4

VORTICES AND VORTEX WAKES

Vortex principles provide an alternative way to derive the same laws that underlie the work and power calculations of Chapter 3. A flying bird leaves tracks in the air in the form of a vortex wake, and this can be examined and used to deduce the forces that the wings exerted on the air, and to estimate the work that they did. Vortex principles are not explicitly used in the *Flight* programme, but are the basis of recent and current wind-tunnel investigations on flight mechanics.

Work and power were discussed in Chapter 3 in terms of linear motions and accelerations in the air, but in the world of real fluids this is at best an approximation. Any process that involves work being done on a fluid invariably involves shear (gradients of velocity) and this in turn results in rotation. *Vortices* appear on every scale wherever work is done on a fluid, whether the scenario is a cup of tea stirred by a spoon, or a hurricane driven by heat from the warm ocean below. When a bird flies by, it leaves vortices behind it in the air, which persist for a while. The vortex wake can be observed and measured, and it carries a record of the work that the bird's wings have done on the air. Vortex concepts are not used explicitly in the *Flight* programme, but some of the most interesting experiments of recent times have approached the same mechanical questions that the programme addresses from a different direction, by looking at the vortex wake, rather than at forces on the wings and in the flight muscles.

What exactly is a vortex, and what rules do vortices obey? Those questions are easily asked, but finding answers that lead to useful predictions about real flows has taken the best efforts of some of the most famous mathematicians of the last two centuries. Leonhard Euler, Daniel Bernoulli, Jean le Rond d'Alembert, Pierre-Simon Laplace, Hermann Helmholtz, Lord Kelvin, Wilhelm Kutta, Nikolai Joukowski and Ludwig Prandtl all made major contributions. The body of theory that they and others built is not exactly simple, but it is an alternative way of looking at the flight of birds and aircraft, which has evolved in parallel with the more direct approach based on considering the impact that every air particle has on the pressure and velocity of its neighbours. In mathematicians' terms, the direct approach amounts to solving the Navier-Stokes equations, which fully describe what the fluid does, but are notoriously difficult to solve for particular cases, whereas vortex concepts are less exact but deliver practically useful results in terms of entities that can be visualised, and obey simple rules. Following the advent of huge computers, it has become practical to solve the Navier-Stokes equations numerically, but vortex concepts still provide a useful and compact way to describe many processes.

An authoritative modern account that covers both approaches can be found in Anderson (1991), and the same author's *History of Aerodynamics* is also highly recommended (Anderson 1997). My objectives in this chapter are limited to attempting to present the basic vortex concepts in an essentially pictorial form. This can be seen as an alternative view of the principles of flight covered in Chapter 3, and also as background to modern experiments on the vortex wakes of birds, and how they relate to calculations of the work done by the wings.

4.1 THE CONCEPT OF THE LINE VORTEX

A *line vortex* is basically a mathematical abstraction that corresponds, under the right conditions, to a physical entity with two components, a *vortex filament*, which is a thin, rotating thread of fluid particles, surrounded by an *induced flow*, which is where the physical effects take place. The induced flow is the visible, whirling vortex, but despite that, it is said to be *irrotational*. This means that although individual particles of fluid may (or may not) circulate around closed paths, they do not rotate on their own axes. Only the fluid particles that make up the vortex filament actually rotate.

Figure 4.1A is a cross section through a line vortex. In this two-dimensional view the point in the middle is a *point vortex*, which is actually a cross section through the vortex filament. The filament can be imagined as a line passing through the point, and extending above and below the page. The filament (only) is where the air is actually rotating, and it *induces* the surrounding air to circulate around it, without the individual particles of air themselves rotating. As the radius from the filament increases, so the tangential speed of the air decreases in each cylindrical shell of air surrounding the rotating filament, in inverse proportion to the radius. If the cross section of the vortex filament were really a point (with zero radius), the speed and the angular velocity within it would have to be infinite, making what mathematicians call a "singularity", a place where the rules break down. However, the induced flow looks essentially the same if the vortex filament is replaced by a spinning core that has a finite radius (Figure 4.1B). The tangential velocity in the core (grey) is zero at the centre, and increases linearly with the radius. The outer surface of the core (at radius 2 in this case) pulls the layer of air in contact with it along, and at larger radii the tangential speed decreases in the same way as it does in the induced flow around the one-dimensional vortex filament of Figure 4.1A. Some of the most useful theoretical results of classical aerodynamics depend on the assumption that real vortices, such as those shed from the wing tips of fixed-wing aircraft, conform to the pattern shown in Figure 4.1B, with a thin core of rotating air surrounded by an irrotational induced flow.

4.2 ⬬ VORTEX CONCEPTS APPLIED TO FIXED WINGS

4.2.1 CIRCULATION AND LIFT

The speed along any of the circular paths in Figure 4.1A or B is constant around the path, and inversely proportional to the radius of the circle. If we integrate the speed around the circumference of one of the circles, the result is the same for any circle. The speed is halved for a bigger circle with twice the circumference, and therefore the integral of speed around the closed path is the same as before. This integral is called the *circulation* and the result is the same for *any* closed path, circular or not, so long as the vortex filament (or the finite core) is entirely contained within it. The circulation therefore has a fixed and measurable value for a particular vortex, and is often called the *strength* of the vortex.

If a cylinder is mounted in a wind tunnel, perpendicular to the wind (coming from the left in Figure 4.2A) the air divides symmetrically

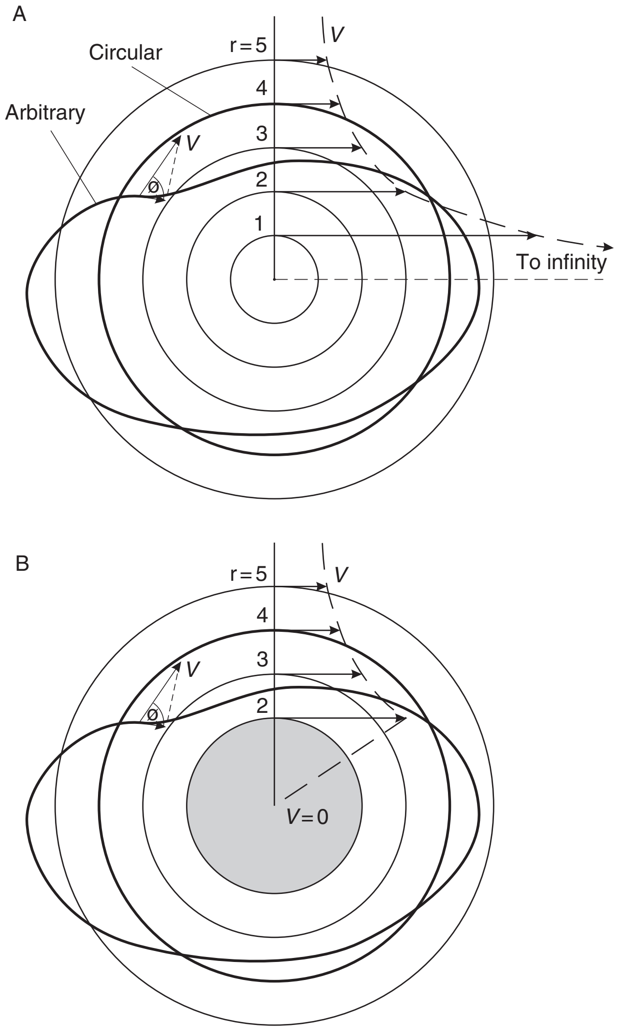

FIGURE 4.1 (A) Cross section through a line vortex. The ''vortex filament'', represented by the point in the middle, is the only region where particles of air actually rotate on their own axes. The ''induced flow'' around the filament is ''irrotational'', meaning that particles of air follow circular paths, but do not rotate on their own axes. Each particle moves at a tangential velocity (V) that is inversely proportional to its radial distance (r) from the filament. The ''circulation'' (Γ) can be found by integrating $V \cos \varphi$ (where φ is the angle between the path and the direction of V) around any closed path that contains the filament. The result is the same whether the path is one of the concentric circles shown (where $\varphi = 0$), or any arbitrary closed path that contains the vortex filament. The value of the circulation so measured is often called the ''strength'' of the vortex. (B) The need for infinite angular velocity in the vortex filament can be avoided by replacing it with a core (grey) whose diameter is finite. Within the core the velocity increases linearly from zero at the centre to the core boundary. Outside the core, the velocity decreases in the same way as the induced flow around the vortex filament in (A). The circulation around any closed path that completely encloses the core is the same as that around an infinitely thin filament.

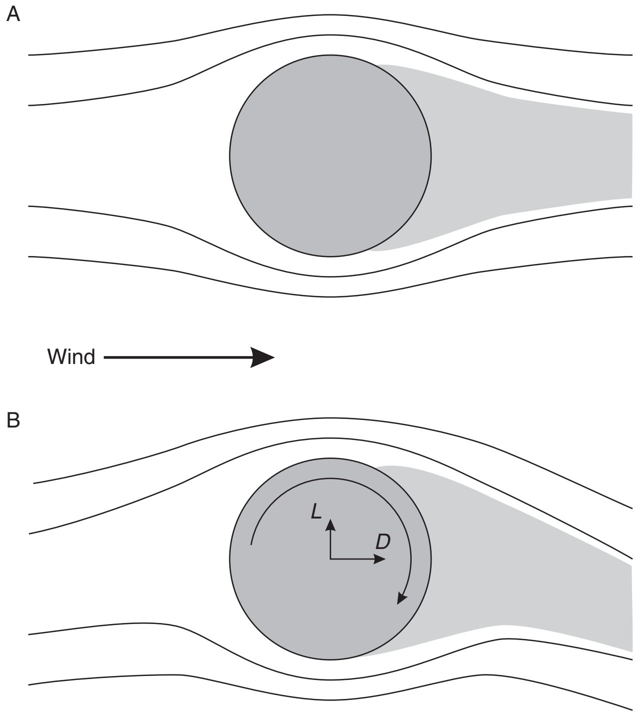

FIGURE 4.2 (A) A cylinder (dark grey) in a wind tunnel is a bluff body from which the flow separates on the downwind side to form a turbulent wake (light grey), resulting in a large amount of drag. (B) If the cylinder is spinning the induced flow of Figure 4.1B is added to the steady wind, resulting in the airflow being deflected in a direction perpendicular to the incident flow. This results in a lift force (*L*), but there is also a large drag force (*D*) due to the turbulent wake.

around it and separates from the surface, forming a turbulent wake on the downwind side (grey). This results in a lot of drag but no lift, since the airflow is slowed down by the obstruction, but not deflected upwards or downwards. If we now set the cylinder spinning as in Figure 4.2B, the effect is to add the circular induced flow of Figure 4.1B to the steady wind of Figure 4.2A. The turbulent wake is still there, along with the drag that results from it, but the induced flow from the vortex that is bound to the spinning cylinder imparts some downwash to the air leaving the cylinder on the right. This in turn produces a lift force on the cylinder (perpendicular to the incident airflow), equal to the rate at which transverse momentum is imparted to the air (Chapter 3, Section 3.1.1).

Lift results when a vortex is added to the steady flow. This is called the Magnus Effect when the vortex is due to a spinning cylinder. The amount of the lift force, per unit length of the cylinder, is directly proportional to the circulation, as defined above, and also to the wind speed, and the air density (Box 4.1). Some asymmetrical shapes develop circulation when exposed to a steady wind, without needing to spin round, and this also produces lift according to the same law. The essential characteristic of an "aerofoil" shape, that is, one that can be used as the cross section of a wing, is that a *bound vortex* develops around it when it is set at a suitable angle of attack to the incident flow, and that the lift force due to the resulting circulation is associated with a much smaller drag force.

BOX 4.1 Lift, circulation and vorticity.

The idea of a vortex

A "vortex" is a construct with properties that may appear somewhat artificial at first sight. It consists of a "vortex filament", which is a one-dimensional line that meanders through the fluid, surrounded by a region of "induced flow". A cross section through the vortex shows the vortex filament as a point, and it is only in this infinitely small region that particles of fluid actually rotate. The surrounding induced flow is said to be "irrotational". Particles of fluid may travel in closed paths around the vortex, or they may stream past, faster on one side than the other, but they do not actually spin around on their own axes.

Variable definitions for this box

C	A constant
L'	Lift per unit span
r	Distance from vortex filament
s	Distance along integration path
V	Local fluid velocity
V_∞	Free stream fluid velocity
Γ	Circulation
φ	Angle between local velocity vector and path of integration
ρ_∞	Free stream fluid density

Circulation

"Circulation" is a property that is measured in the irrotational flow around the vortex filament, not in the vortex filament itself. Figure 4.1 shows a section through an isolated vortex, in which each particle of air (outside the vortex filament) is moving in a circular path around the filament, at a speed (V) which is inversely proportional to the radius (r) from the filament. In other words,

$$V = \frac{C}{r}, \tag{1}$$

where C is a constant. This distribution of velocity is not as arbitrary as it looks, as it expresses the condition that the flow is irrotational. We can define the *circulation* (Γ) around any particular circular path that encloses the vortex filament as the integral of the velocity around that path:

BOX 4.1 *Continued.*

$$\Gamma = (\oint V \mathrm{d}s), \tag{2}$$

where s is distance along the circular path, and the symbol "$\oint$" refers to integration around the closed path. For this case,

$$\Gamma = \left(\frac{C}{r}\right) \times (2\pi r) = 2\pi C, \tag{3}$$

in other words, the circulation is the same around any circle, irrespective of the radius. More generally, for a closed path of any shape that contains the vortex filament, the component of the local velocity along the path at any point is $V \cos \varphi$, where φ is the angle between the local velocity vector and the path of integration, and the circulation is:

$$\Gamma = (\oint V \cos\varphi \mathrm{d}s), \tag{4}$$

The result of the integration is the same (Equation 3) for any closed path that contains the vortex filament. This is still true if the vortex filament is inside (or on the surface of) a body of arbitrary shape, such as a wing cross section, provided that the path of integration encloses the body, and the flow outside the body is irrotational. The circulation has the dimensions of length-squared/time (L^2T^{-1}), and it is a property of the vortex, often called its "strength".

Bound vortex on a wing

An *aerofoil* shape is one which, when immersed in a steady flow of fluid, and set at a suitable angle of attack to the incident flow, develops a vortex around it, such that the fluid velocity on one side of the shape is higher than the free-stream velocity, and that on other side is lower. This vortex is forced into existence on a wing by the "Kutta condition" (see main text), which expresses the effect of viscous forces that equalise the speed at which the fluid leaves the upper and lower surfaces at the trailing edge. Although this type of flow does not involve particles of air moving in closed curves, but only differences in speed on the two sides of a wing, it can be seen as the combination of a free stream whose velocity is V_∞ and a vortex of strength Γ. The lift on the wing, that is, the component of force at right angles to the free stream can be calculated directly from the Kutta-Joukowski theorem, which states that:

$$L' = \rho_\infty V_\infty \Gamma, \tag{5}$$

where ρ_∞ is the free-stream density. The primed variable L' stands for the lift per unit span of the wing. The total lift is obtained by integrating L' across the span from one wing tip to the other.

Vorticity

Unlike circulation, *vorticity* is not a property of a vortex. It is a "field variable" like pressure and density, with a continuous distribution that can be mapped in a region of fluid, and may vary with time. Vorticity is a vector quantity equal to the "curl" of the velocity, which is itself a function of the

BOX 4.1 *Continued.*

partial derivatives of velocity in the three directions of space. Vorticity can also be seen as circulation per unit area, as measured in a plane that is perpendicular to the axes of the vortices. Its dimensions are those of inverse time (L^2T^{-1}/L^2). The intuitive meaning of the vorticity at any point in the measurement plane is that it is twice the angular velocity of a fluid particle at that point. One of the more arcane properties of a vortex as defined above is that the vorticity is infinite in the vortex filament, and zero everywhere else, which is another way of saying that the induced flow is irrotational.

Where there is shear, viscous effects lead to distributed vorticity that is not confined to identifiable filaments, and can be mapped. For example, where air flows over the surface of a wing, the boundary layer can be seen as a "vortex sheet", a layer of very small vortices with their axes lying along the wing span, transverse to the flow. At the trailing edge of the wing, the flow above the wing has an inward component of velocity due to the reduced pressure there, while that below the wing has an outward component of velocity. Where the two layers merge as they leave the trailing edge, this lateral motion produces a free vortex sheet with its axes aligned back along the flight path. These vortices are of course the same as the horseshoe vortices that are shed from the trailing edge (Figure 4.3B), and eventually roll up (behind a fixed wing) to form a pair of concentrated trailing vortices (Figure 4.4).

4.2.2 THE KUTTA CONDITION

How is a bound vortex initiated and maintained around an object like a wing, which is moving along, but not spinning around? If we consider an aircraft just starting its take-off run, the flow around a cross section of the wing first follows a pattern like that in Figure 4.3A. There is a discontinuity where the air sliding along the lower surface comes to the sharp trailing edge, and doubles back on to the upper surface. Then it doubles back again, to merge with the air coming over the upper surface as the flow leaves the wing along the same line that it followed when it approached. This pattern would persist if the viscosity of the air were zero. In the real world, the zigzag path of the air around the trailing edge results in strong local shear, which in turn produces viscous forces that force the flow to speed up over the upper surface, and slow down below. This is equivalent to adding circulation to the steady flow. The circulation around the wing builds up until the air flowing off the upper surface merges smoothly with that coming from below (Figure 4.3B). This is the "Kutta condition", and it prescribes the strength of the bound vortex that forms, and hence also the

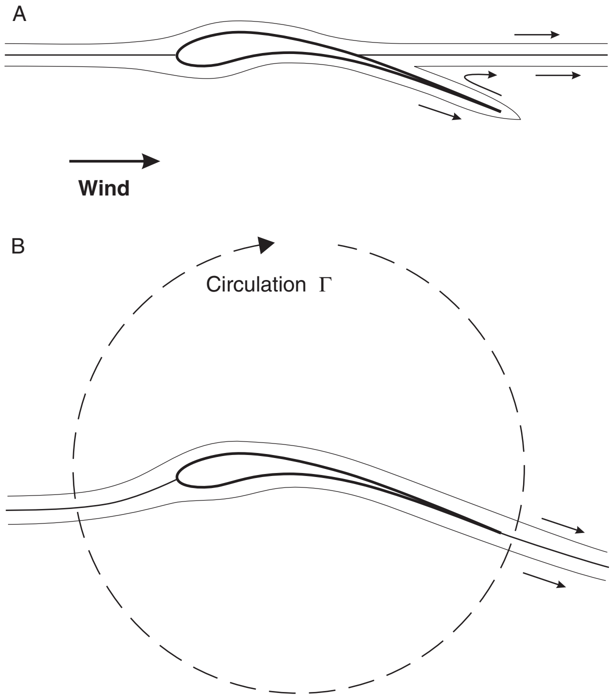

FIGURE 4.3 (A) A fluid with no viscosity flowing around the lower side of a wing would double back around the trailing edge and leave from a stagnation point displaced on to the upper surface, following the same line by which it approached. Although the surface pressure would vary on different parts of the wing, the variations would cancel one another, producing no net lift. There would also be no net drag (d'Alembert's paradox). (B) In reality, viscous forces in the zone of strong shear around the trailing edge force the flow over the upper surface to speed up and that on the lower surface to slow down, until the fluid from both surfaces leaves the trailing edge smoothly at the same speed (the Kutta condition). This amounts to forcing a circulation (Γ) to be added to the flow, which causes upwash in the air approaching the wing, and downwash as it leaves the trailing edge. This in turn gives rise to the lift. There could be neither circulation nor lift without viscosity.

amount of lift that develops. The physical meaning of the bound vortex is that the air flows faster past the top surface of the wing than past the lower surface. Air molecules do not circulate in closed paths around the wing, because the steady component of the flow is always faster than the circulating flow due to the vortex.

4.2.3 HELMHOLTZ'S LAWS

To see how this works in a three-dimensional wing, we need a couple of general properties of line vortices that were discovered by Hermann

Helmholtz in the nineteenth century. The first of Helmholtz's laws is that the circulation (strength) of a line vortex, defined as above, is the same at every cross section along the vortex, however long it is. The second is that a line vortex cannot end in the fluid. It can end by butting against a solid surface, or it can bend round and join on to itself as a vortex ring, but it cannot just end. Helmholtz's laws imply that a line vortex, once present, lasts for ever, and conversely, if no vortex already exists, it is not possible for one to start.

The resolution of this paradox lies in the small print. The conditions that Helmholtz assumed when he derived his laws included the assumption of an "inviscid" fluid, which is one whose density is finite, but whose viscosity is zero. In practice, Helmholtz's laws describe the behaviour of vortices in viscous fluids like air or water rather well, so long as the flow does not contain any regions of strong shear, meaning regions where the speed changes sharply over a short distance in the fluid. There is always shear wherever the fluid slides along a solid surface. The layer of fluid in contact with the surface sticks to it without slip, and viscous forces tend to hold back the layers of fluid sliding past above. Conversely, the motion of the fluid tends to pull the surface along with it. For fluids of low viscosity like air and water, these effects are confined to a thin *boundary layer* next to the surface. It is here that rotation is introduced into the fluid, in the form of a *vortex sheet*, rolling along the surface. Once a vortex is carried away from the solid surface by the flow, forces due to viscosity become negligible, and the vortex behaves (more or less) according to Helmholtz's laws.

4.2.4 THE THREE-DIMENSIONAL FIXED WING

Once a bound vortex has formed on a wing of finite span, it is forbidden to end in the fluid. It cannot just stop at the wing tips. It bends round to form a pair of trailing vortices, whose strength is the same as that of the bound vortex, leading back along the flight path from the wing tips, to the point on the runway where the lift developed. There they are joined together by a "starting vortex", which has the same strength as the bound vortex, but the opposite direction of rotation. A fixed-wing aircraft that flies from one airport to another actually creates an elongated, rectangular vortex ring, closed at one end by the starting vortex which is left behind on the departure runway, and at the other by a "stopping vortex", which is left on the landing runway, when the bound vortex is shed from the wing on landing.

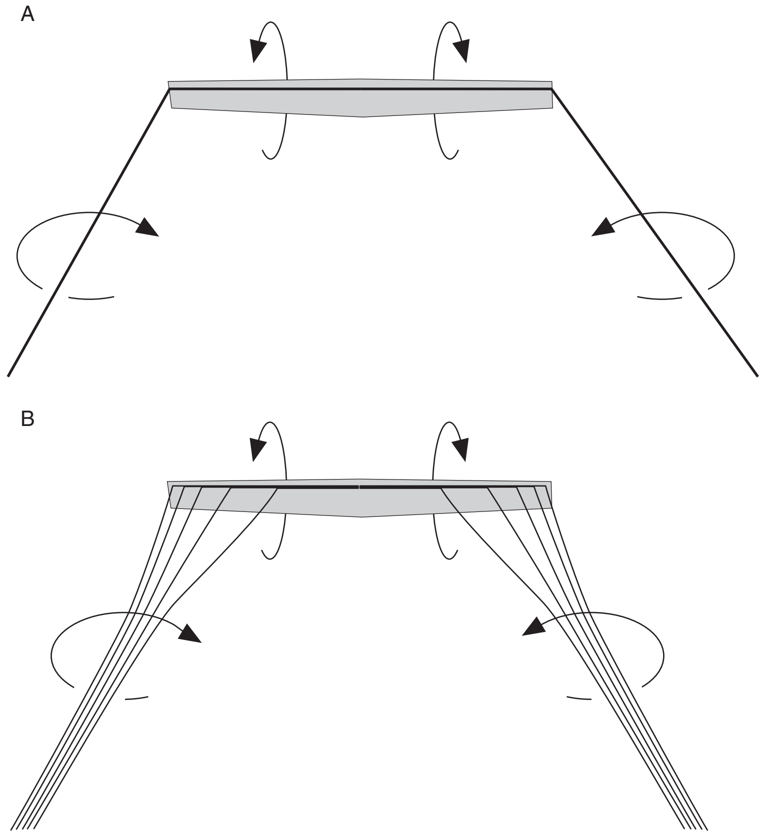

FIGURE 4.4 (A) A wing (grey) seen from above and behind. The lift per unit span is proportional to the strength of the bound vortex (black line), which cannot end at the wing tips, and bends back to form two parallel trailing vortices. (B) The number of vortex filaments making up the bound vortex is here shown as highest in the middle, decreasing as filaments are shed one by one from the trailing edge. In reality a continuous ''vortex sheet'' is shed from the trailing edge, but it rolls up a short distance downstream, to form a pair of trailing vortices of the same strength as in (A) but closer together.

When an aircraft is cruising along at a constant speed in level flight, its weight is balanced by the lift associated with the bound vortex on the wing. A pair of trailing vortices whose strength is the same as that of the bound vortex, stretch out behind (Figure 4.4A). The starting vortex has been left far behind, and for practical purposes the trailing vortices extend backwards to infinity. They grow continuously as the aircraft moves forwards creating new vortex at the front, and leaving the existing vortices behind where they persist, convecting slowly downwards as each vortex is carried down by the induced flow of the other and (in the case of large aircraft) creating a hazard to other aircraft that may happen to fly through them. As the trailing vortices grow, downward momentum is continuously added to the downwash zone between them (Figure 4.4A), at a rate which has to account for the lift force (Chapter 3). At the same time, work is needed to set each newly created section of the vortex pair spinning. The rate at which this work has to be done can be calculated from the strength and spacing of the vortex pair, and the forward speed. It is the same as the induced power required from the aircraft's engine, as calculated in a different way in Chapter 3, Box 3.1. The rates at which momentum and energy appear in the wake can be measured, and used to deduce the forces acting on the aircraft, and the power that is being supplied by the engine. The persisting vortex wake is a kind of "footprint" in the air. It contains a record of the forces that the aircraft (or bird) has applied to the air, and the work that has been done by the engine or flapping wings.

A single "horseshoe vortex" as shown in Figure 4.4A would imply that the circulation of the bound vortex is the same at every cross section from one wing tip to the other, which is not usually the case. Typically, the circulation is strongest in the middle, and tapers off to zero at the wing tips. As we pass outwards from the wing root towards the wing tip, the circulation around each cross section gets less, and this means that a part of the vortex bound to the wing root must have bent round and left the wing as a trailing vortex. In Figure 4.4B, the bound vortex near the wing root is made up from five horseshoe vortices, all of equal strength. Together, they make a "lifting line" along the wing, whose strength decreases in steps, as the vortex filaments are shed one by one from the trailing edge. By making the number of vortex filaments larger, and the strength of each smaller, the strength of the lifting line can be made to decline smoothly to zero at the wing tip, while the circulation is shed as a continuous vortex sheet from the trailing edge of the wing. Such a sheet is unstable, and soon rolls up into a concentrated, tightly wound vortex, whose circulation is the same as that due to the lifting line at its strongest point. The end result, as far as the cross-sectional view of the wake is concerned, a short distance behind the wing, is nearly the same as for a single horseshoe vortex. There are two trailing vortices, each with the same strength as the strongest part of the lifting line, but they are a little closer together than they would be if the lifting line were of constant strength (Figure 4.5). The wake of a gliding kestrel was measured and analysed by Spedding (1987a), and found to conform closely to this pattern.

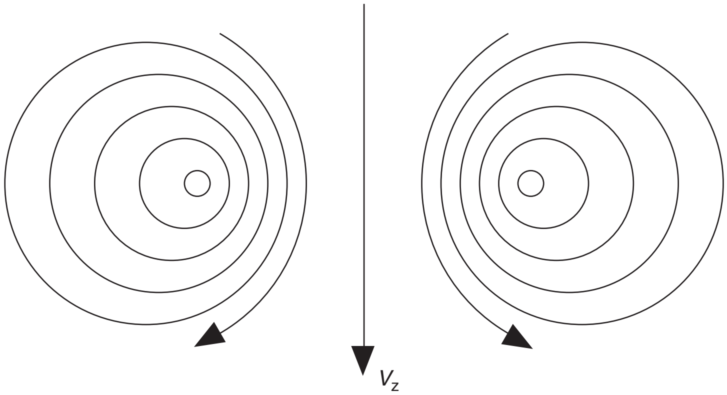

FIGURE 4.5 A cross section through the wake a short distance downstream from the wing of Figure 4.4B. Each vortex of the pair induces additional downward flow on the near side of the other, so that the downward velocity (V_z) in the centre is stronger than either vortex would produce on its own. Also, each vortex makes the other convect downwards at a velocity $V_z/4$. The upward velocity on the outside of each vortex is less than it would be without the other vortex. As the vortex pair grows along the direction of flight, net downward momentum is added at a rate that must balance the aircraft's weight. The rate at which work is done (induced power), must account for all of the motion irrespective of its direction.

4.3 LIFTING-LINE THEORY APPLIED TO FLAPPING WINGS

4.3.1 VORTEX RINGS AND TRAILING VORTICES IN BIRDS' WAKES

Prandtl's lifting-line theory assumes that the vortex sheet, which forms where the air slides over the surface of the wing, can be regarded as the combined effect of a large number of vortex filaments which individually obey Helmholtz's laws, and which, once shed from the wing, roll up into the pair of trailing vortices which are actually observed. The theory gives a good account of the properties of fixed wings, and this indicates that the flow over such wings does indeed consist of vortex filaments surrounded by irrotational induced flow, as outlined above. If Helmholtz's laws are true in general, then one would expect that the theory could be extended to cover flapping wings, a notion with great creative possibilities, which has given rise to a minor industry in recent years.

Rayner (1979a,b) postulated a simple type of wake for a pair of flapping wings, in which the wings are assumed to build up a circulation during the downstroke, and to be completely unloaded (zero circulation) during the upstroke. As the wing tips separate at the beginning of the downstroke, vortices begin to stream from their tips as soon as the circulation becomes established. These join to become a single vortex connecting the wing tips, which grows in length as the wing tips move apart. At the end of the downstroke, the circulation ceases and the two ends of the vortex are shed from the wings. Being forbidden to end in the fluid, they join together to form a complete vortex ring, which is left behind as the wings are repositioned for the next downstroke. Just as an aircraft creates one long, rectangular vortex ring per flight, so a slow-flying bird creates one vortex ring per wingbeat, comprising starting and stopping vortices, connected by very short trailing vortices. The momentum of the vortex ring, divided by the wingbeat period, gives the average lift force, which must be equal to the bird's weight if the flight path is horizontal on average. Likewise, the energy of the vortex ring, divided by the wingbeat period, is the induced power.

In the first quantitative study of the wake of a flying bird, Spedding (1986) trained a jackdaw to fly very slowly through a cloud of tiny helium-filled soap bubbles, and mapped the motion of the air in three dimensions by taking stereoscopic multiple-flash photographs. He observed vortex rings which resembled Rayner's predictions, but found that he could only account for about half the momentum needed to support the jackdaw's weight (Spedding et al. 1984). The reason for this "momentum deficit" remained a mystery for another twenty years. Spedding (1987b) also observed a different type of wake, which had not been predicted, in a kestrel which had been trained to fly through a bubble cloud at a normal cruising speed. In this case, the circulation of the bound vortex did not drop to zero during the upstroke, in fact it did not change at all. As the wing tips moved up and down, they streamed a pair of continuous vortices of constant strength, as a fixed wing would do, implying that the lift *per unit span* did not change. However, the wing tips moved in during the upstroke, as the bird reduced its wing span by flexing the elbow and wrist joints, and out to full span during the downstroke. By varying its wing span in this way, the bird developed more lift during the downstroke than during the upstroke, which is necessary to produce a net forward force over the wingbeat cycle, to balance drag forces. This "concertina" wing motion is invariably seen in high-speed films of birds in fast flapping flight.

4.3.2 BIRDS DO NOT NEED GAITS

Spedding's observations were misinterpreted by others to imply that birds must either use a "vortex-ring gait" at low speeds, or a "constant-circulation gait" at cruising speeds, and must "shift" from one to the other as they speed up or slow down, in much the same way that a horse shifts back and forth between a walk and a trot. Although this idea has been widely repeated, there is actually no evidence for it, and there is no known reason why a bird cannot change smoothly from one type of wake to the other as it speeds up, without any discontinuous shift. In the hypothetical sequence of Figure 4.6, discrete vortex rings form at very low speeds, because the local airspeed over the wing during the upstroke is so low that the circulation cannot be maintained, and the wing has to be unloaded (Figure 4.6A). Once the bird accelerates to a modest forward speed, a small amount of circulation (and lift) can be maintained during the upstroke (Figure 4.6B). A stopping vortex forms to close the ring as the wing is partially unloaded at the end of the downstroke, but it does not contain the full amount of circulation that comes off the wing in the form of trailing vortices during the downstroke. The difference remains in the form of weaker trailing vortices that continue, closer together, during the upstroke. As the forward speed continues to increase (Figure 4.6C), the trailing vortices become stronger during the upstroke, and the "rungs" of the ladder become weaker until finally they disappear altogether, leaving a pair of trailing vortices of constant strength, but variable spacing. No gait shift is required.

4.4 ⬬ WIND TUNNEL STUDIES OF BIRD WAKES

4.4.1 DPIV EXPERIMENTS

Experimental studies of bird wakes entered a new phase with a series of papers by Spedding et al. (2003a,b), Rosén et al. (2004) and Hedenström et al. (2005) on the wakes of small birds flying in a wind tunnel, observed by digital particle imaging velocimetry (DPIV). Like the helium-bubble method, this technique depends on tracking particles in the air, but in the wind tunnel the bird is stationary and the air streams past, carrying any vortex structures in the wake along with it. The particles were tiny liquid droplets introduced into the circulating air stream by a fog generator, and they were illuminated by a thin light sheet coming from a pulsed laser. The light sheet illuminated a vertical plane aligned along the direction of the air flow behind the bird, and the particles in it were photographed from the side. By statistically comparing two photographs, separated by a short time interval, variations of velocity in the plane of the light sheet could be mapped. In slow flight, when the bird was generating vortex rings, the starting and stopping vortices of each ring could be identified and measured, but the structure of the "trailing" parts of the vortex structure had to

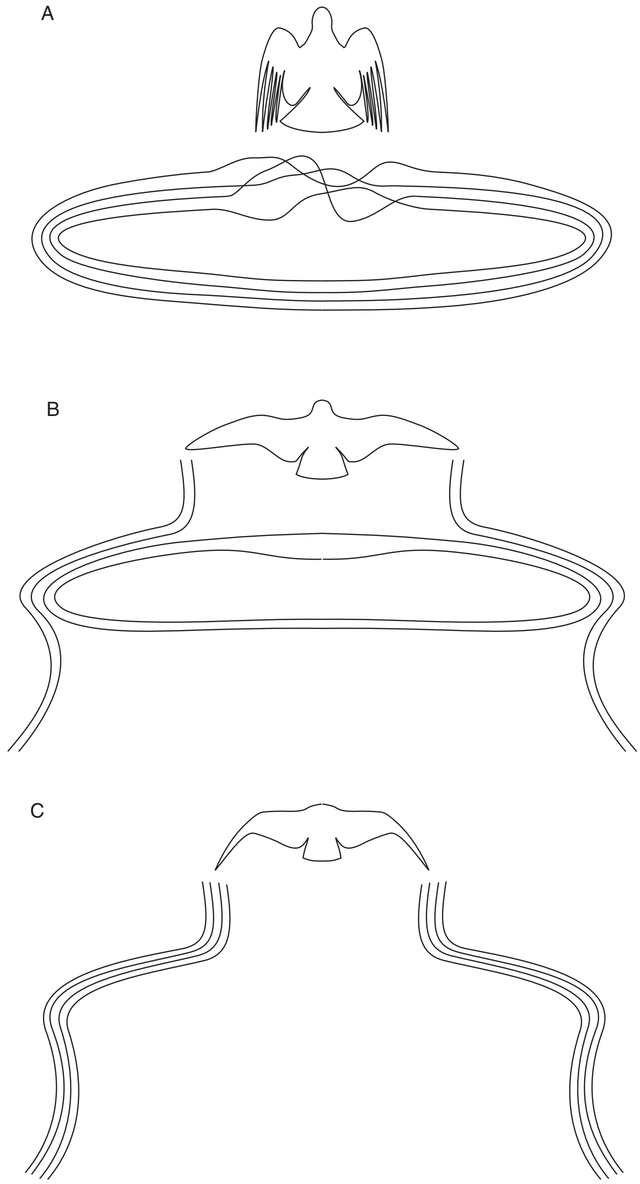

FIGURE 4.6 Schematic bird wakes, seen from behind and slightly above the bird, which is shown during the upstroke of flapping flight, at three different speeds. A (observed). In very slow flight the wing is unloaded during the upstroke, shown by separated flight feathers. The starting vortex formed during the previous downstroke, shown as a bundle of vortex filaments, is shed from the wing when the lift collapses at the end of the downstroke, and its ends join to make a free vortex ring. B (hypothetical). At a moderate forward speed, some lift continues during the upstroke, and some vortex filaments continue to stream from the wing tips, while others are shed and join up to form a vortex

be inferred by comparing maps in which the illuminated plane varied from the centre line to beyond the wing tip.

The experiments of Spedding et al. (2003b) on a thrush nightingale covered a speed range from well below the thrush nightingale's estimated minimum power speed (V_{mp}) to about 2 m s^{-1} above V_{mp}, and the results demonstrated a couple of important points. In the first place, there was no suggestion of any discontinuous "shift" between two or more different "gaits" at different speeds. The bird could fly at any speed between minimum and maximum, adjusting its wake structure continuously to suit the speed. Secondly, the results at intermediate speeds did not display a regular "ladder" structure as proposed in Figure 4.6B. A well-defined starting vortex could be identified at the beginning of each downstroke, but the stopping vortex was less well defined, and weaker than expected. When the results were analysed in terms of vortex filaments and irrotational flow, there was a momentum deficit as observed in the earlier helium-bubble experiments. Hedenström (2006) has described these experimental wakes, with diagrams of their inferred structure.

4.4.2 DISTRIBUTED VORTICITY

In the case of a vortex filament, which is the only part of a line vortex where the air actually rotates, the *vorticity* can be defined as twice the angular velocity of the filament. The induced flow surrounding the filament is irrotational, meaning that its vorticity is everywhere zero. However, vorticity can also be measured at a point in the fluid as the "curl" of the three-dimensional velocity. This is a vector quantity, derived from the local gradients of velocity in the three dimensions of space. Spedding et al. (2003a,b) and Hedenström et al. (2005) used the DPIV technique to map the magnitude and direction of the vorticity in the space immediately downstream of the bird, and found that the vorticity was not wholly confined to well-defined vortex filaments. Starting and stopping vortices could be seen at each wingbeat, but they were not of equal strength. The flow around them was not irrotational, but

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ring. The combined effect is a pair of continuous wing tip vortices, which move out as the wings are fully extended during the downstroke, and in as they are flexed in the upstroke, together with a series of transverse vortices circulating alternately in opposite directions. C (observed). At higher speeds, the transverse starting and stopping vortices dwindle in strength and eventually disappear, leaving only the trailing vortices, undulating up and down, and moving in during the upstroke and out during the downstroke. The circulation of the bound and trailing vortices remains constant. The lift *per unit span* is therefore also constant, but the lift is reduced during the upstroke by shortening the wing span.

contained distributed vorticity that was not concentrated into thin filaments. When this distributed vorticity was taken into account, the momentum deficit disappeared.

It seems that real, flapping-flight bird wakes do not necessarily consist of structures made up of vortex filaments obeying Helmholtz's laws, as postulated by Rayner (1979a,b,c), or as shown in Figure 4.6. There are identifiable vortex cores, but the flow around them is not, in general, irrotational. This means, unfortunately, that it is not practical to calculate the momentum and energy of the wake by elaborating Prandtl's lifting-line theory, as this depends on representing the wake as an array of line vortices, which individually conform to Helmholtz's laws. Although it should be possible in principle to estimate the three main components of mechanical power (outlined in Chapter 3) from a quantitative analysis of these wakes, a full accounting of the momentum and energy will have to include distributed vorticity that is not confined to thin vortex cores. This is difficult, and is a major challenge for theorists. It may also underlie some of the performance differences between bird wings and their artificial counterparts, especially their resistance to boundary-layer separation (below).

4.4.3 IMPLICATIONS OF WAKE STUDIES FOR FLIGHT PERFORMANCE CALCULATIONS

The power calculation used in *Flight*, and described in Chapter 3 for flapping wings, does not explicitly take account of the structure of the vortex wake. If it were possible to do that, the effects would include changing the method of calculating the induced power. The present method depends on the much-derided concept, introduced in Chapter 3, of an "actuator disc", which is swept out by the pair of flapping wings, and imparts a downward induced velocity to the air passing through it. The actuator disc is imagined as adding a constant downward velocity to the whole of a circular tube of air passing through the disc, whose diameter is the same as the wing span. In this it resembles a fixed wing with an elliptical spanwise lift distribution which, according to Prandtl's lifting-line theory, produces a constant downwash velocity across the span, from one wing tip to the other. This case is discussed at length in every aeronautical textbook, together with the proof that this particular lift distribution results in a lower induced drag than any other. Despite its somewhat artificial appearance, the actuator disc predicts the same amount of induced power as a wake like that of Figure 4.7, coming from a fixed wing with an elliptical spanwise lift distribution. The formula for induced power, derived from the actuator disc in Chapter 3, Box 3.1, is exactly the same as the standard formula for the induced drag of a fixed wing with elliptical lift distribution, if the drag is multiplied by the speed to get the induced power (Box 4.2).

4.4.4 BETTER ESTIMATES OF THE INDUCED POWER FACTOR

Because an elliptical lift distribution produces the lowest induced drag that is possible with a fixed wing, corrections to induced drag or induced power calculations take the form of multiplying the calculated drag or power by a number that is somewhat greater (but not much greater) than 1, to account for losses due to deviations from the assumed constant downwash velocity across the span. In *Flight*, the default value for the induced drag factor in gliding flight is 1.1, and that for the induced power factor in flapping flight is 1.2. These values are essentially guesses based on aeronautical experience. The power curve calculation in *Flight* currently identifies two speeds that characterise a particular bird, as defined by its mass, wing span and wing area, taking account of the strength of gravity and the air density. These are the minimum power speed (V_{mp}) and the maximum range speed (V_{mr}) which define the lower and upper limits of the speed range in which

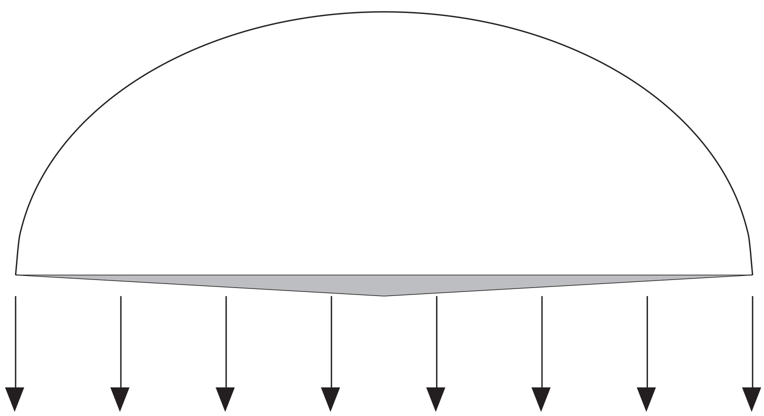

FIGURE 4.7 A fixed wing (grey) seen from behind. The strength of the bound vortex, shown by the curve above the wing, is greatest in the middle, declining to zero at the wing tips. The curve represents the local circulation, and also the lift per unit span, at each point along the span. If (and only if) this curve is half of an ellipse as shown, then the downwash velocity immediately behind the wing (arrows), caused by the shedding of a vortex sheet as in Figure 4.4B, is constant across the span. According to Prandtl's lifting-line theory, this ''elliptical lift distribution'' gives a lower induced drag coefficient than any other, for a given lift coefficient.

BOX 4.2 The actuator disc versus vortex wakes.

Variable definitions for this box

B	Wing span
C_{Dind}	Induced drag coefficient
C_{L}	Lift coefficient
D_{ind}	Induced drag
g	Acceleration due to gravity
k	Induced drag (or power) factor
m	Bird's all-up mass
P_{ind}	Induced power in horizontal flight
R_{a}	Aspect ratio
V_{t}	True airspeed
S_{wing}	Wing area
ρ	Air density

Induced drag from lifting-line theory

The induced drag of a fixed wing can be deduced from Prandtl's classical lifting-line theory, which begins by representing the circulation around the wing as a stack of horseshoe vortices as shown in Figure 4.3B. The total circulation at any particular point along the span depends on the number of vortex filaments still bound to the wing at that point. By specifying where vortex filaments are shed, the theorist can define a particular "lift distribution", in which the strength of the circulation is plotted along the wing span, from one wing tip to the other (Figure 4.6). The same graph also represents the lift-per-unit-span at each point along the span, since this is proportional to the circulation. All aeronautical textbooks (such as the excellent one by Anderson 1991) give a detailed account of the "elliptical" lift distribution, in which the graph is one half of an ellipse. This particular lift distribution is "ideal" in the sense that it produces less induced drag than any other, for a given amount of lift. It also has another (related) characteristic, that the downwash velocity immediately behind the wing, shown by the vertical arrows in Figure 4.6, is constant across the span. According to the lifting-line theory, the coefficient of induced drag (C_{Dind}) for this special case depends only on the lift coefficient (C_{L}) and the aspect ratio (R_{a}):

$$C_{\mathrm{Dind}} = \frac{C_L^2}{\pi R_{\mathrm{a}}}. \tag{1}$$

The aspect ratio (Chapter 1) is

$$R_{\mathrm{a}} = \frac{B^2}{S_{\mathrm{wing}}}, \tag{2}$$

where B is the wing span, and S_{wing} is the wing area. For a fixed-wing aircraft in level flight, the lift force must balance the weight (mg), and the lift coefficient is therefore

$$C_{\mathrm{L}} = \frac{2mg}{(\rho V_{\mathrm{t}}^2 S_{\mathrm{wing}})}, \tag{3}$$

BOX 4.2 *Continued.*

where ρ is the air density, and V_t is the true airspeed. Substituting this expression for C_L in Equation (1) makes the induced drag coefficient

$$C_{Dind} = \frac{4(mg)^2}{(\pi\rho^2 B^2 V_t^4 S_{wing})}. \tag{4}$$

The induced drag itself (D_{ind}) is

$$\begin{aligned} D_{ind} &= \frac{C_{Dind}\rho V_t^2 S_{wing}}{2} \\ &= \frac{2(mg)^2}{(\pi\rho B^2 V_t^2)} \end{aligned} \tag{5}$$

Since the induced drag for an elliptical lift distribution, given by Equation (5), is the lowest that is possible, the induced drag of a real wing has to be multiplied by an *induced drag factor* (k), which is a number that is never less than 1, to account for any deviations from the ideal lift distribution, and from constant downwash velocity along the span:

$$D_{ind} = \frac{2k(mg)^2}{(\pi\rho B^2 V_t^2)}. \tag{6}$$

This formula is used when calculating a bird's glide polar in *Flight* (Chapter 10, Box 10.1) to find the induced drag. The default value assigned to k in gliding is 1.1.

Induced power from the actuator disc

The starting point for calculating the induced *power* in flapping flight comes from helicopter theory rather than fixed-wing theory. A helicopter rotor sweeps out a circular disc, which can be approximated as an "actuator disc", whose theoretical property is that the air pressure increases in a stepwise manner as the air passes through the disc. This pressure step imparts a downward induced velocity to a tube of air that flows through the disc. The cross-sectional area of this tube is assumed to be that of a circle, whose diameter is the same as the rotor diameter in the case of a helicopter, or the wing span in the case of a bird (even though the wings do not sweep out the full area of the disc). The derivation of the induced power (P_{ind}) needed to support the weight (mg) is given in Chapter 3, Box 3.1, and the result is

$$P_{ind} = \frac{2(mg)^2}{(V_t \pi B^2 \rho)}. \tag{7}$$

This is for an "ideal" actuator disc which produces a constant induced velocity from edge to edge of its circular area, which is also the condition for minimum induced power to support a given weight. In practice, variations of induced velocity across the disc require an "induced power factor" (k), which is a number whose value is never less than 1. The default value of k in *Flight* is 1.2 for flapping flight, and the formula used for induced power in the power curve calculation is

BOX 4.2 *Continued.*

$$P_{\text{ind}} = \frac{2k(mg)^2}{(V_t \pi B^2 \rho)}. \tag{8}$$

The reader who believes that it is more "realistic" to calculate induced power from the vortex wake than from an actuator disc should notice that the induced drag for a fixed wing with an elliptical lift distribution (from Equation 5), when multiplied by the speed to get the corresponding power, gives exactly the same result as the induced power from Equation (8). The lifting line and actuator disc calculations are actually different views of the same theory. The basic assumption in both cases is that the downwash velocity is constant across the wing span.

The " bow-tie" fallacy

The advantage of the actuator disc approach for flapping flight is that it requires no information about the wings except the wing span. It does not require that the airspeed "seen" by a point on the wing is the same as the forward speed of the whole aircraft or bird, or that the local airspeed has to be constant across the span, and this permits a massive simplification in considering rotary or flapping wings. The calculation does not even require the angle through which the wings are flapped to be specified, although it does require the "disc area", through which air passes as it is accelerated downwards. If a bird or insect flaps its wings through an angle that is less than the full 180° available to each wing, then the wings sweep out a double sector shaped like a bow tie, and some authors have assumed that this "swept area" should be used instead of the area of the full circle, when calculating the induced power. If this were so, the argument of Chapter 3 Box 3.1 would require that the induced velocity would have to be inversely proportional to the swept area. Extending this line of thought to a fixed wing, in which no area is swept at all, the induced velocity would have to be infinite. However, we know from the lifting-line theory, which has been part of the bedrock of theory for generations of aeronautical engineers, that the induced velocity for a fixed (non-flapping) wing is not infinite, but the same as that for a circular actuator disc, whose diameter is the same as the wing span. Common sense is misleading in this case. The "bow-tie" concept is wrong.

most birds fly most of the time, and for which the calculations are reasonably robust. Speeds lower than V_{mp} are reserved for unsteady activities, such as landing, taking off and hawking for flying insects, while speeds higher than V_{mr} may be used, if they are used at all, by predatory birds such as falcons and skuas for pursuing other birds.

Improved understanding of the vortex wakes may make it possible to identify these characteristic speeds with particular types of wake structure, which can themselves be associated with particular values of a variable such as the reduced frequency (Box 4.3) that summarises the wing motion in a single number. If this proves to be feasible, then it may be possible to go further, and predict a value for the induced power factor (k) from the reduced frequency. When *Flight* calculates a power curve, it would then calculate the reduced frequency at each speed, and recalculate k, instead of using a fixed value for k at all speeds, as it does at present.

BOX 4.3 Reduced frequency and Strouhal number.

Wind-tunnel experimenters tend to describe different wake geometries in terms of the forward speed at which they are observed, but obviously the same wake pattern, if it is seen in different species, is not likely to occur at the same speed. To achieve a more general description, wake patterns will need to be connected with some variable that involves the ratio of the wingbeat frequency to the speed. To make such a variable dimensionless, a reference length is also required. Two dimensionless variables, the reduced frequency and the Strouhal number, which involve different reference lengths, have been introduced by theorists for various purposes.

Variable definitions for this box

A Wingtip amplitude
c_m Mean chord
f Wingbeat frequency
f_{red} Reduced frequency
St Strouhal number
V_t True airspeed

The "reduced frequency" (f_{red}) is defined by Spedding (1992) as

$$f_{red} = \frac{\pi f c_m}{V_t}, \tag{1}$$

where f is the wingbeat frequency, c_m is the mean chord (ratio of wing area to wing span) and V_t is the true airspeed. It characterises the wake geometry, being equal to the ratio of the distance that the bird travels forwards in one wingbeat cycle to the mean chord. High values of the reduced frequency (high frequency, low speed) indicate rapid changes of flow geometry through the wingbeat cycle, and the likelihood that unsteady aerodynamic effects will need to be considered, whereas a low reduced frequency (low frequency, high speed) indicates that quasi-steady aerodynamics may give a satisfactory account of the flow.

"Strouhal number" (St) is a related dimensionless variable. A version of it used by Nudds et al. (2004) is:

$$St = \frac{fA}{V_t}, \tag{2}$$

BOX 4.3 *Continued.*

where A is the wingtip amplitude, defined as the vertical linear excursion of the wing tip above and below its position with wings level. This is the same as Equation (1), except that A is substituted for πc_{m}. Some care is needed with the term "amplitude". The traditional mathematical usage refers to a sine wave, in which the value of some quantity swings in each cycle from zero to $+A$, then down to $-A$, before returning to zero. A is the amplitude, and on this definition, the difference between the positive and negative peaks (the "peak-to-peak swing") is $2A$. However, some authors define the "amplitude" as the peak-to-peak swing, and others neglect to mention what exactly they mean by the term.

The Strouhal number gives an indication of the angle with which the wing tip moves up and down, relative to the flight path, whereas the reduced frequency does not, and this could be seen as an advantage for describing wake geometries. The practical difficulty with the Strouhal number is that the wingtip amplitude is difficult to measure. Bird wing tips are thin and pointed, and are apt to disappear in photographs when seen edge-on. Measuring wingtip amplitude is challenging in the wind tunnel, and impractical in the field, where the observer has no control over the camera geometry. The reader should not be unduly impressed by papers in which large numbers of Strouhal numbers have been calculated from published observations, whose original authors were not paying special attention to measuring wingtip amplitudes, and were not measuring them in a standard way. The reduced frequency is less susceptible to such uncertainties, as it depends on the wingbeat frequency, which is easy to measure from video, and on the standard morphological variables defined in Chapter 1.

4.5 FEATHERED WINGS

There are two possible reasons why bird wakes might have different characteristics from those of fixed wings. The differences might be inherent in the flapping motion, and in that case one would expect bats to show similar wakes, with distributed vorticity and ill-defined vortex structures. Another possibility is that the feathered surface of birds' wings and bodies is responsible, and if that were the case, the wakes of birds and bats would differ. It has been noted elsewhere in this book that the boundary layer appears to remain attached far more tenaciously over a living bird's body, than over the same body at the same Reynolds number when dead and frozen, or over a smooth-surfaced model of similar shape (Chapter 15, Box 15.4). It appears from the DPIV studies that the wings of small birds are also more resistant to flow separation than model aircraft wings at a similar scale, but not much is known about the proneness or otherwise of bat wings and bodies to boundary-layer separation. Hedenström et al. (2007) have recently studied the wakes of a small nectar-feeding bat by the DPIV technique (above), but their measurements were limited to speeds from the minimum power speed downwards, which were said to be the normal speed range for the species. It may be that Microchiroptera generally are specialised for manoeuvrable flight in the unstable speed range below V_{mp}, and that they differ in that respect from the majority of birds. This would complicate comparisons between bird and bat wakes, but be that as it may, two interesting possibilities have been raised. Birds (but not bats) may be able to suppress boundary-layer separation by exploiting some property of the feathered surface, or alternatively, both birds and bats may be able to do this in some way that does not depend on feathers. Either way, new aerodynamic principles are likely to be involved.