Principles of Flow

3

3.1 Introduction

In almost any investigation involving groundwater, questions arise about how much water is moving and how fast it is flowing. Typical questions in contamination remediation studies are: "What should the well discharge be to capture the entire plume of contaminated water?" or "How long will it take for the contaminated groundwater to reach a nearby stream?" Regarding water supply issues, you might hear questions like "If we pump 5 million gallons/day from this well field, will it dry up a nearby wetland and nearby domestic wells?" or "How much discharge can we hope to get from a well 100 ft deep in this aquifer at this location?" The answers to such questions are based on groundwater flow analyses, which in turn are based on some straightforward physical principles that govern subsurface flow. An empirical relationship called Darcy's law and conservation of mass form the basis for many useful hand calculations and computer simulations that can be made to analyze groundwater flow. Principles of flow are also covered in books by Freeze and Cherry (1979), Todd (1980), Domenico and Schwartz (1998), and Fetter (2001).

3.2 Darcy's Law and Hydraulic Conductivity

In 1856, French engineer Henry Darcy was working for the city of Dijon, France on a project involving the use of sand to filter the water supply. He performed laboratory experiments to examine the factors that govern the rate of water flow through sand (Darcy, 1856; Freeze, 1994). The results of his experiments defined basic empirical principles of groundwater flow that are embodied in an equation now known as Darcy's law.

Darcy's apparatus consisted of a sand-filled column with an inlet and an outlet similar to that illustrated in Figure 3.1. Two manometers (essentially very small piezometers) measure the hydraulic head at two points within the column (h_1 and h_2). The sample is saturated, and a steady flow of water is forced through at a discharge rate Q [L^3/T].

Darcy found through repeated experiments with a specific sand that Q was proportional to the head difference Δh between the two manometers and inversely proportional to ($\propto$) the distance between manometers Δs:

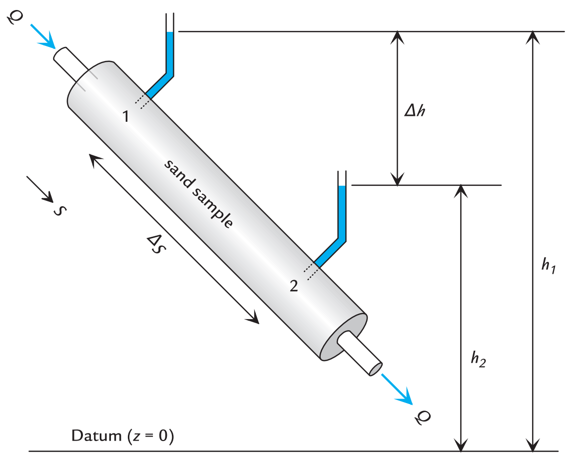

Figure 3.1 Schematic illustrating steady flow through a sand sample. The manometers measure heads h_1 and h_2 at locations 1 and 2 within the column. The s coordinate direction runs parallel to the column.

$$Q \propto \Delta h, \qquad Q \propto \frac{1}{\Delta s} \tag{3.1}$$

Obviously, Q is also proportional to the cross-sectional area of the column A.

Combining these observations and writing an equation in differential form gives **Darcy's law** for one-dimensional flow:

$$Q_s = -K_s \frac{dh}{ds} A \tag{3.2}$$

where Q_s is discharge in the s direction. The constant of proportionality K_s is the **hydraulic conductivity** in the s direction, a property of the geologic medium. Hydraulic conductivity is a measure of the ease with which a medium transmits water; higher K_s materials transmit water more easily than low K_s materials. The term *hydraulic conductivity* is sometimes abbreviated to just *conductivity*. The minus sign on the right side of this equation is necessary because head decreases in the direction of flow. If there is flow in the positive s direction, Q_s is positive and dh/ds is negative. Conversely, when flow is in the negative s direction, Q_s is negative and dh/ds is positive.

Consider the units of the entities in Darcy's law. Head h and the coordinate s both have length units, so dh/ds is dimensionless. The dimensionless quantity dh/ds represents the rate that head changes in the s direction, and is known as the **hydraulic gradient**. The dimensions of Q_s are $[L^3/T]$ and of A are $[L^2]$, so the hydraulic conductivity K_s is $[L/T]$.

Rocks or soils with small pores allow only slow migration of water while materials with larger, less constricted pores permit more rapid migration. Water traveling through small, constricted pores must shear itself more in the process of traveling a given distance than water traveling through larger pores. More shearing in the water causes more viscous resistance and slower flow. Other factors being equal, the average velocity of groundwater migration is proportional to K. Hydraulic conductivity is an empirical constant measured in laboratory or field experiments.

Another term, *permeability*, has historically been synonymous with *hydraulic conductivity*, but now its usage is associated with *intrinsic permeability*, a related property that is not specific to the fluid water (see Section 3.3).

Table 3.1 lists some typical ranges of hydraulic conductivity values for common rocks and soils. These ranges are mostly based on data compiled by Davis (1969) and

Table 3.1 Typical Values of Hydraulic Conductivity

Material	K(cm/sec)	Source
Gravel	10^{-1} to 100	1
Clean sand	10^{-4} to 1	1
Silty sand	10^{-5} to 10^{-1}	1
Silt	10^{-7} to 10^{-3}	1
Glacial till	10^{-10} to 10^{-4}	1
Clay	10^{-10} to 10^{-6}	1,2
Limestone and dolomite	10^{-7} to 1	1
Fractured basalt	10^{-5} to 1	1
Sandstone	10^{-8} to 10^{-3}	1
Igneous and metamorphic rock	10^{-11} to 10^{-2}	1
Shale	10^{-14} to 10^{-8}	2

Sources: (1) Freeze and Cherry (1979); (2) Neuzil (1994).

summarized by Freeze and Cherry (1979). Hydraulic conductivity varies over a tremendous range, 12 orders of magnitude, in common geologic materials. The wide variations of fracture width and frequency in crystalline rocks account for the huge ranges in observed hydraulic conductivities in such rock. Where carbonate rocks have been eroded by dissolution, fractures widen to form large openings and talk of "underground rivers" is not just mythology. Some basalts are also very conductive due to open columnar joints and voids at the bases and tops of successive lava flows. Groundwater flow velocities in basalts and limestones can be extremely high compared to velocities in more typical geologic materials where the pore sizes are on the order of millimeters or smaller.

The most common units for hydraulic conductivity are meters/day and feet/day for field studies, and cm/sec for laboratory studies. Inconsistent units such as gallons/day/foot^2 have been used in older irrigation and water supply studies. For conversion factors, see Appendix A.

Example 3.1 A sample of silty sand is tested in a laboratory experiment just like that illustrated in Figure 3.1. The column has an inside diameter of 10 cm and the length between manometers is $\Delta s = 25$ cm. With a steady flow of $Q = 1.7$ cm^3/min, the head difference between the manometers is $\Delta h = 15$ cm. Calculate the hydraulic conductivity K_s.

This is a direct application of Eq. 3.2, with a little rearrangement at the beginning to isolate K_s:

$$\begin{aligned} K_s &= -Q_s \frac{1}{A} \frac{ds}{dh} \\ &= -1.7 \text{ cm}^3/\text{min} \left(\frac{1}{\pi (5 \text{ cm})^2} \right) \left(\frac{25 \text{ cm}}{-15 \text{ cm}} \right) \\ &= 0.036 \text{ cm/min} \\ &= 6.0 \times 10^{-4} \text{ cm/sec} \end{aligned}$$

The sign of ds/dh is negative because as s increases, h decreases (see Figure 3.1).