace{1cm} (13.3.2d)
- $\sum x_{dps} \leq y_{gs}$, $d = 1, 2, \ldots, D$, $s = 1, 2, \ldots, S$ \hspace{1cm} (13.3.2e)
- $\sum \theta_{eq} \leq y_{gs}$, $d = 1, 2, \ldots, D$, $s = 1, 2, \ldots, S$ \hspace{1cm} (13.3.2f)
- $R_s \leq \sum \gamma_{ds} \log X_{ds}$, $s = 1, 2, \ldots, S$. \hspace{1cm} (13.3.2g)
Design of Resilient Two-layer PF Networks: Reconfiguration in both Layer-1 & Layer-2

\textbf{Link-Path Formulation}

\textbf{Flow Reconfiguration in Layers 1 and 2}

\textbf{variables}
- \(x_{dp}s\) flow allocated to path \(p\) of demand \(d\) in situation \(s\)
- \(X_{ds}\) total flow allocated to demand \(d\) in situation \(s\), \(X = (X_1, X_2, \ldots, X_D)\) for \(s = 1, 2, \ldots, S\)
- \(y_{gs}\) capacity of link \(g\) in situation \(s\)
- \(z_{ps}\) flow allocated to path \(p\) of link \(s\) in situation \(s\)
- \(w_g\) capacity of link \(g\)
- \(R_s\) logarithmic revenue in situation \(s\), \(R = (R_1, R_2, \ldots, R_S)\)
- \(R\) vector \(R = (R_1, R_2, \ldots, R_S)\) of the revenues sorted in the non-decreasing order

\textbf{objective}
- \text{maximize lexicographically} \(R\)

\textbf{constraints}
- \(\sum_{g} y_{gs}x_{dp}s \leq B\) \hspace{1cm} \((13.3.3a)\)
- \(\sum_{p} x_{dp}s = X_{ds}\) \hspace{1cm} \((13.3.3b)\)
- \(h_{ds} \leq X_{ds} \leq H_{ds}\) \hspace{1cm} \((13.3.3c)\)
- \(\sum_{e} E_{eg}y_{es}x_{dp}s \leq y_{es}\) \hspace{1cm} \((13.3.3d)\)
- \(\sum_{g} w_gz_{pg}s \leq y_{es}\) \hspace{1cm} \((13.3.3e)\)
- \(\sum_{g} \sum_{s} y_{es}z_{pg}s \leq w_gz_{pg}\) \hspace{1cm} \((13.3.3f)\)
- \(R_s \leq \sum_{g} R_GG_{gfs}\) \hspace{1cm} \((13.3.3g)\)
- \(R_{gs} \leq \sum_{g} R_{gs}G_{gfs}\) \hspace{1cm} \((13.3.3h)\)