

$\frac{\partial^2 w}{\partial x^2} = \frac{1}{k} \frac{\partial w}{\partial t}, \quad 0 < x < a, 0 < t$ $w(0,t) = 0, w(a,t) = 0, \quad 0 < t$ $w(x,0) = g(x), \quad 0 < x < a$	<i>Are the partial differential equation and boundary conditions homogeneous? Check.</i>
Look for solutions in the form $w(x,t) = \phi(x)T(t)$ with neither factor identically zero.	<i>Why require that neither factor be zero?</i>
Then $\phi''(x)T(t) = \frac{1}{k} \phi(x)T'(t)$	<i>Where did the equation come from?</i>
$\frac{\phi''(x)}{\phi(x)} = \frac{T'(t)}{kT(t)}, \quad 0 < x < a, 0 < t$	<i>Where did the equation come from?</i>
Therefore, both sides of the equation above are constant.	<i>Why are both sides constant?</i>
Also, $\phi(0)T(t) = 0, \phi(a)T(t) = 0, \quad 0 < t$	<i>Where are these equations from?</i>
Therefore, $\phi(0) = 0, \phi(a) = 0$	<i>How can we conclude this?</i>
Choosing the constant $= p^2$ gives $\frac{\phi''}{\phi} = p^2 > 0 \text{ and } \phi(0) = 0, \phi(a) = 0$ and this doesn't work.	<i>Why not?</i>
Choosing the constant $= 0$ gives $\frac{\phi''}{\phi} = 0 \text{ and } \phi(0) = 0, \phi(a) = 0$ and this doesn't work either.	<i>Why not?</i>
Choosing the constant $= -\lambda^2$ gives $\begin{cases} \phi'' + \lambda^2 \phi = 0, & 0 < x < a \\ \phi(0) = 0, \phi(a) = 0 \end{cases}$	<i>What is the name for this kind of problem? What is to be found? Solve it.</i>

<p>Solution:</p> $\lambda_n = \frac{n\pi}{a}, \quad \phi_n(x) = \sin(\lambda_n x),$ $n = 1, 2, 3, \dots$	<p><i>Where did this come from? What are names for <math>\phi</math> and <math>\lambda</math>?</i></p>
$T_n(t) = \exp(-\lambda_n^2 kt)$	<p><i>Where did this come from? Can you derive it?</i></p>
$w_n(x, t) = \sin(\lambda_n x) \exp(-\lambda_n^2 kt)$	<p><i>Where did this come from? What properties does the function have?</i></p>
$w(x, t) = \sum_{n=1}^{\infty} b_n \sin(\lambda_n x) \exp(-\lambda_n^2 kt)$	<p><i>How did the sum and the b's come in? What properties does w have?</i></p>
<p>Next, satisfy</p> $\sum_{n=1}^{\infty} b_n \sin(\lambda_n x) = g(x), \quad 0 < x < a$	<p><i>Where did this come from?</i></p>
<p>Then <math>b_n</math> must be</p> $b_n = \frac{2}{a} \int_0^a g(x) \sin\left(\frac{n\pi x}{a}\right) dx$	<p><i>Justify this equation. Why are the limits on the integration as shown?</i></p>
<p>Done!</p>	<p><i>How can we say this? What is the solution of the problem stated at the beginning?</i></p>