RAT

A general-purpose computer program for RAdiative Transfer

Version 1.0

User Manual

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Preface

At the invitation of Professor Michael F. Modest, I have started writing this version of the manual for a general-purpose computer program for radiative transfer problems. This version of the program is meant to serve as a starting point for you to further develop the finite-volume method for your own application.

This version of the computer code is the result of my work at the University of Minnesota. Special thanks go to Prof. Suhas V. Patankar for his valuable training, guidance and patience. His direct participation in the writing of this manual would definitely have made it more comprehensive and enlightening. Although he is not involved with the detail writing, his influence can be seen throughout this manual. I have followed the format of his latest book (Computation of Conduction and Duct Flow Heat Transfer, Taylor & Francis, 1991) in writing this manual.

Thanks also go to Dr. HaeOk S. Lee, who introduced me to the fascinating field of radiation heat transfer and for her guidance, especially during the initial part of my work in radiation heat transfer. I thank Prof. Roy. S. Amano for introducing me to computational fluid dynamics, which opened doors for me to pursue my knowledge in the field. I am indebt to Prof. Ephraim M. Sparrow. His support has resulted in this work on radiation heat transfer. I have benefited from numerous discussions with Dr. J. P. Moder over the years.

I would like to thank Mr. Prasenjit Rath for writing and typing the narration to some of the example problems. Lastly, but certain not least, I am grateful to my wife for her support and patience throughout the years.

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CHAPTER

ONE

INTRODUCTION

1.1 Purpose of the Manual

This manual provides examples on how to use a two-dimensional general-purpose computer program for RAdiative Transfer; called RAT hereafter. Although RAT can be used to model a variety of radiative transfer problems in two-dimensional framework, three examples are shown in this version of the manual.

1.2 Capabilities and Limitations of RAT

This version of RAT is written to solve the *steady-state* form of the radiative transfer equation using the finite-volume method of Chai et al. (1994a) and Chai and Patankar (2000). It is designed for *Cartesian* coordinates. Irregular geometries with vertical or horizontal surfaces can be handled using the procedure proposed by Chai et al. (1994b). For this class of irregular geometries, the irregularities are captured exactly and no additional approximations are introduced by using the current version of RAT. Irregular geometries with inclined surfaces (this includes geometries with curved surfaces) can also be modeled using the procedure of Chai et al. (1994b). The inclined surfaces are however, approximated using staircase-like irregular geometries consist of vertical and horizontal surfaces. As a result, additional approximations are introduced in the modeling of inclined or curved surfaces. These types of irregular geometries can be modeled more accurately using a more advanced approach (Chai et. al., 1995). This will however, makes RAT more difficult to understand and use. As a result, a simpler version of RAT is included here.

Other than the above mentioned restriction, RAT is quite general. It can handle absorbing-emitting and scattering medium. Isotropic and anisotropic scattering can be modeled. Selected mie-scattering phase functions are incorporated into RAT. Black and diffusely reflecting walls can be modeled. Symmetry boundary condition is incorporated in RAT. Inhomogeneous medium and radiative equilibrium condition can be modeled. Both SI or English units can be used with RAT, as long as a consistent set of units is used. For ease of use, the current version of RAT sets the Stefan-Boltzmann constant, σ to $W/m^2\text{-}K^4$. As a result, by default the length, mass and time must be in *m*, *kg*, and *sec*. Other units can be used by changing the value (and thus the units) of σ.

1.3 Structure of RAT

There are two main modules in RAT. These are the *invariant* portion and the *adaptation* part. As the name implies, you should not have to change the *invariant* portion of the program for almost all of your problems which fall within the general capabilities of RAT. This part contains the solution procedure (using the FV method). The *adaptation* part of RAT is where you provide the problem-specific information; such as geometry, optical properties, boundary conditions, phase functions, output etc. This manual provides three example adaptations for you to get started on using RAT.

RAT is written using FORTRAN 77. Some newer features of FORTRAN are not exploited in this version of RAT. This is done intentionally so that RAT can be run using almost all compilers and computers without modifications.

1.4 How to run RAT

Four modules are needed to run RAT. These are PARAM.FOR, COMMON.FOR, RAT.FOR and ADAPT.FOR. In this nomenclature, RAT.FOR and ADAPT.FOR are the *invariant* part and the *adaptation* portion of the program. COMMON.FOR contains all the common block related variables. PARAM.FOR contains the parameters for the program.

For case-sensitive compilers and/or operating systems, the first two files, namely, PARAM.FOR and COMMON.FOR must be stored in *upper*-case. The other two modules can be in either upper or lower case. You must compile and link both RAT.FOR and ADAPT.FOR to create an executable file. The results can then be obtained by running the executable file. Note that since RAT.FOR does not change from problem-to-problem, you will need to compile it once. However, you should recompile ADAPT.FOR every time you make changes to it.

It is important that the parameters in PARAM.FOR are set properly. The meanings of the parameters are given in the nomenclature (Appendix B).

CHAPTER

TWO

SAMPLE PROBLEMS

2.1 Black, Square Enclosure with Absorbing and Isothermal Medium (Example 1)

2.1-1 Problem Description

The problem under consideration is a steady-state radiation in participating hot medium surrounded by a black enclosure of square shape as shown in Fig. 2.1.1. The hot medium (at T_g) is assumed absorbing, emitting but non-scattering. The boundaries are at a prescribed temperature T_w. The medium has an uniform absorptivity κ. For the present problem the following values are used.

$$T_w = 0\text{ K}, \qquad \varepsilon_w = 1, \qquad \kappa = 10\ m^{-1}, \qquad T_g = \left(\frac{1}{\sigma}\right)^{1/4} \tag{2.1.1}$$

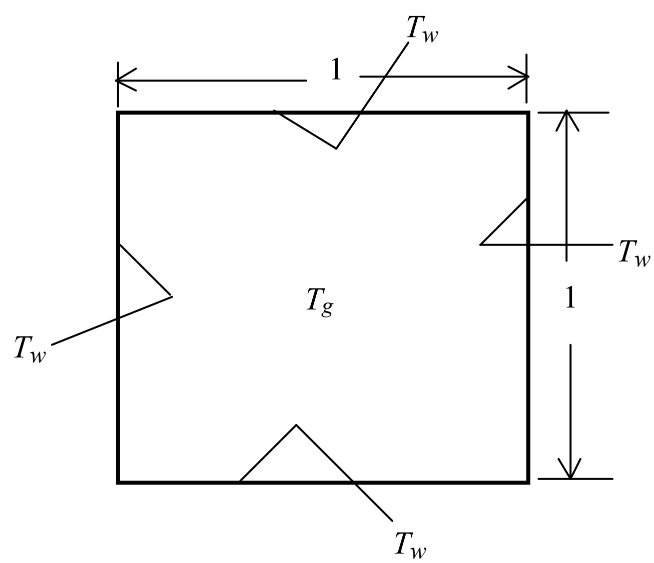


Fig. 2.1.1 Radiation in an absorbing and isothermal medium.

where σ is Stefan-Boltzmann constant. Our aim is to calculate the total irradiation and boundary radiative heat flux distribution.

2.1-2 Design of ADAPT

GRID. The title of the field printout is set to 'G' through `TITLE (1)`. The output file (`PROB1.DAT`) is then specified via `OPEN`. By default, the angular domains are $0 \le \theta \le \pi$ and $0 \le \phi \le 2\pi$. These are specified in `DEFLT` using the variables `TL` and `PL` respectively. Two control angles represented by `NCVLT = 2` are taken in the θ-direction and 4 control angles represented by `NCVLP = 4` are taken in the ϕ-direction respectively. The default values of `POWERT = 1` and `POWERP = 1` are used to generate angular grids with uniform $\Delta\theta$ and $\Delta\phi$. The boundaries of the control angles are calculated by calling `QUAD`. The spatial domains are $0 \le x \le 1$ and $0 \le y \le 1$ which are specified through `XL` and `YL` respectively. Ten control volumes are used in the x and y directions which are represented by `NCVLX = 10` and `NCVLY = 10` respectively. The default values of `POWERX = 1` and `POWERY = 1` are used. As a result, an uniform spatial grid is created by calling `EZGRID`.

START. Numerical values of all boundary conditions as given in Eq. (2.1.1) are set here. The maximum number of iterations for the present problem are set as `LAST = 20`. The value of absorption coefficient is taken as `ALPHA = 10`. Then we fill `T(I,J)` array by `TEM`, which serves as the temperature of hot gases at all *interior* control volumes. Boundary temperature is kept at the default value as all boundaries are at absolute zero temperature for the present problem.

LC. Inhomogeneous medium is set here. The present problem is homogeneous as absorption coefficient of the medium is constant here. The homogeneous absorption coefficient was specified in START.

OUTPUT. For each iteration `(ITER)` the value of an actual intensity, `F(I,J,L,M)` at the center of the enclosure is printed and maximum difference in magnitude of the intensity from previous iteration, `DMAX` (which is printed as `DIFF-MAX`) over all control volumes and control angles is printed for verifying the convergence of the solution. The default value of `EROR = 1.E-6` is used in this example for satisfying the convergence criteria. When the maximum number of iterations has been reached (`ITER = LAST`), or when the maximum change of the radiative intensities between two successive iterations falls below the prescribed value `(KSTOP = 1)`, the boundary heat fluxes and incident radiation energy are calculated by calling `HFLUX`. Then incident radiation energy, `G(I,J)` is nondimensionalised at all control volume nodes by dividing it by 4. All grid related variables and non-dimensional incident radiation energy at control volume nodes are printed by calling `PRINT`. `QTOP` represents the net radiative heat flux at the top boundary and `QBOT` represents the net radiative heat flux at the bottom boundary. `X(I)` represents the value of `X` at grid location `I`. Magnitude of `QTOP` and magnitude of `QBOT` are printed here for different `X(I)`. `QLEFT` represents the net radiative heat flux at the left boundary and `QRITE` represents the net radiative heat flux at the right boundary. `Y(J)` represents the value of `Y` at grid location `J`. Magnitude of `QLEFT` and magnitude of `QRITE` are printed here for different `Y(J)`.

GAMSOR. Irregular geometries and radiative equilibrium conditions are set here. Since in our present problem geometry of the enclosure is not irregular and it is in non-radiative equilibrium condition, hence no operation is performed here.

2.1-3 Additional Fortran Names

DMAX	maximum of $\left| \left(I - I_{OLD} \right) / I \right|$ calculated over all control volumes and control angles
LAST	maximum number of iterations
ALPHA	absorption coefficient
STFAN	Stefan-Boltzmann constant
G(I,J)	incident radiation
TEM	non-dimensional gas temperature inside the enclosure
QTOP	net radiative heat flux at top boundary
QBOT	net radiative heat flux at bottom boundary
QLEFT	net radiative heat flux at left boundary
QRITE	net radiative heat flux at right boundary

2.1-4 Listing of ADAPT for Example 1

```
C*********************************************************************
      SUBROUTINE ADAPT
C*********************************************************************
      INCLUDE 'PARAM.FOR'
      INCLUDE 'COMMON.FOR'
C*********************************************************************
C  PROBLEM 1: BLACK, SQUARE ENCLOSURE WITH ABSORBING
C                   AND ISOTHERMAL MEDIUM
C*********************************************************************
      ENTRY GRID
C
      TITLE(1)=' G '
      OPEN(7,FILE='PROB1.DAT')
C
      NCVLP=4
      NCVLT=2
C
      CALL QUAD
C
      NCVLX=10
      NCVLY=10
C
      XL=1.
      YL=1.
C
      CALL EZGRID
C
      RETURN
C*********************************************************************
      ENTRY START
C
      LAST=20
      ALPHA=10.
C
      TEM=(1./STFAN)**(1./4.)
```

```
C
      DO 110 J=2,M2
         DO 111 I=2,L2
            T(I,J)=TEM
111      CONTINUE
110    CONTINUE
C
      RETURN
C*******************************************************************
      ENTRY LC
C
      RETURN
C*******************************************************************
      ENTRY OUTPUT
C
      IF(ITER.EQ.0) WRITE(6,500)
      WRITE(6,501) ITER, F(L1/2,M1/2,2,2),DMAX
      IF(ITER.EQ.0) WRITE(7,500)
      WRITE(7,501) ITER, F(L1/2,M1/2,2,2),DMAX
C
      IF(KSTOP.EQ.1.OR.ITER.EQ.LAST) THEN
         PAUSE
         CALL HFLUX
C
       DO 521 J=1,M1
          DO 522 I=1,L1
            G(I,J)=G(I,J)/4.
522       CONTINUE
521    CONTINUE
C
         CALL PRINT
         PAUSE
C
         WRITE(6,502)
         WRITE(7,502)
C
         DO 510  I=2,L2
            WRITE(6,503) X(I),ABS(QPY(I,M1)-QMY(I,M1)),
1                         ABS(QPY(I,1)-QMY(I,1))
            WRITE(7,503) X(I),ABS(QPY(I,M1)-QMY(I,M1)),
1                         ABS(QPY(I,1)-QMY(I,1))
510      CONTINUE
C
         WRITE(6,504)
         WRITE(7,504)
C
         DO 511 J=2,M2
            WRITE(6,503) Y(J),ABS(QPX(1,J)-QMX(1,J)),
1                         ABS(QPX(L1,J)-QMX(L1,J))
            WRITE(7,503) Y(J),ABS(QPX(1,J)-QMX(1,J)),
1                         ABS(QPX(L1,J)-QMX(L1,J))
511      CONTINUE
C
      ENDIF
C
 500  FORMAT(/3X,'ITER',8X,'F',12X,'DIFF-MAX'/1X,48('*'))
 501  FORMAT(3X,I3,2(3X,1PE12.3))
 502  FORMAT(/8X,'X',10X,'QTOP',7X,'QBOT'/1X,39('*'))
 503  FORMAT(1X,3(1PE12.3))
 504  FORMAT(/8X,'Y',9X,'QLEFT',7X,'QRITE'/1X,39('*'))
C
      RETURN
C*******************************************************************
      ENTRY GAMSOR
```

```
C
      RETURN
      END
C*****************************************************************
```

2.1-5 Results for Example 1

```
   ITER        F            DIFF-MAX
 **********************************************
    0      0.000E+00      0.000E+00
    1      3.158E-01      1.000E+00
    2      3.158E-01      0.000E+00

 I =     1        2        3        4        5        6        7
 X = 0.00E+00 5.00E-02 1.50E-01 2.50E-01 3.50E-01 4.50E-01 5.50E-01
 XU= 0.00E+00 0.00E+00 1.00E-01 2.00E-01 3.00E-01 4.00E-01 5.00E-01

 I =     8        9       10       11       12
 X = 6.50E-01 7.50E-01 8.50E-01 9.50E-01 1.00E+00
 XU= 6.00E-01 7.00E-01 8.00E-01 9.00E-01 1.00E+00

 J =     1        2        3        4        5        6        7
 Y = 0.00E+00 5.00E-02 1.50E-01 2.50E-01 3.50E-01 4.50E-01 5.50E-01
 YV= 0.00E+00 0.00E+00 1.00E-01 2.00E-01 3.00E-01 4.00E-01 5.00E-01

 J =     8        9       10       11       12
 Y = 6.50E-01 7.50E-01 8.50E-01 9.50E-01 1.00E+00
 YV= 6.00E-01 7.00E-01 8.00E-01 9.00E-01 1.00E+00

 L =     1        2        3        4
 TH = 0.00E+00 7.85E-01 2.36E+00 3.14E+00

 M =     1        2        3        4        5        6
 PH = 0.00E+00 7.85E-01 2.36E+00 3.93E+00 5.50E+00 6.28E+00

 ******           G            ******
         --------------------

  I =     1        2        3        4        5        6        7
  J
 12     0.00E+00 4.17E-01 4.72E-01 4.91E-01 4.97E-01 4.99E-01 4.99E-01
 11     4.17E-01 7.08E-01 7.95E-01 8.21E-01 8.29E-01 8.32E-01 8.32E-01
 10     4.72E-01 7.95E-01 8.98E-01 9.30E-01 9.40E-01 9.43E-01 9.43E-01
  9     4.91E-01 8.21E-01 9.30E-01 9.65E-01 9.76E-01 9.79E-01 9.79E-01
  8     4.97E-01 8.29E-01 9.40E-01 9.76E-01 9.88E-01 9.91E-01 9.91E-01
  7     4.99E-01 8.32E-01 9.43E-01 9.79E-01 9.91E-01 9.95E-01 9.95E-01
  6     4.99E-01 8.32E-01 9.43E-01 9.79E-01 9.91E-01 9.95E-01 9.95E-01
  5     4.97E-01 8.29E-01 9.40E-01 9.76E-01 9.88E-01 9.91E-01 9.91E-01
  4     4.91E-01 8.21E-01 9.30E-01 9.65E-01 9.76E-01 9.79E-01 9.79E-01
  3     4.72E-01 7.95E-01 8.98E-01 9.30E-01 9.40E-01 9.43E-01 9.43E-01
  2     4.17E-01 7.08E-01 7.95E-01 8.21E-01 8.29E-01 8.32E-01 8.32E-01
  1     0.00E+00 4.17E-01 4.72E-01 4.91E-01 4.97E-01 4.99E-01 4.99E-01

  I =     8        9       10       11       12
  J
 12     4.97E-01 4.91E-01 4.72E-01 4.17E-01 0.00E+00
 11     8.29E-01 8.21E-01 7.95E-01 7.08E-01 4.17E-01
 10     9.40E-01 9.30E-01 8.98E-01 7.95E-01 4.72E-01
  9     9.76E-01 9.65E-01 9.30E-01 8.21E-01 4.91E-01
  8     9.88E-01 9.76E-01 9.40E-01 8.29E-01 4.97E-01
  7     9.91E-01 9.79E-01 9.43E-01 8.32E-01 4.99E-01
  6     9.91E-01 9.79E-01 9.43E-01 8.32E-01 4.99E-01
  5     9.88E-01 9.76E-01 9.40E-01 8.29E-01 4.97E-01
  4     9.76E-01 9.65E-01 9.30E-01 8.21E-01 4.91E-01
```

```
 3     9.40E-01 9.30E-01 8.98E-01 7.95E-01 4.72E-01
 2     8.29E-01 8.21E-01 7.95E-01 7.08E-01 4.17E-01
 1     4.97E-01 4.91E-01 4.72E-01 4.17E-01 0.00E+00

        X            QTOP         QBOT
***************************************
   5.000E-02   8.333E-01   8.333E-01
   1.500E-01   9.444E-01   9.444E-01
   2.500E-01   9.814E-01   9.814E-01
   3.500E-01   9.936E-01   9.936E-01
   4.500E-01   9.972E-01   9.972E-01
   5.500E-01   9.972E-01   9.972E-01
   6.500E-01   9.936E-01   9.936E-01
   7.500E-01   9.814E-01   9.814E-01
   8.500E-01   9.444E-01   9.444E-01
   9.500E-01   8.333E-01   8.333E-01

        Y           QLEFT        QRITE
***************************************
   5.000E-02   8.333E-01   8.333E-01
   1.500E-01   9.444E-01   9.444E-01
   2.500E-01   9.814E-01   9.814E-01
   3.500E-01   9.936E-01   9.936E-01
   4.500E-01   9.972E-01   9.972E-01
   5.500E-01   9.972E-01   9.972E-01
   6.500E-01   9.936E-01   9.936E-01
   7.500E-01   9.814E-01   9.814E-01
   8.500E-01   9.444E-01   9.444E-01
   9.500E-01   8.333E-01   8.333E-01
```

2.1-6 Discussion of Results

It can be seen that the solution is converged in one iteration. This is because the medium is non-scattering and the walls are non-reflecting walls. In the result lists for the present problem, along x-direction `X(I)` and `XU(I)` represents the value of `X` at grid location `I` and the value of `X` for the corresponding control volume face. Similarly, along y-direction `Y(J)` and `YV(J)` represents the value of `Y` at grid location `J` and value of `Y` for the corresponding control volume face. Angular grid contains grid related information in θ and ϕ-directions respectively. In the θ-direction `TH(L)` represents the value of θ at the grid location `L` and in the ϕ-direction `PH(M)` represents the value of ϕ at the grid location `M`. The final field printout of incident radiation energy shows that the effect of hot gases inside the enclosure is to create a maximum irradiation (incident radiation energy), `G(I,J)` at the center of the enclosure whose non-dimensional value is numerically evaluated by finite-volume method as 0.995. The distribution of irradiation inside the medium is plotted as shown in Fig. 2.1.2 for absorption coefficients, $\kappa = 10\ m^{-1}$ and $\kappa = 1\ m^{-1}$. As we move towards the boundary from the center of the enclosure the magnitude of irradiation decreases and we get minimum irradiation at the boundaries. It was also noted that the irradiation is symmetrical about the vertical and horizontal centerlines of the enclosure. From Fig. 2.1.2 it is seen that as the value of absorption coefficient, κ decreases the magnitude of the incident radiation energy decreases. It is because the magnitude of intensity decreases along the path of travel as absorption coefficient decreases. The field printout of boundary heat fluxes shows that the net radiative heat fluxes are maximum at the center of each boundary and are symmetrical about the center of the boundaries.

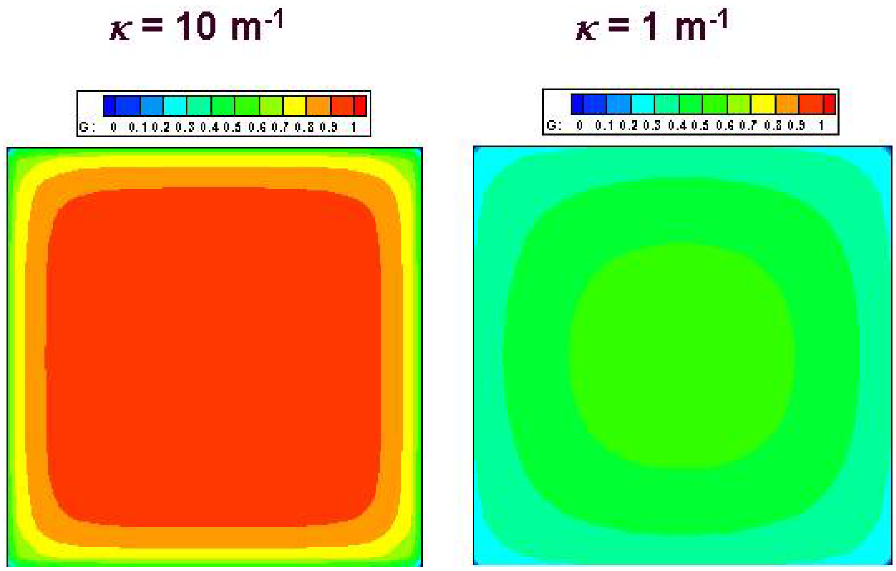


Fig. 2.1.2 Distribution of incident radiation energy for absorption coefficients, $\kappa = 10\ m^{-1}$ and $\kappa = 1\ m^{-1}$.

2.1-7 Final Remarks

The distribution of incident radiation energy and wall heat fluxes in a square enclosure are presented in this section. The medium inside the enclosure is absorbing and emitting. It was noted that the solution converged well after one iteration since the walls are non-reflecting and temperature of all the four walls are known. Effect of imposing the symmetry condition at the boundaries can also be studied for this present problem which is discussed in the next example.

2.2 Black, Square Enclosure with Absorbing and Isothermal Medium with Symmetry Condition at Right and Bottom Boundaries (Example 2)

2.2-1 Problem Description

The problem under consideration is same as Example 1. Due to symmetries, one-quarter of the domain in Example 1 is simulated. It is a steady-state radiation problem in participating hot medium surrounded by a black enclosure of square shape as shown in Fig. 2.2.1. The medium is assumed absorbing, emitting but non-scattering. Left and top walls are at a prescribed temperature T_w. The medium has an uniform absorptivity κ. For the present problem the following values are used.

$$T_W = 0\text{ K},\qquad \varepsilon_W = 1,\qquad \kappa = 10\ m^{-1},\qquad T_g = \left(\frac{1}{\sigma}\right)^{1/4} \tag{2.2.1}$$

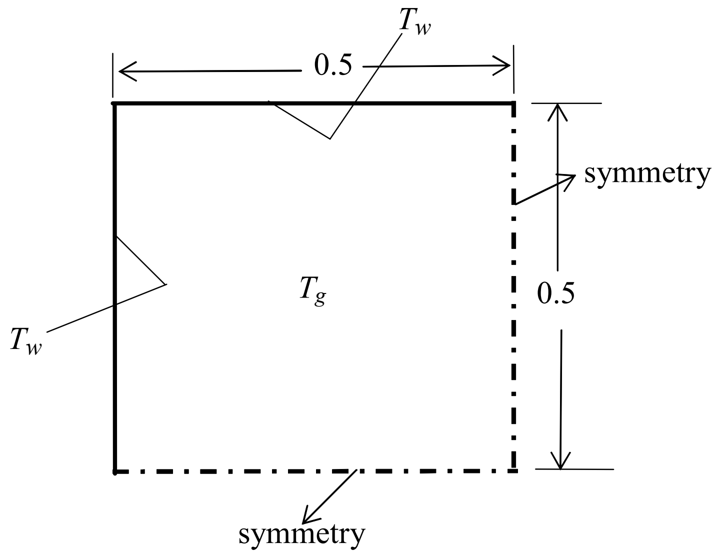


Fig. 2.2.1 Radiation in an absorbing and isothermal medium.

where σ is Stefan-Boltzmann constant. Our aim is to calculate the total irradiation and boundary radiative heat flux distribution.

2.2-2 Design of ADAPT

GRID. The title of the field printout is set to 'G' through `TITLE (1)`. The output file (`PROB2.DAT`) is then specified via `OPEN`. By default, the angular domains are $0 \le \theta \le \pi$ and $0 \le \phi \le 2\pi$. These are specified in `DEFLT` using the variables `TL` and `PL` respectively. Two control angles represented by `NCVLT = 2` are taken in the θ-direction and 4 control angles represented by `NCVLP = 4` are taken in the ϕ-direction respectively. The default value of `POWERT = 1` and `POWERP = 1` are used to generate angular grids with uniform $\Delta\theta$ and $\Delta\phi$. The boundaries of the control angles are calculated by calling `QUAD`. The spatial domains are $0 \le x \le 0.5$ and $0 \le y \le 0.5$ which are specified through `XL` and `YL` respectively. Five control volumes are used in the x and y directions which are represented by `NCVLX = 5` and `NCVLY = 5` respectively. The default values of `POWERX = 1` and `POWERY = 1` are used. As a result, an uniform spatial grid is created by calling `EZGRID`.

START. Numerical values of all boundary conditions as given in Eq. (2.2.1) are set here. The maximum number of iterations for the present problem are set as `LAST = 20`. The value of absorption coefficient is taken as `ALPHA = 10`. Then we fill `T(I,J)` array by `TEM`, which serves as the temperature of hot gases at all *interior* control volumes. `KBCL1(J) = 2` and `KBCJ1(I) = 2` are set for symmetry along east (`KBCL1`) and south (`KBCJ1`) boundaries respectively. Boundary temperature is kept at the default value as west and north boundaries are at absolute zero temperature for the present problem.

LC. Inhomogeneous medium is set here. The present problem is homogeneous as absorption coefficient of the medium is constant here. The homogeneous absorption coefficient was specified in START.

OUTPUT. For each iteration (ITER) the value of an actual intensity, F(I,J,L,M) at the center of the enclosure is printed and maximum difference in magnitude of the intensity from previous iteration, DMAX (which is printed as DIFF-MAX) over all control volumes and control angles is printed for verifying the convergence of the solution. The default value of EROR = 1.E-6 is used in this example for satisfying the convergence criteria. When the maximum number of iterations has been reached (ITER = LAST), or when the maximum change of the radiative intensities between two successive iterations falls below the prescribed value (KSTOP = 1), the boundary heat fluxes and incident radiation energy are calculated by calling HFLUX. Then incident radiation energy, G(I,J) is nondimensionalised at all control volume nodes by dividing it by 4. All grid related variables and non-dimensional incident radiation energy at control volume nodes are printed by calling PRINT. QTOP represents the net radiative heat flux at the top boundary and QBOT represents the net radiative heat flux at the bottom boundary. X(I) represents the value of X at grid location I. Magnitude of QTOP and magnitude of QBOT are printed here for different X(I). QLEFT represents the net radiative heat flux at the left boundary and QRITE represents the net radiative heat flux at the right boundary. Y(J) represents the value of Y at grid location J. Magnitude of QLEFT and magnitude of QRITE are printed here for different Y(J).

GAMSOR. Irregular geometries and radiative equilibrium conditions are set here. Since in our present problem geometry of the enclosure is not irregular and it is in non-radiative equilibrium condition, hence no operation is performed here.

2.2-3 Additional Fortran Names

DMAX	maximum of $\left|\left(I - I_{OLD}\right)/I\right|$ calculated over all control volumes and control angles
LAST	maximum number of iterations
KBCL1(J)	set for type of boundary condition along east boundary
KBCJ1(I)	set for type of boundary condition along south boundary
ALPHA	absorption coefficient
STFAN	Stefan-Boltzmann constant
G(I,J)	incident radiation
TEM	non-dimensional gas temperature inside the enclosure
QTOP	net radiative heat flux at top boundary
QBOT	net radiative heat flux at bottom boundary
QLEFT	net radiative heat flux at left boundary
QRITE	net radiative heat flux at right boundary

2.2-4 Listing of ADAPT for Example 2

```
C*******************************************************************
      SUBROUTINE ADAPT
C*******************************************************************
      INCLUDE 'PARAM.FOR'
      INCLUDE 'COMMON.FOR'
C*******************************************************************
C  PROBLEM 2: BLACK, SQUARE ENCLOSURE WITH ABSORBING
C             AND ISOTHERMAL MEDIUM WITH SYMMETRIES
C             AT THE RIGHT AND BOTTOM BOUNDARIES
C*******************************************************************
C
      ENTRY GRID
C
      TITLE(1)=' G '
      OPEN(7,FILE='PROB2.DAT')
C
      NCVLP=4
      NCVLT=2
C
      CALL QUAD
C
      NCVLX=5
      NCVLY=5
C
      XL=0.5
      YL=0.5
C
      CALL EZGRID
C
      RETURN
C*******************************************************************
      ENTRY START
C
      LAST=20
      ALPHA=10.
C
      TEM=(1./STFAN)**(1./4.)
C
      DO 110 J=2,M2
         DO 111 I=2,L2
            T(I,J)=TEM
            KBCL1(J)=2
            KBCJ1(I)=2
111      CONTINUE
110   CONTINUE
C
      RETURN
C*******************************************************************
      ENTRY LC
C
      RETURN
C*******************************************************************
      ENTRY OUTPUT
C
      IF(ITER.EQ.0) WRITE(6,500)
      WRITE(6,501) ITER, F(L1/2,M1/2,2,2),DMAX
      IF(ITER.EQ.0) WRITE(7,500)
      WRITE(7,501) ITER, F(L1/2,M1/2,2,2),DMAX
C
      IF(KSTOP.EQ.1.OR.ITER.EQ.LAST) THEN
         PAUSE
         CALL HFLUX
```

```
C
    DO 521 J=1,M1
       DO 522 I=1,L1
          G(I,J)=G(I,J)/4.
522    CONTINUE
521 CONTINUE
C
         CALL PRINT
         PAUSE
C
         WRITE(6,502)
         WRITE(7,502)
C
         DO 510 I=2,L2
            WRITE(6,503) X(I),ABS(QPY(I,M1)-QMY(I,M1)),
1                     ABS(QPY(I,1)-QMY(I,1))
            WRITE(7,503) X(I),ABS(QPY(I,M1)-QMY(I,M1)),
1                     ABS(QPY(I,1)-QMY(I,1))
510      CONTINUE
C
         WRITE(6,504)
         WRITE(7,504)
C
         DO 511 J=2,M2
            WRITE(6,503) Y(J),ABS(QPX(1,J)-QMX(1,J)),
1                        ABS(QPX(L1,J)-QMX(L1,J))
            WRITE(7,503) Y(J),ABS(QPX(1,J)-QMX(1,J)),
1                        ABS(QPX(L1,J)-QMX(L1,J))
511      CONTINUE
C
      ENDIF
C
 500  FORMAT(/3X,'ITER',8X,'F',12X,'DIFF-MAX'/1X,48('*'))
 501  FORMAT(3X,I3,2(3X,1PE12.3))
 502  FORMAT(/8X,'X',10X,'QTOP',7X,'QBOT'/1X,39('*'))
 503  FORMAT(1X,3(1PE12.3))
 504  FORMAT(/8X,'Y',9X,'QLEFT',7X,'QRITE'/1X,39('*'))
C
      RETURN
C*******************************************************************
      ENTRY GAMSOR
C
      RETURN
      END
C*******************************************************************
```

2.2-5 Results for Example 2

```
   ITER         F           DIFF-MAX
 ************************************************
     0       0.000E+00      0.000E+00
     1       2.586E-01      1.000E+00
     2       2.829E-01      3.998E-01
     3       2.829E-01      1.648E-01
     4       2.829E-01      0.000E+00

 I =     1        2        3        4        5        6        7
 X = 0.00E+00 5.00E-02 1.50E-01 2.50E-01 3.50E-01 4.50E-01 5.00E-01
 XU= 0.00E+00 0.00E+00 1.00E-01 2.00E-01 3.00E-01 4.00E-01 5.00E-01

 J =     1        2        3        4        5        6        7
 Y = 0.00E+00 5.00E-02 1.50E-01 2.50E-01 3.50E-01 4.50E-01 5.00E-01
 YV= 0.00E+00 0.00E+00 1.00E-01 2.00E-01 3.00E-01 4.00E-01 5.00E-01
```

```
 L =      1        2        3        4
TH = 0.00E+00 7.85E-01 2.36E+00 3.14E+00

M =      1        2        3        4        5        6
PH = 0.00E+00 7.85E-01 2.36E+00 3.93E+00 5.50E+00 6.28E+00

******            G            ******
        --------------------

 I =      1        2        3        4        5        6        7
 J
 7    0.00E+00 4.17E-01 4.72E-01 4.91E-01 4.97E-01 4.99E-01 0.00E+00
 6    4.17E-01 7.08E-01 7.95E-01 8.21E-01 8.29E-01 8.32E-01 8.32E-01
 5    4.72E-01 7.95E-01 8.98E-01 9.30E-01 9.40E-01 9.43E-01 9.43E-01
 4    4.91E-01 8.21E-01 9.30E-01 9.65E-01 9.76E-01 9.79E-01 9.79E-01
 3    4.97E-01 8.29E-01 9.40E-01 9.76E-01 9.88E-01 9.91E-01 9.91E-01
 2    4.99E-01 8.32E-01 9.43E-01 9.79E-01 9.91E-01 9.95E-01 9.95E-01
 1    0.00E+00 8.32E-01 9.43E-01 9.79E-01 9.91E-01 9.95E-01 0.00E+00

       X           QTOP        QBOT
***************************************
   5.000E-02   8.333E-01   0.000E+00
   1.500E-01   9.444E-01   5.960E-08
   2.500E-01   9.814E-01   5.960E-08
   3.500E-01   9.936E-01   5.960E-08
   4.500E-01   9.972E-01   5.960E-08

       Y          QLEFT        QRITE
***************************************
   5.000E-02   9.972E-01   5.960E-08
   1.500E-01   9.936E-01   5.960E-08
   2.500E-01   9.814E-01   5.960E-08
   3.500E-01   9.444E-01   5.960E-08
   4.500E-01   8.333E-01   0.000E+00
```

2.2-6 Discussion of Results

It can be seen that the solution is not converged in one iteration. This is because of the fact that due to the unknown temperature of right and bottom boundaries, intensities are unknown in these boundaries. Hence, iteration procedure starts by guessing intensity in the boundaries where temperature is unknown. In the result lists for the present problem, along *x*-direction `X(I)` and `XU(I)` represents the value of `X` at grid location `I` and the value of `X` for the corresponding control volume face. Similarly, along *y*-direction `Y(J)` and `YV(J)` represents the value of `Y` at grid location `J` and value of `Y` for the corresponding control volume face. Angular grid contains grid related information in θ and ϕ-directions respectively. In the θ-direction `TH(L)` represents the value of θ at the grid location `L` and in the ϕ-direction `PH(M)` represents the value of ϕ at the grid location `M`. The final field printout of incident radiation energy shows that the effect of hot gases inside the enclosure is to create a maximum irradiation (incident radiation energy), `G(I,J)` along east and south walls of the enclosure whose maximum value is 0.995 which is same as we got from example 1. This is because the present problem is same as previous example, but we are solving only the one-forth of the previous example. The distribution of irradiation inside the medium is plotted as shown in Fig. 2.2.2 for absorption coefficients, $\kappa = 10\ m^{-1}$ and $\kappa = 1\ m^{-1}$. As we move towards north-west boundary from south-east boundary of the enclosure the magnitude of irradiation decreases and we get minimum irradiation in north and west boundaries. From Fig. 2.2.2 it is seen that as the value of absorption coefficient, κ decreases the magnitude incident radiation energy decreases. It is because the magnitude of the intensity decreases along the path of travel as absorption coefficient decreases. The field printout of boundary heat fluxes shows that the net radiative heat fluxes are maximum at the top and left boundaries and minimum at bottom and right boundaries. This is because the bottom and right boundaries are hot. Hence, heat flux due to its own boundary temperature is existing which is nullifying the heat flux reaching to that boundary from all other boundaries. Therefore, the absolute value of net radiative heat flux decreases.

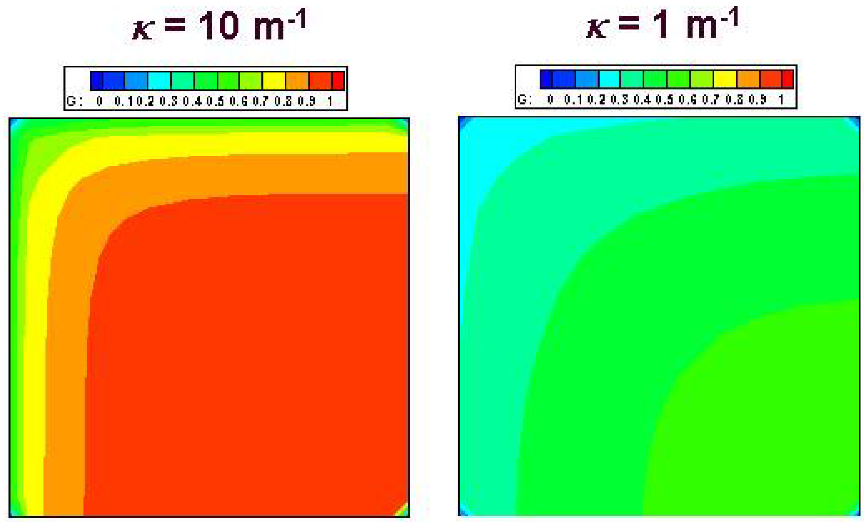


Fig. 2.2.2 Distribution of incident radiation energy for absorption coefficients, $\kappa = 10\ m^{-1}$ and $\kappa = 1\ m^{-1}$.

2.2-7 Final Remarks

Effect of imposing the symmetry boundary condition on distribution of incident radiation energy and wall heat fluxes has been studied in this example problem. Here, the solution is not converged in one iteration as intensity at the symmetry boundaries are unknown due to unknown temperature on these boundaries. Till now, the examples we have discussed where medium is assumed non-scattering. Effect of imposing the scattering medium is also studied which is discussed in the next example.

2.3 Black, Square Enclosure with Absorbing and Anisotropically Scattering Medium and Hot Bottom Wall (Example 3)

2.3-1 Problem Description

The problem under consideration is steady-state radiation in an absorbing and anisotropically scattering medium surrounded by a black enclosure of square shape as shown in Fig. 2.3.1. The bottom wall is kept hot and other three walls are kept at absolute zero temperature. The medium has an uniform absorptivity κ and scattering coefficient σ_s. For the present problem the following values are used.

$$T_{bottom} = \left(\frac{1}{\sigma}\right)^{1/4}, \ T_{top} = T_{left} = T_{right} = 0 \text{ K}, \ \ T_g = 0 \text{ K}, \ \varepsilon_w = 1, \ \ \kappa = 0.5 \ m^{-1}, \ \sigma_s = 0.5 \ m^{-1} \quad (2.3.1)$$

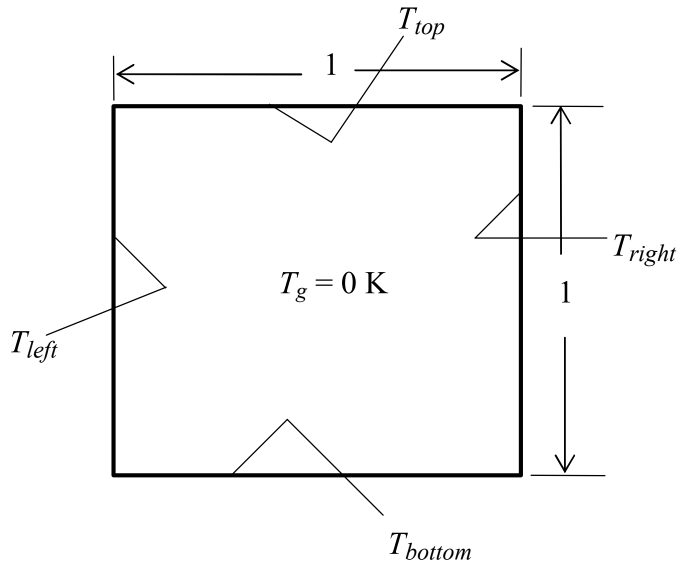


Fig. 2.3.1 Radiation in an absorbing and anisotropically scattering medium.

where σ is Stefan-Boltzmann constant. Our aim is to calculate the total irradiation and boundary radiative heat flux distribution.

2.3-2 Design of ADAPT

GRID. The title of the field printout is set to 'G' through `TITLE (1)`. The output file (`PROB3.DAT`) is then specified via `OPEN`. By default, the angular domains are $0 \le \theta \le \pi$ and $0 \le \phi \le 2\pi$. These are specified in `DEFLT` using the variables `TL` and `PL` respectively. Two control angles represented by `NCVLT = 2` are taken in the θ-direction and 8 control angles represented by `NCVLP = 8` are taken in the ϕ-direction respectively. The default value of `POWERT = 1` and `POWERP = 1` are used to generate angular grids with uniform $\Delta\theta$ and $\Delta\phi$. The boundaries of the control angles are calculated by calling `QUAD`. The spatial domains are $0 \le x \le 1$ and $0 \le y \le 1$ which are specified through `XL` and `YL` respectively. Five control volumes are used in the x and y directions which are represented by `NCVLX = 5` and `NCVLY = 5` respectively. The default values of `POWERX = 1` and `POWERY = 1` are used. As a result, an uniform spatial grid is created by calling `EZGRID`.

START. Numerical values of all boundary conditions as given in Eq. (2.3.1) are set here. The maximum number of iterations for the present problem are set as `LAST = 20`. Anisotropic scattering medium is set as `KISO = 0`. Back scattering is specified by `KPHASE = 7`. The value of absorption coefficient and scattering coefficient are taken as `ALPHA = 0.5` and `SIG = 0.5` respectively. Then we fill `T(I,1)` array by `TEM`, which serves as the temperature of hot bottom wall. Other boundary temperatures are kept at the default value as all the boundaries except the bottom boundary are at absolute zero temperature for the present problem.

LC. Inhomogeneous medium is set here. The present problem is homogeneous as absorption coefficient and scattering coefficient of the medium is constant here. The homogeneous absorption coefficient and scattering coefficient was specified in START.

OUTPUT. For each iteration (ITER) the value of an actual intensity, F(I,J,L,M) at the center of the enclosure is printed and maximum difference in magnitude of the intensity from previous iteration, DMAX (which is printed as DIFF-MAX) over all control volumes and control angles is printed for verifying the convergence of the solution. The default value of EROR = 1.E-6 is used in this example for satisfying the convergence criteria. When the maximum number of iterations has been reached (ITER = LAST), or when the maximum change of the radiative intensities between two successive iterations falls below the prescribed value (KSTOP = 1), the boundary heat fluxes and incident radiation energy are calculated by calling HFLUX. All grid related variables and incident radiation energy at control volume nodes are printed by calling PRINT. QTOP represents the net radiative heat flux at the top boundary and QBOT represents the net radiative heat flux at the bottom boundary. X(I) represents the value of X at grid location I. Magnitude of QTOP and magnitude of QBOT are printed here for different X(I). QLEFT represents the net radiative heat flux at the left boundary and QRITE represents the net radiative heat flux at the right boundary. Y(J) represents the value of Y at grid location J. Magnitude of QLEFT and magnitude of QRITE are printed here for different Y(J).

GAMSOR. Irregular geometries and radiative equilibrium conditions are set here. Since in our present problem geometry of the enclosure is not irregular and it is in non-radiative equilibrium condition, hence no operation is performed here.

2.3-3 Additional Fortran Names

DMAX	maximum of $\left|\left(I - I_{OLD}\right)/I\right|$ calculated over all control volumes and control angles
LAST	maximum number of iterations
ALPHA	absorption coefficient
SIG	scattering coefficient
KISO	set for type of scattering
KPHASE	set for back scattering
STFAN	Stefan-Boltzmann constant
G(I,J)	incident radiation energy
TEM	non-dimensional temperature of bottom wall
QTOP	net radiative heat flux at top boundary
QBOT	net radiative heat flux at bottom boundary
QLEFT	net radiative heat flux at left boundary
QRITE	net radiative heat flux at right boundary

2.3-4 Listing of ADAPT for Example 3

```
C*********************************************************************
      SUBROUTINE ADAPT
C*********************************************************************
      INCLUDE 'PARAM.FOR'
      INCLUDE 'COMMON.FOR'
C*********************************************************************
C  PROBLEM 3: BLACK, SQUARE ENCLOSURE WITH PARTICIPATING MEDIUM
C             AND HOT BOTTOM WALL
C*********************************************************************
C
      ENTRY GRID
C
      TITLE(1)=' G '
      OPEN(7,FILE='PROB3.DAT')
C
      NCVLP=8
      NCVLT=2
C
      CALL QUAD
C
      NCVLX=5
      NCVLY=5
C
      XL=1.
      YL=1.
C
      CALL EZGRID
C
      RETURN
C*********************************************************************
      ENTRY START
C
      LAST=20
      KISO=0
      KPHASE=7
      ALPHA=0.5
      SIG=0.5
C
      TEM=(1./STFAN)**(1./4.)
C
      DO 11 I=2,L1-1
         T(I,1)=TEM
11    CONTINUE
C
      RETURN
C*********************************************************************
C
      ENTRY LC
      RETURN
C*********************************************************************
      ENTRY OUTPUT
C
      IF(ITER.EQ.0) WRITE(6,500)
      WRITE(6,501) ITER, F(L1/2,M1/2,2,2),DMAX
      IF(ITER.EQ.0) WRITE(7,500)
      WRITE(7,501) ITER, F(L1/2,M1/2,2,2),DMAX
C
      IF(KSTOP.EQ.1.OR.ITER.EQ.LAST) THEN
         CALL HFLUX
         CALL PRINT
C
```

```
         WRITE(6,502)
         WRITE(7,502)
C
         DO 510 I=2,L2
            WRITE(6,503) X(I),ABS(QPY(I,M1)-QMY(I,M1)),
1                             ABS(QPY(I,1)-QMY(I,1))
            WRITE(7,503) X(I),ABS(QPY(I,M1)-QMY(I,M1)),
1                             ABS(QPY(I,1)-QMY(I,1))
510      CONTINUE
C
         WRITE(6,504)
         WRITE(7,504)
C
         DO 511 J=2,M2
            WRITE(6,503) Y(J),ABS(QPX(1,J)-QMX(1,J)),
1                             ABS(QPX(L1,J)-QMX(L1,J))
            WRITE(7,503) Y(J),ABS(QPX(1,J)-QMX(1,J)),
1                             ABS(QPX(L1,J)-QMX(L1,J))
511      CONTINUE
C
      ENDIF
C
 500  FORMAT(/3X,'ITER',8X,'F',12X,'DIFF-MAX'/1X,48('*'))
 501  FORMAT(3X,I3,2(3X,1PE12.3))
 502  FORMAT(/8X,'X',10X,'QTOP',7X,'QBOT'/1X,39('*'))
 503  FORMAT(1X,3(1PE12.3))
 504  FORMAT(/8X,'Y',9X,'QLEFT',7X,'QRITE'/1X,39('*'))
C
      RETURN
C*******************************************************************
C
      ENTRY GAMSOR
C
      RETURN
      END
C*******************************************************************
```

2.3-5 Results for Example 3

```
   ITER         F             DIFF-MAX
 ************************************************
     0      0.000E+00      0.000E+00
     1      4.147E-02      1.000E+00
     2      5.099E-02      1.000E+00
     3      5.363E-02      2.027E-01
     4      5.413E-02      5.372E-02
     5      5.424E-02      1.312E-02
     6      5.426E-02      3.220E-03
     7      5.427E-02      7.630E-04
     8      5.427E-02      1.801E-04
     9      5.427E-02      4.190E-05
    10      5.427E-02      9.730E-06
    11      5.427E-02      2.277E-06
    12      5.427E-02      8.697E-07

 I =       1        2        3        4        5        6        7
 X = 0.00E+00 1.00E-01 3.00E-01 5.00E-01 7.00E-01 9.00E-01 1.00E+00
 XU= 0.00E+00 0.00E+00 2.00E-01 4.00E-01 6.00E-01 8.00E-01 1.00E+00

 J =       1        2        3        4        5        6        7
 Y = 0.00E+00 1.00E-01 3.00E-01 5.00E-01 7.00E-01 9.00E-01 1.00E+00
 YV= 0.00E+00 0.00E+00 2.00E-01 4.00E-01 6.00E-01 8.00E-01 1.00E+00
```

```
L =      1         2         3         4
TH = 0.00E+00 7.85E-01 2.36E+00 3.14E+00

M =      1         2         3         4         5         6         7
PH = 0.00E+00 3.93E-01 1.18E+00 1.96E+00 2.75E+00 3.53E+00 4.32E+00

M =      8         9        10
PH = 5.11E+00 5.89E+00 6.28E+00

******             G            ******
        --------------------

 I =     1         2         3         4         5         6         7
 J
 7    0.00E+00 2.24E-01 2.65E-01 2.81E-01 2.65E-01 2.24E-01 0.00E+00
 6    1.89E-01 2.43E-01 2.90E-01 3.07E-01 2.90E-01 2.43E-01 1.89E-01
 5    2.68E-01 3.58E-01 4.29E-01 4.54E-01 4.29E-01 3.58E-01 2.68E-01
 4    3.82E-01 5.32E-01 6.34E-01 6.66E-01 6.34E-01 5.32E-01 3.82E-01
 3    5.52E-01 8.12E-01 9.54E-01 9.94E-01 9.54E-01 8.12E-01 5.52E-01
 2    8.05E-01 1.30E+00 1.47E+00 1.52E+00 1.47E+00 1.30E+00 8.05E-01
 1    0.00E+00 2.15E+00 2.19E+00 2.20E+00 2.19E+00 2.15E+00 0.00E+00

       X           QTOP        QBOT
**************************************
   1.000E-01   1.463E-01   9.214E-01
   3.000E-01   1.754E-01   9.022E-01
   5.000E-01   1.866E-01   8.966E-01
   7.000E-01   1.754E-01   9.022E-01
   9.000E-01   1.463E-01   9.214E-01

       Y          QLEFT       QRITE
**************************************
   1.000E-01   3.829E-01   3.829E-01
   3.000E-01   2.453E-01   2.453E-01
   5.000E-01   1.577E-01   1.577E-01
   7.000E-01   1.033E-01   1.033E-01
   9.000E-01   6.834E-02   6.834E-02
```

2.3-6 Discussion of Results

It can be seen that the solution is not converged in one iteration. This is because the medium is scattering. Hence, at each node source function is unknown. Therefore, at the beginning we are guessing the source function. On the basis of guess value of source function intensity is calculated at the next iteration and it is compared with intensity calculated from previous iteration. This iterative process will continue till the maximum difference in intensity between two successive iterations will be less than or equal to `EROR`. Here it is seen that after twelve iterations solution is converged. In the result lists for the present problem, along x direction `X(I)` and `XU(I)` represents the value of `X` at grid location `I` and the value of `X` for the corresponding control volume face. Similarly, along y direction `Y(J)` and `YV(J)` represents the value of `Y` at grid location `J` and value of `Y` for the corresponding control volume face. Angular grid contains grid related information in θ and ϕ-directions respectively. In the θ-direction `TH(L)` represents the value of θ at the grid location `L` and in the ϕ-direction `PH(M)` represents the value of ϕ at the grid location `M`. The final field printout of incident radiation energy shows that the effect of hot bottom wall is to create maximum irradiation (incident radiation energy), `G(I,J)` at the bottom of the enclosure which is numerically evaluated by finite volume method as 2.2. The distribution of irradiation inside the medium is plotted for backward and forward scattering as shown in Fig. 2.3.2 for absorption coefficient, $\kappa = 0.5\ m^{-1}$

and scattering coefficient, σ_s = 0.5 m^{-1}. As we go away from the bottom boundary of the enclosure towards the top, the magnitude of irradiation decreases and we get minimum irradiation at the top boundary. It is also seen that backward scattering gives more irradiation at the bottom wall compared to forward scattering. The field printout of boundary heat fluxes shows that the net radiative heat flux at the bottom wall is maximum and is symmetrical about the center of the bottom boundary. Net radiative heat fluxes at left and right boundaries are same and increases along these walls as we come closer to the hot bottom wall.

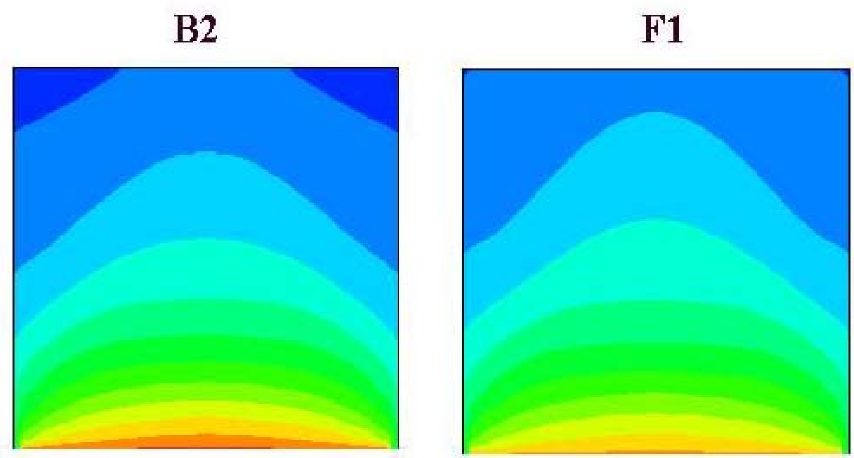


Fig. 2.3.2 Distribution of incident radiation energy for backward and forward scattering with absorption coefficient, $\kappa = 0.5$ m^{-1} and scattering coefficient, $\sigma_s = 0.5$ m^{-1}.

2.3-7 Final Remarks

Effect of scattering (forward and backward) on incident radiation energy and wall heat fluxes are discussed here. It was noted that the intensity is not converged after one iteration unlike Example 1. This is because of the fact that the source function due to in-scattering is unknown at each control volume nodes.

2.4 Black, Square Enclosure with Inhomogeneous Absorbing and Anisotropically Scattering Medium and Hot Bottom Wall (Example 4)

2.4-1 Problem Description

The problem under consideration is steady-state radiation in an *inhomogeneous* absorbing and anisotropically scattering medium surrounded by a black enclosure of square shape as shown in Fig. 2.4.1. The bottom wall is kept hot and other three walls are kept at absolute zero temperature. The absorption coefficients κ_1 and κ_2 and scattering coefficients $\sigma_{s,1}$ and $\sigma_{s,2}$ are shown in Fig. 2.4.1. For the present problem the following values are used.

$$T_{bottom} = \left(\frac{1}{\sigma}\right)^{1/4}, \quad T_{top} = T_{left} = T_{right} = 0 \text{ K}, \qquad T_g = 0 \text{ K}, \qquad \varepsilon_w = 1 \tag{2.4.1}$$

$$\kappa_1 = 0.5\ m^{-1}, \qquad \sigma_{s,1} = 0.5\ m^{-1} \qquad \kappa_2 = 10\ \kappa_1 \qquad \sigma_{s,2} = 10\ \sigma_{s,2} \tag{2.4.2}$$

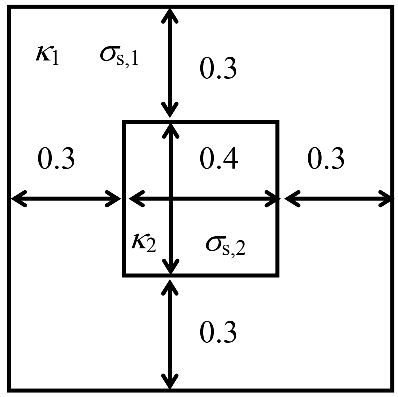


Fig. 2.4.1 Radiation in an inhomogeneous absorbing and anisotropically scattering medium.

where σ is Stefan-Boltzmann constant. Our aim is to calculate the total irradiation and boundary radiative heat flux distribution.

2.4-2 Design of ADAPT

GRID. The title of the field printout is set to 'G' through `TITLE (1)`. The output file (`PROB4.DAT`) is then specified via `OPEN`. By default, the angular domains are $0 \le \theta \le \pi$ and $0 \le \phi \le 2\pi$. These are specified in `DEFLT` using the variables `TL` and `PL` respectively. Four control angles represented by `NCVLT = 4` are taken in the θ-direction and 8 control angles represented by `NCVLP = 8` are taken in the ϕ-direction respectively. The default value of `POWERT = 1` and `POWERP = 1` are used to generate angular grids with uniform $\Delta\theta$ and $\Delta\phi$. The boundaries of the control angles are calculated by calling `QUAD`. The spatial domains are $0 \le x \le 1$ and $0 \le y \le 1$ which are specified through `XL` and `YL` respectively. Ten control volumes are used in the x and y directions which are represented by `NCVLX = 10` and `NCVLY = 10` respectively. The default values of `POWERX = 1` and `POWERY = 1` are used. As a result, an uniform spatial grid is created by calling `EZGRID`.

START. Numerical values of all boundary conditions as given in Eqs. (2.4.1) and (2.4.2) are set here. The maximum number of iterations for the present problem are set as `LAST = 30`. Anisotropic scattering medium is set as `KISO = 0`. Forward scattering (F1 phase function) is specified by `KPHASE = 2`. The value of absorption coefficient κ_1 and scattering coefficient $\sigma_{s,1}$ are taken as `ALPHA = 0.5` and `SIG = 0.5` respectively. Then we fill `T(I,1)` array by `TEM`, which serves as the temperature of hot bottom wall. Other boundary temperatures are kept at the default value as all the boundaries except the bottom boundary are at absolute zero temperature for the present problem.

LC. Inhomogeneous medium is set here. For $0.3 \le x \le 0.7$ and $0.3 \le y \le 0.7$, the absorption coefficient κ_2 and scattering coefficient $\sigma_{s,2}$ are specified by modifying the `CAPPA(I,J)` and `SIGMA(I,J)` arrays respectively. The absorption coefficient and scattering coefficient for the remainder of the domain remain unchanged and are equal to κ_1 and $\sigma_{s,1}$ respectively.

OUTPUT. For each iteration (ITER) the value of an actual intensity, F(I,J,L,M) at the center of the enclosure is printed and maximum difference in magnitude of the intensity from previous iteration, DMAX (which is printed as DIFF-MAX) over all control volumes and control angles is printed for verifying the convergence of the solution. The default value of EROR = 1.E-6 is used in this example for satisfying the convergence criteria. When the maximum number of iterations has been reached (ITER = LAST), or when the maximum change of the radiative intensities between two successive iterations falls below the prescribed value (KSTOP = 1), the boundary heat fluxes and incident radiation energy are calculated by calling HFLUX. All grid-related variables and incident radiation energy at control volume nodes are printed by calling PRINT. QTOP represents the net radiative heat flux at the top boundary and QBOT represents the net radiative heat flux at the bottom boundary. X(I) represents the value of X at grid location I. Magnitude of QTOP and magnitude of QBOT are printed here for different X(I). QLEFT represents the net radiative heat flux at the left boundary and QRITE represents the net radiative heat flux at the right boundary. Y(J) represents the value of Y at grid location J. Magnitude of QLEFT and magnitude of QRITE are printed here for different Y(J).

GAMSOR. Irregular geometries and radiative equilibrium conditions are set here. Since in our present problem geometry of the enclosure is not irregular and it is in non-radiative equilibrium condition, hence no operation is performed here.

2.4-3 Additional Fortran Names

DMAX	maximum of $\left|\left(I - I_{OLD}\right)/I\right|$ calculated over all control volumes and control angles
LAST	maximum number of iterations
ALPHA	absorption coefficient
SIG	scattering coefficient
KISO	set for type of scattering
KPHASE	set for back scattering
STFAN	Stefan-Boltzmann constant
G(I,J)	incident radiation energy
TEM	non-dimensional temperature of bottom wall
QTOP	net radiative heat flux at top boundary
QBOT	net radiative heat flux at bottom boundary
QLEFT	net radiative heat flux at left boundary
QRITE	net radiative heat flux at right boundary

2.4-4 Listing of ADAPT for Example 4

```
C***********************************************************************
      SUBROUTINE USER
C***********************************************************************
      INCLUDE 'PARAM.FOR'
```

```
      INCLUDE 'COMMON.FOR'
C*********************************************************************
C  PROBLEM 4: BLACK, SQUARE ENCLOSURE WITH INHOMOGENOUS MEDIUM
C             AND HOT BOTTOM WALL
C*********************************************************************
C
      ENTRY GRID
C
      TITLE(1)=' G '
      OPEN(7,FILE='PROB4.DAT')
C
      NCVLP=8
      NCVLT=4
C
      CALL QUAD
C
      NCVLX=10
      NCVLY=10
C
      XL=1.
      YL=1.
C
      CALL EZGRID
C
      RETURN
C*********************************************************************
C
      ENTRY START
C
      LAST=30
      KISO=0
      KPHASE=2
      ALPHA=0.5
      SIG=0.5
C
      TEM=(1./STFAN)**(1./4.)
C
      DO 11 I=2,L1-1
         T(I,1)=TEM
 11   CONTINUE
C
      RETURN
C*********************************************************************
C
      ENTRY LC
C
      DO 100 J=1,M1
         DO 101 I=1,L1
            IF(X(I).GT.0.3.AND.X(I).LT.0.7.AND.
     1         Y(J).GT.0.3.AND.Y(J).LT.0.7) THEN
               CAPPA(I,J)=10*ALPHA
               SIGMA(I,J)=10*SIG
            ENDIF
 101     CONTINUE
 100   CONTINUE
C
      RETURN
C*********************************************************************
C
      ENTRY OUTPUT
C
      IF(ITER.EQ.0) WRITE(6,500)
      WRITE(6,501) ITER, F(L1/2,M1/2,2,2),DMAX
      IF(ITER.EQ.0) WRITE(7,500)
```

```
      WRITE(7,501) ITER, F(L1/2,M1/2,2,2),DMAX
C
      IF(KSTOP.EQ.1.OR.ITER.EQ.LAST) THEN
         CALL HFLUX
         CALL PRINT
C
         WRITE(6,502)
         WRITE(7,502)
C
         DO 510 I=2,L2
            WRITE(6,503) X(I),ABS(QPY(I,M1)-QMY(I,M1)),
     1                   ABS(QPY(I,1)-QMY(I,1))
            WRITE(7,503) X(I),ABS(QPY(I,M1)-QMY(I,M1)),
     1                   ABS(QPY(I,1)-QMY(I,1))
 510     CONTINUE
C
         WRITE(6,504)
         WRITE(7,504)
C
         DO 511 J=2,M2
            WRITE(6,503) Y(J),ABS(QPX(1,J)-QMX(1,J)),
     1                   ABS(QPX(L1,J)-QMX(L1,J))
C
            WRITE(7,503) Y(J),ABS(QPX(1,J)-QMX(1,J)),
     1                   ABS(QPX(L1,J)-QMX(L1,J))
 511     CONTINUE
C
      ENDIF
C
 500  FORMAT(/3X,'ITER',8X,'F',12X,'DIFF-MAX'/1X,48('*'))
 501  FORMAT(3X,I3,2(3X,1PE12.3))
 502  FORMAT(/8X,'X',10X,'QTOP',7X,'QBOT'/1X,39('*'))
 503  FORMAT(1X,3(1PE12.3))
 505  FORMAT(1X,2(F10.5))
 504  FORMAT(/8X,'Y',9X,'QLEFT',7X,'QRITE'/1X,39('*'))
C
      RETURN
C*******************************************************************
      ENTRY GAMSOR
C
      RETURN
      END
C*******************************************************************
```

2.4-5 Results for Example 4

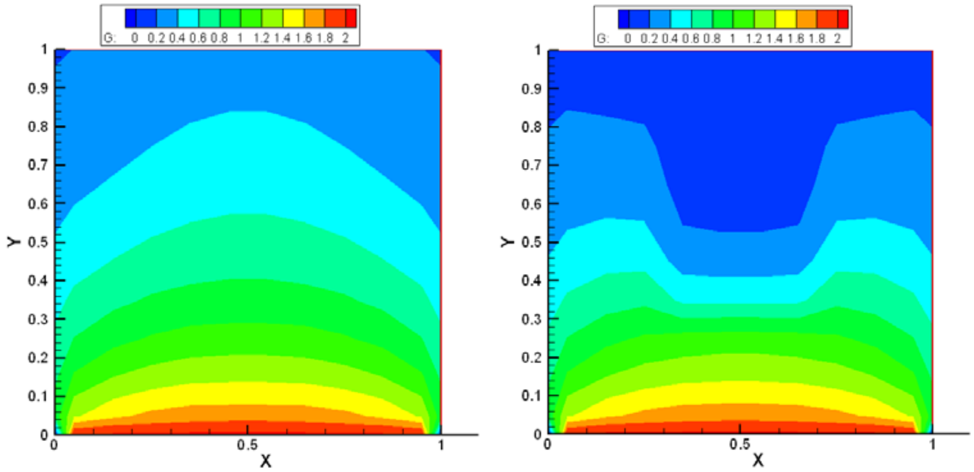
```
   ITER        F             DIFF-MAX
 ************************************************
     0      0.000E+00      0.000E+00
     1      1.724E-03      1.000E+00
     2      8.829E-03      1.000E+00
     3      1.191E-02      5.533E-01
     4      1.282E-02      2.462E-01
     5      1.307E-02      9.393E-02
     6      1.314E-02      3.203E-02
     7      1.316E-02      1.018E-02
     8      1.316E-02      3.085E-03
     9      1.316E-02      9.060E-04
    10      1.316E-02      2.606E-04
    11      1.316E-02      7.393E-05
    12      1.316E-02      2.086E-05
    13      1.316E-02      5.856E-06
```

```
   14      1.316E-02      1.757E-06
   15      1.316E-02      5.856E-07


I =     1        2        3        4        5        6        7
X = 0.00E+00 5.00E-02 1.50E-01 2.50E-01 3.50E-01 4.50E-01 5.50E-01
XU= 0.00E+00 0.00E+00 1.00E-01 2.00E-01 3.00E-01 4.00E-01 5.00E-01

I =     8        9       10       11       12
X = 6.50E-01 7.50E-01 8.50E-01 9.50E-01 1.00E+00
XU= 6.00E-01 7.00E-01 8.00E-01 9.00E-01 1.00E+00


J =     1        2        3        4        5        6        7
Y = 0.00E+00 5.00E-02 1.50E-01 2.50E-01 3.50E-01 4.50E-01 5.50E-01
YV= 0.00E+00 0.00E+00 1.00E-01 2.00E-01 3.00E-01 4.00E-01 5.00E-01

J =     8        9       10       11       12
Y = 6.50E-01 7.50E-01 8.50E-01 9.50E-01 1.00E+00
YV= 6.00E-01 7.00E-01 8.00E-01 9.00E-01 1.00E+00


L =     1        2        3        4        5        6
TH = 0.00E+00 3.93E-01 1.18E+00 1.96E+00 2.75E+00 3.14E+00


M =     1        2        3        4        5        6        7
PH = 0.00E+00 3.93E-01 1.18E+00 1.96E+00 2.75E+00 3.53E+00 4.32E+00

M =     8        9       10
PH = 5.11E+00 5.89E+00 6.28E+00

******           G            ******
        --------------------

 I =     1        2        3        4        5        6        7
 J
12    0.00E+00 1.59E-01 1.45E-01 1.39E-01 1.14E-01 1.07E-01 1.07E-01
11    1.47E-01 1.60E-01 1.46E-01 1.40E-01 1.15E-01 1.08E-01 1.08E-01
10    1.80E-01 1.98E-01 1.88E-01 1.78E-01 1.24E-01 1.08E-01 1.08E-01
 9    2.20E-01 2.45E-01 2.43E-01 2.30E-01 1.28E-01 1.05E-01 1.05E-01
 8    2.69E-01 3.04E-01 3.16E-01 3.03E-01 1.27E-01 1.02E-01 1.02E-01
 7    3.32E-01 3.82E-01 4.14E-01 4.07E-01 1.94E-01 1.69E-01 1.69E-01
 6    4.13E-01 4.87E-01 5.44E-01 5.51E-01 3.17E-01 2.98E-01 2.98E-01
 5    5.19E-01 6.27E-01 7.19E-01 7.54E-01 5.61E-01 5.56E-01 5.56E-01
 4    6.42E-01 8.08E-01 9.43E-01 1.03E+00 1.07E+00 1.09E+00 1.09E+00
 3    7.63E-01 1.03E+00 1.20E+00 1.29E+00 1.34E+00 1.36E+00 1.36E+00
 2    8.96E-01 1.37E+00 1.56E+00 1.65E+00 1.69E+00 1.71E+00 1.71E+00
 1    0.00E+00 2.04E+00 2.04E+00 2.04E+00 2.05E+00 2.05E+00 2.05E+00

 I =     8        9       10       11       12
 J
12    1.14E-01 1.39E-01 1.45E-01 1.59E-01 0.00E+00
11    1.15E-01 1.40E-01 1.46E-01 1.60E-01 1.47E-01
10    1.24E-01 1.78E-01 1.88E-01 1.98E-01 1.80E-01
 9    1.28E-01 2.30E-01 2.43E-01 2.45E-01 2.20E-01
 8    1.27E-01 3.03E-01 3.16E-01 3.04E-01 2.69E-01
 7    1.94E-01 4.07E-01 4.14E-01 3.82E-01 3.32E-01
 6    3.17E-01 5.51E-01 5.44E-01 4.87E-01 4.13E-01
 5    5.61E-01 7.54E-01 7.19E-01 6.27E-01 5.19E-01
 4    1.07E+00 1.03E+00 9.43E-01 8.08E-01 6.42E-01
 3    1.34E+00 1.29E+00 1.20E+00 1.03E+00 7.63E-01
 2    1.69E+00 1.65E+00 1.56E+00 1.37E+00 8.96E-01
 1    2.05E+00 2.04E+00 2.04E+00 2.04E+00 0.00E+00
```

```
        X           QTOP        QBOT
***************************************
   5.000E-02   1.147E-01   9.871E-01
   1.500E-01   1.045E-01   9.852E-01
   2.500E-01   1.003E-01   9.838E-01
   3.500E-01   8.282E-02   9.825E-01
   4.500E-01   7.765E-02   9.819E-01
   5.500E-01   7.765E-02   9.819E-01
   6.500E-01   8.282E-02   9.825E-01
   7.500E-01   1.003E-01   9.838E-01
   8.500E-01   1.045E-01   9.852E-01
   9.500E-01   1.147E-01   9.871E-01

        Y          QLEFT       QRITE
***************************************
   5.000E-02   4.433E-01   4.433E-01
   1.500E-01   3.683E-01   3.683E-01
   2.500E-01   2.985E-01   2.985E-01
   3.500E-01   2.263E-01   2.263E-01
   4.500E-01   1.669E-01   1.669E-01
   5.500E-01   1.248E-01   1.248E-01
   6.500E-01   9.582E-02   9.582E-02
   7.500E-01   7.571E-02   7.571E-02
   8.500E-01   6.096E-02   6.096E-02
   9.500E-01   4.945E-02   4.945E-02
```

2.4-6 Discussion of Results

It can be seen that the solution is not converged in one iteration. This is because the medium scatters energy. Hence, at each node source function is unknown. Therefore, at the beginning we are guessing the source function. On the basis of guess value of source function intensity is calculated at the next iteration and it is compared with intensity calculated from previous iteration. This iterative process will continue till the maximum difference in intensity between two successive iterations will be less than or equal to `EROR`. Here it is seen that after fifteen iterations solution is converged. In the result lists for the present problem, along x direction `X(I)` and `XU(I)` represents the value of `X` at grid location `I` and the value of `X` for the corresponding control volume face. Similarly, along y direction `Y(J)` and `YV(J)` represents the value of `Y` at grid location `J` and value of `Y` for the corresponding control volume face. Angular grid contains grid related information in θ and ϕ-directions respectively. In the θ-direction `TH(L)` represents the value of θ at the grid location `L` and in the ϕ-direction `PH(M)` represents the value of ϕ at the grid location `M`. The final field printout of incident radiation energy shows that the effect of hot bottom wall is to create maximum irradiation (incident radiation energy), `G(I,J)` at the bottom of the enclosure which is numerically evaluated by finite volume method as 2.05. The distributions of irradiation inside the medium for homogeneous and inhomogeneous media are shown for a forward scattering phase function (F1) in Fig. 2.4.2. Due to the large absorption coefficient in the middle of the inhomogeneous medium, there is a sharp decrease in the incident radiation. It should be noted that the incident radiation for the homogeneous medium with F1 scattering phase function shown in Figs. 2.3.2 and 2.4.2 are different. This is due to the different spatial and angular grids employed in the two examples. The field printout of boundary heat fluxes shows that the net radiative heat flux at the bottom wall is maximum and is symmetrical about the center of the bottom boundary. Net radiative heat fluxes at left and right boundaries are same and increases along these walls as we come closer to the hot bottom wall.



Homogeneous medium Inhomogeneous medium

Fig. 2.4.2 Distributions of incident radiation energy

2.4-7 Final Remarks

Effect of inhomogeneous optical properties on incident radiation energy and wall heat fluxes are discussed here.

2.5 Black, Square Enclosure with Absorbing-Emitting Medium with a radiative source (Example 5)

2.5-1 Problem Description

The problem under consideration is steady-state radiation in an absorbing-emitting medium with a radiative source. In this example, the radiative source is specified as $q_{gen} = 5\,kW/m^3$. The medium intensity can be calculated from

$$\nabla \cdot q = q_{gen} = \kappa(4\pi I_b - G) \tag{2.5.1}$$

Once the blackbody intensity is obtained from Eq. (2.5.1), the gas temperature can be calculated using

$$E_b = \pi I_b = \sigma T_g^4 \tag{2.5.2}$$

The boundary conditions are specified as

$y = 0$ $\qquad$ $T = 1200\,\text{K}$ $\qquad$ (2.5.3a)

$y = 1m$ $\qquad$ $T = 400\,\text{K}$ $\qquad$ (2.5.3b)

Others $T = 800\,\text{K}$ (2.5.3c)

Our aim is to calculate the temperature of the medium.

2.5-2 Design of ADAPT

GRID. The title of the field printout is set to 'G' through `TITLE (1)`. The output file (`PROB5.DAT`) is then specified via `OPEN`. By default, the angular domains are $0 \le \theta \le \pi$ and $0 \le \phi \le 2\pi$. These are specified in `DEFLT` using the variables `TL` and `PL` respectively. Four control angles represented by `NCVLT = 4` are taken in the θ-direction and 8 control angles represented by `NCVLP = 8` are taken in the ϕ-direction respectively. The default value of `POWERT = 1` and `POWERP = 1` are used to generate angular grids with uniform $\Delta\theta$ and $\Delta\phi$. The boundaries of the control angles are calculated by calling `QUAD`. The spatial domains are $0 \le x \le 1$ and $0 \le y \le 1$ which are specified through `XL` and `YL` respectively. Ten control volumes are used in the x and y directions which are represented by `NCVLX = 10` and `NCVLY = 10` respectively. The default values of `POWERX = 1` and `POWERY = 1` are used. As a result, an uniform spatial grid is created by calling `EZGRID`.

START. Numerical values of all boundary conditions as given in Eq. (2.5.3) are set here. The maximum number of iterations for the present problem are set as `LAST = 30`. The value of absorption coefficient κ is taken as `ALPHA = 0.5`. The radiative heat source is specified as `QGEN`. Then we fill the boundary temperatures according to Eq. (2.5.3).

LC. Since the medium is homogeneous, no addition treatment is done here.

OUTPUT. For each iteration (`ITER`) the temperature of the medium is calculated using Eq. (2.5.2). The value of the temperature, `T(I,J)` at the center of the enclosure and maximum difference in magnitude of the intensity from previous iteration, `DMAX` (which is printed as `DIFF-MAX`) over all control volumes and control angles are printed for verifying the convergence of the solution. The default value of `EROR = 1.E-6` is used in this example for satisfying the convergence criteria. When the maximum number of iterations has been reached (`ITER = LAST`), or when the maximum change of the radiative intensities between two successive iterations falls below the prescribed value (`KSTOP = 1`), the boundary heat fluxes and incident radiation energy are calculated by calling `HFLUX`. All grid-related variables and incident radiation energy at control volume nodes are printed by calling `PRINT`. The user is of course free to use the temperature as the convergence monitoring variable. You will need to write this algorithm yourself in `ENTRY OUTPUT`.

GAMSOR. The effect of the radiative source given by Eq. (2.5.1) is incorporated in here. The incident radiation G is calculated by calling `HFLUX`. The intensities of all *internal* control volumes are then calculated according to Eq. (2.5.1). Radiative equilibrium is a degenerate case of this example with $q_{gen} = 0$.

2.5-3 Additional Fortran Names

DMAX	maximum of $\left\|\left(I - I_{OLD}\right)/I\right\|$ calculated over all control volumes and control angles
LAST	maximum number of iterations
ALPHA	absorption coefficient
STFAN	Stefan-Boltzmann constant
G(I,J)	incident radiation energy
QGEN	radiative heat source

2.5-4 Listing of ADAPT for Example 5

```
C*******************************************************************
      SUBROUTINE USER
C*******************************************************************
      INCLUDE 'PARAM.FOR'
      INCLUDE 'COMMON.FOR'
C*******************************************************************
C  PROBLEM 5: BLACK, SQUARE ENCLOSURE WITH THE MEDIUM
C             SUBJECTED TO A RADIATIVE HEAT SOURCE
C*******************************************************************
C
      ENTRY GRID
C
      TITLE(1)=' G '
      OPEN(7,FILE='PROB5.DAT')
C
      NCVLP=8
      NCVLT=4
C
      CALL QUAD
C
      NCVLX=10
      NCVLY=10
C
      XL=1.
      YL=1.
C
      CALL EZGRID
C
      RETURN
C*******************************************************************
C
      ENTRY START
C
      LAST=30
      ALPHA=0.5
      QGEN=5000
C
      DO 100 J=2,M2
         T(1,J)=800
         T(L1,J)=800
 100  CONTINUE
C
      DO 101 I=2,L2
         T(I,1)=1200
         T(I,M1)=400
 101  CONTINUE
C
      RETURN
C*******************************************************************
C
```

```
      ENTRY LC
C
      RETURN
C*********************************************************************
C
      ENTRY OUTPUT
C
      DO 200 J=2,M2
         DO 201 I=2,L2
            T(I,J)=(PI*RIB(I,J)/STFAN)**0.25
 201     CONTINUE
 200   CONTINUE
C
      IF(ITER.EQ.0) WRITE(6,500)
      WRITE(6,501) ITER, T(L1/2,M1/2),DMAX
      IF(ITER.EQ.0) WRITE(7,500)
      WRITE(7,501) ITER, T(L1/2,M1/2),DMAX
C
      IF(KSTOP.EQ.1.OR.ITER.EQ.LAST) THEN
         CALL HFLUX
         CALL PRINT
      ENDIF
C
 500  FORMAT(/3X,'ITER',8X,'T',12X,'DIFF-MAX'/1X,48('*'))
 501  FORMAT(3X,I3,2(3X,1PE12.3))
C
      RETURN
C*********************************************************************
      ENTRY GAMSOR
C
      CALL HFLUX
C
      DO 300 J=2,M2
         DO 301 I=2,L2
            RIB(I,J)=(QGEN/CAPPA(I,J)+G(I,J))/(4.*PI)
 301     CONTINUE
 300  CONTINUE
C
      RETURN
      END
C*********************************************************************
```

2.5-5 Results for Example 5

```
   ITER        T             DIFF-MAX
 ************************************************
     0      0.000E+00      0.000E+00
     1      4.582E+02      1.000E+00
     2      8.733E+02      9.055E-01
     3      9.327E+02      2.185E-01
     4      9.481E+02      6.207E-02
     5      9.523E+02      1.768E-02
     6      9.534E+02      4.985E-03
     7      9.537E+02      1.394E-03
     8      9.538E+02      3.882E-04
     9      9.539E+02      1.079E-04
    10      9.539E+02      2.996E-05
    11      9.539E+02      8.328E-06
    12      9.539E+02      2.460E-06
    13      9.539E+02      8.153E-07
```

```
I =      1        2        3        4        5        6        7
X = 0.00E+00 5.00E-02 1.50E-01 2.50E-01 3.50E-01 4.50E-01 5.50E-01
XU= 0.00E+00 0.00E+00 1.00E-01 2.00E-01 3.00E-01 4.00E-01 5.00E-01

I =      8        9       10       11       12
X = 6.50E-01 7.50E-01 8.50E-01 9.50E-01 1.00E+00
XU= 6.00E-01 7.00E-01 8.00E-01 9.00E-01 1.00E+00


J =      1        2        3        4        5        6        7
Y = 0.00E+00 5.00E-02 1.50E-01 2.50E-01 3.50E-01 4.50E-01 5.50E-01
YV= 0.00E+00 0.00E+00 1.00E-01 2.00E-01 3.00E-01 4.00E-01 5.00E-01

J =      8        9       10       11       12
Y = 6.50E-01 7.50E-01 8.50E-01 9.50E-01 1.00E+00
YV= 6.00E-01 7.00E-01 8.00E-01 9.00E-01 1.00E+00


L =      1        2        3        4        5        6
TH = 0.00E+00 3.93E-01 1.18E+00 1.96E+00 2.75E+00 3.14E+00


M =      1        2        3        4        5        6        7
PH = 0.00E+00 3.93E-01 1.18E+00 1.96E+00 2.75E+00 3.53E+00 4.32E+00

M =      8        9       10
PH = 5.11E+00 5.89E+00 6.28E+00

******           G            ******
        --------------------

 I =     1        2        3        4        5        6        7
 J
12    0.00E+00 8.17E+04 8.44E+04 8.82E+04 9.18E+04 9.39E+04 9.39E+04
11    1.05E+05 9.63E+04 9.51E+04 9.72E+04 1.00E+05 1.02E+05 1.02E+05
10    1.11E+05 1.09E+05 1.09E+05 1.12E+05 1.16E+05 1.18E+05 1.18E+05
 9    1.19E+05 1.19E+05 1.22E+05 1.26E+05 1.31E+05 1.33E+05 1.33E+05
 8    1.26E+05 1.30E+05 1.35E+05 1.41E+05 1.45E+05 1.48E+05 1.48E+05
 7    1.35E+05 1.41E+05 1.48E+05 1.55E+05 1.60E+05 1.62E+05 1.62E+05
 6    1.45E+05 1.54E+05 1.64E+05 1.71E+05 1.76E+05 1.78E+05 1.78E+05
 5    1.55E+05 1.69E+05 1.82E+05 1.90E+05 1.94E+05 1.96E+05 1.96E+05
 4    1.67E+05 1.87E+05 2.03E+05 2.12E+05 2.17E+05 2.19E+05 2.19E+05
 3    1.79E+05 2.09E+05 2.29E+05 2.40E+05 2.45E+05 2.48E+05 2.48E+05
 2    1.90E+05 2.40E+05 2.61E+05 2.72E+05 2.77E+05 2.79E+05 2.79E+05
 1    0.00E+00 2.94E+05 2.97E+05 2.98E+05 2.99E+05 2.99E+05 2.99E+05

 I =     8        9       10       11       12
 J
12    9.18E+04 8.82E+04 8.44E+04 8.17E+04 0.00E+00
11    1.00E+05 9.72E+04 9.51E+04 9.63E+04 1.05E+05
10    1.16E+05 1.12E+05 1.09E+05 1.09E+05 1.11E+05
 9    1.31E+05 1.26E+05 1.22E+05 1.19E+05 1.19E+05
 8    1.45E+05 1.41E+05 1.35E+05 1.30E+05 1.26E+05
 7    1.60E+05 1.55E+05 1.48E+05 1.41E+05 1.35E+05
 6    1.76E+05 1.71E+05 1.64E+05 1.54E+05 1.45E+05
 5    1.94E+05 1.90E+05 1.82E+05 1.69E+05 1.55E+05
 4    2.17E+05 2.12E+05 2.03E+05 1.87E+05 1.67E+05
 3    2.45E+05 2.40E+05 2.29E+05 2.09E+05 1.79E+05
 2    2.77E+05 2.72E+05 2.61E+05 2.40E+05 1.90E+05
 1    2.99E+05 2.98E+05 2.97E+05 2.94E+05 0.00E+00
```

2.5-6 Discussion of Results

As expected, the solution did not converge in one iteration. This is because the blackbody intensity of the medium is not known. Hence, the source function is unknown. Therefore, at the beginning we are guessing the source function. On the basis of guess value of source function intensity is calculated at the next iteration and it is compared with intensity calculated from previous iteration. This iterative process will continue till the maximum difference in intensity between two successive iterations will be less than or equal to `EROR`. Here it is seen that after thirteen iterations solution is converged. In the result lists for the present problem, along x direction `X(I)` and `XU(I)` represents the value of `X` at grid location `I` and the value of `X` for the corresponding control volume face. Similarly, along y direction `Y(J)` and `YV(J)` represents the value of `Y` at grid location `J` and value of `Y` for the corresponding control volume face. Angular grid contains grid related information in θ and ϕ-directions respectively. In the θ-direction `TH(L)` represents the value of θ at the grid location `L` and in the ϕ-direction `PH(M)` represents the value of ϕ at the grid location `M`. Figure 2.5.1 shows the temperature distribution due to the effect of the radiative heat source. The black lines show that locations of the centers of the control volumes for this uniformly divided spatial domain.

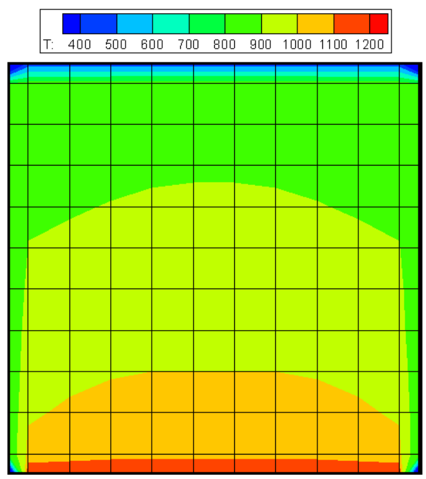


Fig. 2.5.1 Temperature distribution due to the radiative source.

2.5-7 Final Remarks

This example shows how radiative heat source is modeled using RAT. The next example shows the same problem modeled using non-uniform spatial grids.

2.6 Black, Square Enclosure with Absorbing-Emitting Medium with a radiative source (Example 6)

2.6-1 Problem Description

The physical parameters of this problem are identical to that of Example 5. This problem demonstrates the use of non-uniform spatial grids. The problem under consideration is steady-state radiation in an absorbing-emitting medium with a radiative source. In this example, the radiative source is specified as $q_{gen} = 5\,kW/m^3$. The medium intensity can be calculated from

$$\nabla \cdot q = q_{gen} = \kappa(4\pi I_b - G) \tag{2.6.1}$$

Once the blackbody intensity is obtained from Eq. (2.6.1), the gas temperature can be calculated using

$$E_b = \pi I_b = \sigma T_g^4 \tag{2.6.2}$$

The boundary conditions are specified as

$$y = 0 \qquad T = 1200\,\text{K} \tag{2.6.3a}$$

$$y = 1m \qquad T = 400\,\text{K} \tag{2.6.3b}$$

$$\text{Others} \qquad T = 800\,\text{K} \tag{2.6.3c}$$

Our aim is to calculate the temperature of the medium.

2.6-2 Design of ADAPT

GRID. The title of the field printout is set to 'G' through `TITLE (1)`. The output file (`PROB6.DAT`) is then specified via `OPEN`. By default, the angular domains are $0 \le \theta \le \pi$ and $0 \le \phi \le 2\pi$. These are specified in `DEFLT` using the variables `TL` and `PL` respectively. Four control angles represented by `NCVLT = 4` are taken in the θ-direction and 8 control angles represented by `NCVLP = 8` are taken in the ϕ-direction respectively. The default value of `POWERT = 1` and `POWERP = 1` are used to generate angular grids with uniform $\Delta\theta$ and $\Delta\phi$. The boundaries of the control angles are calculated by calling `QUAD`. The spatial domain is divided into two zones in the x direction through `NZX` = 2. `XZONE(1)` specifies the length of the first zone to 0.5 which is half the size of the enclosure. The first zone is divided into five control volumes using `NCVX(1)` = 5. The widths of the control volumes are arranged to *expand* in the positive x direction using `POWRX(1)` = 1.5. Similar to the first zone, the width of the second zone is set to 0.5 using `XZONE(2)`. Five control volumes fill this space and is set using `NCVX(2)` = 5. The widths of these control volumes contract towards the wall (in the positive x direction). This is arranged using `POWRX(2) = -1.5`. Note that in this program, positive (> 1.0) POWRX implies expanding grids, while negative (< −1) indicates contracting grids. The same *magnitude* (1.5 in this example) ensures that the meshes are symmetrical about the centerline (in this example). More complete explanation can be found in Patankar (1991). The same concept is used in specifying the grids in the y direction. The meshes are generated by calling `ZGRID` (not `EZGRID` as in the previous example.)

START. Numerical values of all boundary conditions as given in Eq. (2.6.3) are set here. The maximum number of iterations for the present problem are set as LAST = 30. The value of absorption coefficient κ is taken as ALPHA = 0.5. The radiative heat source is specified as QGEN. Then we fill the boundary temperatures according to Eq. (2.6.3).

LC. Since the medium is homogeneous, no addition treatment is done here.

OUTPUT. For each iteration (ITER) the temperature of the medium is calculated using Eq. (2.6.2). The value of the temperature, T(I,J) at the center of the enclosure and maximum difference in magnitude of the intensity from previous iteration, DMAX (which is printed as DIFF-MAX) over all control volumes and control angles are printed for verifying the convergence of the solution. The default value of EROR = 1.E-6 is used in this example for satisfying the convergence criteria. When the maximum number of iterations has been reached (ITER = LAST), or when the maximum change of the radiative intensities between two successive iterations falls below the prescribed value (KSTOP = 1), the boundary heat fluxes and incident radiation energy are calculated by calling HFLUX. All grid-related variables and incident radiation energy at control volume nodes are printed by calling PRINT. The user is of course free to use the temperature as the convergence monitoring variable. You will need to write this algorithm yourself in ENTRY OUTPUT.

GAMSOR. The effect of the radiative source given by Eq. (2.6.1) is incorporated in here. The incident radiation G is calculated by calling HFLUX. The intensities of all *internal* control volumes are then calculated according to Eq. (2.6.1). Radiative equilibrium is a degenerate case of this example with $q_{gen} = 0$.

2.6-3 Additional Fortran Names

DMAX	maximum of $\left|\left(I - I_{OLD}\right)/I\right|$ calculated over all control volumes and control angles
LAST	maximum number of iterations
ALPHA	absorption coefficient
STFAN	Stefan-Boltzmann constant
G(I,J)	incident radiation energy
QGEN	radiative heat source

2.6-4 Listing of ADAPT for Example 6

```
C*********************************************************************
      SUBROUTINE USER
C*********************************************************************
      INCLUDE 'PARAM.FOR'
      INCLUDE 'COMMON.FOR'
C*********************************************************************
C  PROBLEM 6: BLACK, SQUARE ENCLOSURE IN A MEDIUM
C             WITH A RADIATIVE HEAT SOURCE
C*********************************************************************
C
      ENTRY GRID
C
```

```
      TITLE(1)=' G '
      OPEN(7,FILE='PROB6.DAT')
C
      NCVLP=8
      NCVLT=4
C
      CALL QUAD
C
      NZX=2
      XZONE(1)=0.5
      NCVX(1)=5
      POWRX(1)=1.5
      XZONE(2)=0.5
      NCVX(2)=5
      POWRX(2)=-1.5
C
      NZY=2
      YZONE(1)=0.5
      NCVY(1)=5
      POWRY(1)=1.5
      YZONE(2)=0.5
      NCVY(2)=5
      POWRY(2)=-1.5
C
      CALL ZGRID
C
      RETURN
C*******************************************************************
C
      ENTRY START
C
      LAST=30
      ALPHA=0.5
      QGEN=5000
C
      DO 100 J=2,M2
         T(1,J)=800
         T(L1,J)=800
 100  CONTINUE
C
      DO 101 I=2,L2
         T(I,1)=1200
         T(I,M1)=400
 101  CONTINUE
C
      RETURN
C*******************************************************************
C
      ENTRY LC
C
      RETURN
C*******************************************************************
C
      ENTRY OUTPUT
C
      DO 200 J=2,M2
         DO 201 I=2,L2
            T(I,J)=(PI*RIB(I,J)/STFAN)**0.25
 201     CONTINUE
 200  CONTINUE
C
      IF(ITER.EQ.0) WRITE(6,500)
      WRITE(6,501) ITER, T(L1/2,M1/2),DMAX
      IF(ITER.EQ.0) WRITE(7,500)
```

```
      WRITE(7,501) ITER, T(L1/2,M1/2),DMAX
C
      IF(KSTOP.EQ.1.OR.ITER.EQ.LAST) THEN
         CALL HFLUX
         CALL PRINT
      ENDIF
C
 500  FORMAT(/3X,'ITER',8X,'T',12X,'DIFF-MAX'/1X,48('*'))
 501  FORMAT(3X,I3,2(3X,1PE12.3))
C
      RETURN
C*******************************************************************
      ENTRY GAMSOR
C
      CALL HFLUX
C
      DO 300 J=2,M2
         DO 301 I=2,L2
            RIB(I,J)=(QGEN/CAPPA(I,J)+G(I,J))/(4.*PI)
 301     CONTINUE
 300  CONTINUE
C
      RETURN
      END
C*******************************************************************
```

2.6-5 Results for Example 6

```
   ITER        T             DIFF-MAX
 ************************************************
     0      0.000E+00      0.000E+00
     1      4.582E+02      1.000E+00
     2      8.755E+02      9.031E-01
     3      9.359E+02      2.205E-01
     4      9.517E+02      6.284E-02
     5      9.561E+02      1.804E-02
     6      9.573E+02      5.131E-03
     7      9.576E+02      1.449E-03
     8      9.577E+02      4.075E-04
     9      9.577E+02      1.143E-04
    10      9.577E+02      3.198E-05
    11      9.577E+02      9.083E-06
    12      9.577E+02      2.553E-06
    13      9.577E+02      8.696E-07


 I =      1        2        3        4        5        6        7
 X = 0.00E+00 2.24E-02 8.56E-02 1.79E-01 2.95E-01 4.29E-01 5.71E-01
 XU= 0.00E+00 0.00E+00 4.47E-02 1.26E-01 2.32E-01 3.58E-01 5.00E-01

 I =      8        9       10       11       12
 X = 7.05E-01 8.21E-01 9.14E-01 9.78E-01 1.00E+00
 XU= 6.42E-01 7.68E-01 8.74E-01 9.55E-01 1.00E+00


 J =      1        2        3        4        5        6        7
 Y = 0.00E+00 2.24E-02 8.56E-02 1.79E-01 2.95E-01 4.29E-01 5.71E-01
 YV= 0.00E+00 0.00E+00 4.47E-02 1.26E-01 2.32E-01 3.58E-01 5.00E-01

 J =      8        9       10       11       12
 Y = 7.05E-01 8.21E-01 9.14E-01 9.78E-01 1.00E+00
```

```
YV= 6.42E-01 7.68E-01 8.74E-01 9.55E-01 1.00E+00


L =       1        2        3        4        5        6
TH = 0.00E+00 3.93E-01 1.18E+00 1.96E+00 2.75E+00 3.14E+00


M =       1        2        3        4        5        6        7
PH = 0.00E+00 3.93E-01 1.18E+00 1.96E+00 2.75E+00 3.53E+00 4.32E+00

M =       8        9       10
PH = 5.11E+00 5.89E+00 6.28E+00

******              G             ******
        --------------------

 I =     1        2        3        4        5        6        7
 J
12     0.00E+00 8.04E+04 8.18E+04 8.55E+04 9.07E+04 9.47E+04 9.47E+04
11     1.03E+05 9.29E+04 8.87E+04 9.01E+04 9.43E+04 9.79E+04 9.79E+04
10     1.07E+05 1.03E+05 9.99E+04 1.01E+05 1.05E+05 1.09E+05 1.09E+05
 9     1.14E+05 1.13E+05 1.12E+05 1.15E+05 1.20E+05 1.24E+05 1.24E+05
 8     1.23E+05 1.23E+05 1.25E+05 1.30E+05 1.36E+05 1.41E+05 1.41E+05
 7     1.33E+05 1.35E+05 1.40E+05 1.48E+05 1.55E+05 1.59E+05 1.59E+05
 6     1.46E+05 1.49E+05 1.58E+05 1.69E+05 1.77E+05 1.81E+05 1.81E+05
 5     1.60E+05 1.66E+05 1.81E+05 1.95E+05 2.03E+05 2.07E+05 2.07E+05
 4     1.74E+05 1.86E+05 2.07E+05 2.23E+05 2.33E+05 2.38E+05 2.38E+05
 3     1.86E+05 2.09E+05 2.37E+05 2.55E+05 2.65E+05 2.70E+05 2.70E+05
 2     1.93E+05 2.42E+05 2.71E+05 2.84E+05 2.90E+05 2.93E+05 2.93E+05
 1     0.00E+00 2.92E+05 2.95E+05 2.98E+05 2.99E+05 2.99E+05 2.99E+05

 I =     8        9       10       11       12
 J
12     9.07E+04 8.55E+04 8.18E+04 8.04E+04 0.00E+00
11     9.43E+04 9.01E+04 8.87E+04 9.29E+04 1.03E+05
10     1.05E+05 1.01E+05 9.99E+04 1.03E+05 1.07E+05
 9     1.20E+05 1.15E+05 1.12E+05 1.13E+05 1.14E+05
 8     1.36E+05 1.30E+05 1.25E+05 1.23E+05 1.23E+05
 7     1.55E+05 1.48E+05 1.40E+05 1.35E+05 1.33E+05
 6     1.77E+05 1.69E+05 1.58E+05 1.49E+05 1.46E+05
 5     2.03E+05 1.95E+05 1.81E+05 1.66E+05 1.60E+05
 4     2.33E+05 2.23E+05 2.07E+05 1.86E+05 1.74E+05
 3     2.65E+05 2.55E+05 2.37E+05 2.09E+05 1.86E+05
 2     2.90E+05 2.84E+05 2.71E+05 2.42E+05 1.93E+05
 1     2.99E+05 2.98E+05 2.95E+05 2.92E+05 0.00E+00
```

2.6-6 Discussion of Results

As expected, the solution did not converge in one iteration. This is because the blackbody intensity of the medium is not known. Hence, the source function is unknown. Therefore, at the beginning we are guessing the source function. On the basis of guess value of source function intensity is calculated at the next iteration and it is compared with intensity calculated from previous iteration. This iterative process will continue till the maximum difference in intensity between two successive iterations will be less than or equal to `EROR`. Here it is seen that after thirteen iterations solution is converged. In the result lists for the present problem, along x direction `X(I)` and `XU(I)` represents the value of `X` at grid location `I` and the value of `X` for the corresponding control volume face. Similarly, along y direction `Y(J)` and `YV(J)` represents the value of `Y` at grid location `J` and value of `Y` for the corresponding control volume face. Angular grid contains grid related information in θ and ϕ-directions respectively.

In the θ-direction `TH(L)` represents the value of θ at the grid location `L` and in the ϕ-direction `PH(M)` represents the value of ϕ at the grid location `M`. Figure 2.6.1 shows the temperature distribution due to the effect of the radiative heat source. The black lines show that locations of the centers of the control volumes for this uniformly divided spatial domain. Note that the nodes are finer near the four walls.

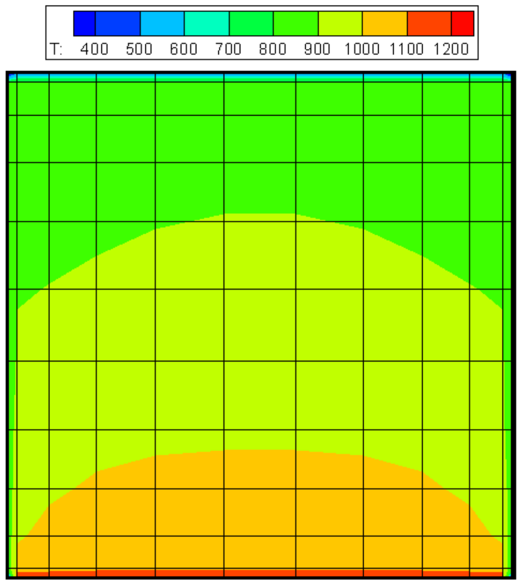


Fig. 2.6.1 Temperature distribution due to the radiative source.

2.6-7 Final Remarks

APPENDIX

A

LISTING OF THE INVARIANT PART OF RAT

A.1 Include File "PARAM.FOR"

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C TWO-DIMENSIONAL FINITE-VOLUME METHOD FOR
C RADIATIVE HEAT TRANSFER IN PARTICIPATING MEDIA
C                                          JOHN C. CHAI
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      PARAMETER (NI=17,NJ=17,NT=6,NP=10,NZMX=4,NPSUB=7)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C  PHI      : ANGLE FROM THE X-AXIS ON THE X-Y PLANE, M , J1
C  THETA    : ANGLE FROM THE Z-AXIS, L, K1
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
```

A.2 Include File "COMMON.FOR"

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      CHARACTER*10 TITLE
      COMMON/BLK01/F(NI,NJ,NT,NP)
    COMMON/BLK02/X(NI),XU(NI),XCV(NI),Y(NJ),YV(NJ),YCV(NJ),VOL(NI,NJ),
     5 THETA(NT),THETAI(NT),PHI(NP),PHII(NP)
    COMMON/BLK03/AX(NJ,NT,NP),AY(NI,NT,NP),DCX(NT,NP),DCY(NT,NP),
     1 DOM(NT,NP)
    COMMON/BLK04/ISOLID(NI,NJ)
    COMMON/BLK05/BM(NI,NJ,NT,NP),SM(NI,NJ,NT,NP),SP(NI,NJ,NT,NP),
     9 SC(NI,NJ,NT,NP),RIB(NI,NJ),T(NI,NJ)
      COMMON/BLK06/L1,L2,L3,M1,M2,M3,K1,K2,K3,J1,J2,J3,ITER,LAST
      COMMON/BLK07/XL,YL,TL,PL,SMALL,BIG,EROR,DMAX
      COMMON/BLK08/KSTOP,KBOUND,KISO,KPHASE,KPNORM
      COMMON/BLK09/RIBI1(NJ,NT,NP),RIBL1(NJ,NT,NP),
     1 RIBJ1(NI,NT,NP),RIBM1(NI,NT,NP)
      COMMON/BLK10/QPY(NI,NJ),QMY(NI,NJ),QPX(NI,NJ),QMX(NI,NJ),G(NI,NJ)
      COMMON/BLK11/CAPPA(NI,NJ),SIGMA(NI,NJ),BETA(NI,NJ)
     1      ,ALPHA,SIG,EPSI1,EPSL1,EPSJ1,EPSM1
     2      ,RHOI1,RHOL1,RHOJ1,RHOM1,STFAN
      COMMON/BLK12/MORDER,A(100),PHASE(NT,NP,NT,NP)
      COMMON/BLK13/PI,PIBY2,PI32,PI4
      COMMON/BLK14/TITLE(16)
      COMMON/BLK15/NCVLX,NCVLY,NCVX(NZMX),NCVY(NZMX),NZX,NZY,NCVLP,NCVLT,
      COMMON/BLK16/POWERX,POWERY,XZONE(NZMX),POWRX(NZMX)
     1       ,YZONE(NZMX),POWRY(NZMX),POWERT,POWERP
      COMMON/BLK17/MPHI1,MPHI2,MPHI3,MPHI4,MP1P1,MP2P1,MP3P1,MP4P1,
     1    LTETA2,LT2P1
      COMMON/BLK18/KBCI1(NJ),KBCL1(NJ),KBCJ1(NI),KBCM1(NI)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
```

A.3 Invariant Part of RAT

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      PROGRAM RAT2D
C***********************************************************************
      INCLUDE 'PARAM.FOR'
      INCLUDE 'COMMON.FOR'
C***********************************************************************
      CALL DEFLT
      CALL GRID
C
      CALL SETUP1
C
      CALL START
C
      CALL SETUP2
C
      CALL LC
C
   CHECK FOR SCATTERING
C
      DO 5 I=2,L2
         DO 6 J=2,M2
            IF(SIGMA(I,J).NE.0.) KSIG=1
 6       CONTINUE
 5    CONTINUE
C
      IF(KSIG.EQ.1) THEN
         IF(KISO.EQ.1) THEN
            CALL ISOTRP
         ELSE
            CALL ANISO
         ENDIF
      ENDIF
C
      CALL OUTPUT
      DO 10 ITER1=1,LAST
         ITER=ITER1
         IF(KSTOP.EQ.1) STOP
         CALL GAMSOR
        CALL BNDRY
        CALL SMBM
         CALL HEART
        CALL BOUND
         CALL OUTPUT
 10   CONTINUE
C
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE DEFLT
C***********************************************************************
      INCLUDE 'PARAM.FOR'
      INCLUDE 'COMMON.FOR'
C***********************************************************************
      LAST=5
      MODE=1
C
      EROR=1.E-6
      SMALL=1.E-10
      BIG=1.E20
C
      PI=4.0*ATAN(1.0)
      PIBY2=PI/2.
      PI32=3.*PI/2.
```

```
      PI4=4.*PI
C
      XL=1.0
      YL=1.0
C
      TL=PI
      PL=2.*PI
C
      POWERX=1.0
      POWERY=1.0
      POWERP=1.0
      POWERT=1.0
C
      NCVLX=5
      NCVLY=5
C
      DO 10 NZ=1,NZMX
         XZONE(NZ)=1.
         YZONE(NZ)=1.
         NCVX(NZ)=5
         NCVY(NZ)=5
         POWRX(NZ)=1.
         POWRY(NZ)=1.
 10   CONTINUE
C
      EPSJ1=1.
      EPSI1=1.
      EPSM1=1.
      EPSL1=1.
C
      ALPHA=0.0
      SIG=0.0
C
      STFAN=5.6696E-8
      KISO=1
C
      KBOUND=0
C
      DO 95 I=1,NI
         DO 96 J=1,NJ
            DO 97 L=1,NT
               DO 98 M=1,NP
                  F(I,J,L,M)=0.
                  BM(I,J,L,M)=0.
                  SM(I,J,L,M)=0.
                  SC(I,J,L,M)=0.
                  SP(I,J,L,M)=0.
                  RIBI1(J,L,M)=0.
                  RIBL1(J,L,M)=0.
                  RIBJ1(I,L,M)=0.
                  RIBM1(I,L,M)=0.
 98            CONTINUE
 97         CONTINUE
            T(I,J)=0.
            RIB(I,J)=0.
            CAPPA(I,J)=ALPHA
            SIGMA(I,J)=SIG
            BETA(I,J)=CAPPA(I,J)+SIGMA(I,J)
            KBCI1(J)=1
            KBCL1(J)=1
            KBCJ1(I)=1
            KBCM1(I)=1
 96      CONTINUE
 95   CONTINUE
```

```
C
      DO 123 L=2,K2
         DO 124 M=2,J2
            DO 125 LL=2,K2
               DO 126 MM=2,J2
                  PHASE(L,M,LL,MM)=0.0
 126           CONTINUE
 125        CONTINUE
 124     CONTINUE
 123  CONTINUE
C
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE SETUP
C***********************************************************************
      INCLUDE 'PARAM.FOR'
      INCLUDE 'COMMON.FOR'
C***********************************************************************

      DC(ARGU)=0.5*ARGU-0.25*SIN(2.*ARGU)
      SIN2(ARG)=(1.-COS(2.*ARG))/2.0
      AM(ARGU)=AMAX1(ARGU,0.0)
C***********************************************************************
      ENTRY SETUP1
C
      L2=L1-1
      L3=L2-1
      M2=M1-1
      M3=M2-1
      K2=K1-1
      K3=K2-1
      J2=J1-1
      J3=J2-1
C
COME  HERE TO CALCULATE X-DIRECTION GRID
C
      X(1)=XU(2)
      DO 5 I=2,L2
         X(I)=0.5*(XU(I+1)+XU(I))
         XCV(I)=XU(I+1)-XU(I)
 5    CONTINUE
      X(L1)=XU(L1)
C
COME  HERE TO CALCULATE Y-DIRECTION GRID
C
      Y(1)=YV(2)
      DO 10 J=2,M2
         Y(J)=0.5*(YV(J+1)+YV(J))
         YCV(J)=YV(J+1)-YV(J)
 10   CONTINUE
      Y(M1)=YV(M1)
C
CALCULATIONS OF CONTROL VOLUME VOLUMES
C
      DO 15 J=2,M2
         DO 16 I=2,L2
            VOL(I,J)=XCV(I)*YCV(J)
 16      CONTINUE
 15   CONTINUE
C
CALCULATIONSOF THETA-DIRECTION GRID
C
      THETA(1)=THETAI(2)
```

```
      DO 20 L=2,K2
         THETA(L)=0.5*(THETAI(L+1)+THETAI(L))
 20   CONTINUE
      THETA(K1)=THETAI(K1)
C
CALCULATIONS OF PHI-DIRECTION GRID
C
      PHI(1)=PHII(2)
      DO 30 M=2,J2
         PHI(M)=0.5*(PHII(M+1)+PHII(M))
 30   CONTINUE
      PHI(J1)=PHII(J1)
C
  CALCULATIONS OF CONTROL ANGLES AND "DIRECTION COSINES"
C
      DO 40 M=2,J2
         PHIM=PHI(M)
         IF(PHIM.LT.PIBY2) MPHI2=M
         IF(PHIM.LT.PI) MPHI3=M
         IF(PHIM.LT.PI32) MPHI4=M
         TERM1=COS(PHII(M+1))-COS(PHII(M))
         TERM2=SIN(PHII(M+1))-SIN(PHII(M))
         TERM3=PHII(M+1)-PHII(M)
         DO 41 L=2,K2
            IF(THETA(L).LT.PIBY2) LTETA2=L
            TERM4=DC(THETAI(L+1))-DC(THETAI(L))
            TERM5=(SIN2(THETAI(L+1))-SIN2(THETAI(L)))/2.
            DCX(L,M)= TERM2*TERM4
            DCY(L,M)=-TERM1*TERM4
            IF(ABS(DCX(L,M)).LT.1.E-5) DCX(L,M)=0.0
            IF(ABS(DCY(L,M)).LT.1.E-5) DCY(L,M)=0.0
            DOM(L,M)=-(COS(THETAI(L+1))-COS(THETAI(L)))*TERM3
 41      CONTINUE
 40   CONTINUE
C
    DO 50 L=2,K2
        DO 51 M=2,J2
            DO 52 J=2,M2
                AX(J,L,M)=ABS(DCX(L,M)*YCV(J))
 52         CONTINUE
            DO 53 I=2,L2
                AY(I,L,M)=ABS(DCY(L,M)*XCV(I))
 53         CONTINUE
 51     CONTINUE
 50 CONTINUE
C
      MP2P1=MPHI2+1
      MP3P1=MPHI3+1
      MP4P1=MPHI4+1
      LT2P1=LTETA2+1
C
CAUTION**** XCV(1),XCV(L1),YCV(1),YCV(M1) ARE NOT USED.  ****
      XCV(1)=SMALL
      XCV(L1)=SMALL
      YCV(1)=SMALL
      YCV(M1)=SMALL
C
    PRINT 2
      PRINT 55
C
 2    FORMAT(//15X,
     1     'COMPUTATION IN  TWO-DIMENSIONAL  CARTESIAN COORDINATES')
 55   FORMAT(14X,56(1H*),//)
C
```

```
      RETURN
C**********************************************************************
      ENTRY SETUP2
C
COME HERE TO SPECIFY BOUNDARY EMISSION FROM THE EAST AND WEST WALLS
C
      DO 60 J=2,M2
        IF(KBCI1(J).EQ.1) THEN
            DO 62 L=2,K2
                DO 63 M=2,MPHI2
                  RIBI1(J,L,M)=STFAN*T(1,J)**4/PI
 63             CONTINUE
                DO 64 M=MP4P1,J2
                  RIBI1(J,L,M)=STFAN*T(1,J)**4/PI
 64             CONTINUE
 62         CONTINUE
        ENDIF
        IF(KBCL1(J).EQ.1) THEN
            DO 65 L=2,K2
                DO 66 M=MP2P1,MPHI4
                  RIBL1(J,L,M)=STFAN*T(L1,J)**4/PI
 66             CONTINUE
 65         CONTINUE
        ENDIF
 60   CONTINUE
C
COME HERE TO SPECIFY BOUNDARY EMISSIONS FROM THE NORTH AND SOUTH WALLS
C
      DO 70 I=2,L2
        IF(KBCJ1(I).EQ.1) THEN
            DO 72 L=2,K2
                DO 73 M=2,MPHI3
                  RIBJ1(I,L,M)=STFAN*T(I,1)**4/PI
 73             CONTINUE
 72         CONTINUE
        ENDIF
        IF(KBCM1(I).EQ.1) THEN
            DO 75 L=2,K2
                DO 76 M=MP3P1,J2
                  RIBM1(I,L,M)=STFAN*T(I,M1)**4/PI
 76             CONTINUE
 75         CONTINUE
        ENDIF
 70   CONTINUE
C
COME  HERE TO SPECIFY EMISSION FROM THE MEDIUM
C
      DO 90 I=2,L2
         DO 91 J=2,M2
            RIB(I,J)=STFAN*T(I,J)**4/PI
            CAPPA(I,J)=ALPHA
            SIGMA(I,J)=SIG
            BETA(I,J)=CAPPA(I,J)+SIGMA(I,J)
 91      CONTINUE
 90   CONTINUE
C
COME  HERE TO UPDATE REFLECTIVITIES
C
      RHOJ1=1-EPSJ1
      RHOI1=1-EPSI1
      RHOM1=1-EPSM1
      RHOL1=1-EPSL1
C
      RETURN
```

```
    END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE HEART
C**************************************************************************
      INCLUDE 'PARAM.FOR'
      INCLUDE 'COMMON.FOR'
C**************************************************************************
      AM(ARGU)=AMAX1(ARGU,0.0)
C**************************************************************************
      KSTOP=1
      DMAX=-1000.
C
COME HERE TO START FROM THE SOUTH-WEST CORNER (I=2, J=2)
    DO 10 J=2,M2
        DO 11 I=2,L2
            DO 12 L=2,K2
                DO 13 M=2,MPHI2
                 VOLM=VOL(I,J)*DOM(L,M)
                    RNUM=AX(J,L,M)*F(I-1,J,L,M)+
     1                    AY(I,L,M)*F(I,J-1,L,M)+
     2                VOLM*SM(I,J,L,M)
                    DENO=AX(J,L,M)+AY(I,L,M)+BM(I,J,L,M)*VOLM
                    FOLD=F(I,J,L,M)
                 F(I,J,L,M)=RNUM/(DENO+SMALL)
                 DIFF=ABS(F(I,J,L,M)-FOLD)/(F(I,J,L,M)+SMALL)
                 DMAX=AMAX1(DMAX,DIFF)
                 IF(DMAX.GT.EROR) KSTOP=0
 13             CONTINUE
 12          CONTINUE
 11       CONTINUE
 10   CONTINUE
C
COME HERE TO START FROM THE SOUTH-EAST CORNER (I=L2, J=2)
    DO 20 J=2,M2
        DO 21 I=L2,2,-1
            DO 22 L=2,K2
                DO 23 M=MP2P1,MPHI3
                 VOLM=VOL(I,J)*DOM(L,M)
                    RNUM=AX(J,L,M)*F(I+1,J,L,M)+
     1                    AY(I,L,M)*F(I,J-1,L,M)+
     2                VOLM*SM(I,J,L,M)
                    DENO=AX(J,L,M)+AY(I,L,M)+BM(I,J,L,M)*VOLM
                    FOLD=F(I,J,L,M)
                 F(I,J,L,M)=RNUM/(DENO+SMALL)
                 DIFF=ABS(F(I,J,L,M)-FOLD)/(F(I,J,L,M)+SMALL)
                 DMAX=AMAX1(DMAX,DIFF)
                 IF(DMAX.GT.EROR) KSTOP=0
 23             CONTINUE
 22          CONTINUE
 21       CONTINUE
 20   CONTINUE
C
COME HERE TO START FROM THE NORTH-WEST CORNER (I=2, J=M2)
    DO 110 J=M2,2,-1
        DO 111 I=2,L2
            DO 112 L=2,K2
                DO 113 M=MP4P1,J2
                 VOLM=VOL(I,J)*DOM(L,M)
                    RNUM=AX(J,L,M)*F(I-1,J,L,M)+
     1                    AY(I,L,M)*F(I,J+1,L,M)+
     2                VOLM*SM(I,J,L,M)
                    DENO=AX(J,L,M)+AY(I,L,M)+BM(I,J,L,M)*VOLM
                    FOLD=F(I,J,L,M)
                 F(I,J,L,M)=RNUM/(DENO+SMALL)
```

```
                  DIFF=ABS(F(I,J,L,M)-FOLD)/(F(I,J,L,M)+SMALL)
                  DMAX=AMAX1(DMAX,DIFF)
                  IF(DMAX.GT.EROR) KSTOP=0
 113            CONTINUE
 112         CONTINUE
 111      CONTINUE
 110   CONTINUE
C
COME HERE TO START FROM THE NORTH-EAST CORNER (I=L2, J=M2)
    DO 120 J=M2,2,-1
        DO 121 I=L2,2,-1
            DO 122 L=2,K2
                DO 123 M=MP3P1,MPHI4
                  VOLM=VOL(I,J)*DOM(L,M)
                     RNUM=AX(J,L,M)*F(I+1,J,L,M)+
     1                     AY(I,L,M)*F(I,J+1,L,M)+
     2                 VOLM*SM(I,J,L,M)
                     DENO=AX(J,L,M)+AY(I,L,M)+BM(I,J,L,M)*VOLM
                     FOLD=F(I,J,L,M)
                  F(I,J,L,M)=RNUM/(DENO+SMALL)
                  DIFF=ABS(F(I,J,L,M)-FOLD)/(F(I,J,L,M)+SMALL)

                  DMAX=AMAX1(DMAX,DIFF)
                  IF(DMAX.GT.EROR) KSTOP=0
 123            CONTINUE
 122         CONTINUE
 121      CONTINUE
 120   CONTINUE
C
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE SMBM
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      INCLUDE 'PARAM.FOR'
      INCLUDE 'COMMON.FOR'
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      DO 11 J=2,M2
         DO 12 I=2,L2
            DO 13 L=2,K2
               DO 14 M=2,J2
                  SMSUM=0.
                  IF(SIGMA(I,J).NE.0.) THEN
                     DO 15 LL=2,K2
                        DO 16 MM=2,J2
                           SMSUM=SMSUM+PHASE(LL,MM,L,M)
     1                          *DOM(LL,MM)*F(I,J,LL,MM)
 16                     CONTINUE
 15                  CONTINUE
                     SMSUM=SMSUM-PHASE(L,M,L,M)*DOM(L,M)
     1                     *F(I,J,L,M)
                  ENDIF
                  BM(I,J,L,M)=CAPPA(I,J)+SIGMA(I,J)*
     1                 (1.-PHASE(L,M,L,M)*DOM(L,M)/PI4)
                  SM(I,J,L,M)=CAPPA(I,J)*RIB(I,J)
     1                 +SIGMA(I,J)*SMSUM/PI4
 14            CONTINUE
 13         CONTINUE
 12      CONTINUE
 11   CONTINUE
C
      RETURN
      END
```

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE BNDRY
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      INCLUDE 'PARAM.FOR'
      INCLUDE 'COMMON.FOR'
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DO 100 I=2,L2
         IF(KBCJ1(I).EQ.1) THEN
            CALL WLSUM(I,1,K2,J2,-1,DCY,F,SSUM)
            DO 102 L=2,K2
               DO 103 M=2,MPHI3
                  F(I,1,L,M)=EPSJ1*RIBJ1(I,L,M)+RHOJ1*SSUM/PI
 103           CONTINUE
 102        CONTINUE
         ELSEIF(KBCJ1(I).EQ.2) THEN
            DO 104 L=2,K2
               DO 105 M=2,MPHI3
                  MM=J2-(M-2)
                  F(I,1,L,M)=F(I,1,L,MM)
 105           CONTINUE
 104        CONTINUE
         ENDIF
C
         IF(KBCM1(I).EQ.1) THEN
            CALL WLSUM(I,M1,K2,J2,1,DCY,F,RNSUM)
            DO 412 L=2,K2
               DO 413 M=MP3P1,J2
                  F(I,M1,L,M)=EPSM1*RIBM1(I,L,M)+RHOM1*RNSUM/PI
 413           CONTINUE
 412        CONTINUE
         ELSEIF(KBCM1(I).EQ.2) THEN
            DO 414 L=2,K2
               DO 415 M=MP3P1,J2
                  MM=MPHI3-(M-MP3P1)
                  F(I,M1,L,M)=F(I,M1,L,MM)
 415           CONTINUE
 414        CONTINUE
         ENDIF
 100  CONTINUE
C
      DO 500 J=2,M2
         IF(KBCI1(J).EQ.1) THEN
            CALL WLSUM(1,J,K2,J2,-1,DCX,F,WSUM)
            DO 501 L=2,K2
               DO 502 M=2,MPHI2
                  F(1,J,L,M)=EPSI1*RIBI1(J,L,M)+RHOI1*WSUM/PI
 502           CONTINUE
               DO 503 M=MP4P1,J2
                  F(1,J,L,M)=EPSI1*RIBI1(J,L,M)+RHOI1*WSUM/PI
 503           CONTINUE
 501        CONTINUE
         ELSEIF(KBCI1(J).EQ.2) THEN
            DO 504 L=2,K2
               DO 505 M=2,MPHI2
                  MM=MPHI3-(M-2)
                  F(1,J,L,M)=F(1,J,L,MM)
 505           CONTINUE
               DO 515 M=MP4P1,J2
                  MM=MPHI4-(M-MP4P1)
                  F(1,J,L,M)=F(1,J,L,MM)
 515           CONTINUE
 504        CONTINUE
         ENDIF
C
```

```
         IF(KBCL1(J).EQ.1) THEN
            CALL WLSUM(L1,J,K2,J2,1,DCX,F,ESUM)
            DO 508 L=2,K2
               DO 509 M=MP2P1,MPHI4
                  F(L1,J,L,M)=EPSL1*RIBL1(J,L,M)+RHOL1*ESUM/PI
 509           CONTINUE
 508        CONTINUE
         ELSEIF(KBCL1(J).EQ.2) THEN
            DO 1510 L=2,K2
               DO 1511 M=MP2P1,MPHI3
                  MM=MPHI2-(M-MP2P1)
                  F(L1,J,L,M)=F(L1,J,L,MM)
 1511          CONTINUE
               DO 1514 M=MP3P1,MPHI4
                  MM=J2-(M-MP3P1)
                  F(L1,J,L,M)=F(L1,J,L,MM)
 1514          CONTINUE
 1510       CONTINUE
         ENDIF
 500  CONTINUE
C
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE BOUND
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      INCLUDE 'PARAM.FOR'
      INCLUDE 'COMMON.FOR'
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DO 10 J=2,M2
         DO 11 L=2,K2
            DO 12 M=MP2P1,MPHI4
               F(1,J,L,M)=F(2,J,L,M)
 12          CONTINUE
             DO 13 M=2,MPHI2
               F(L1,J,L,M)=F(L2,J,L,M)
 13         CONTINUE
             DO 14 M=MP4P1,J2
               F(L1,J,L,M)=F(L2,J,L,M)
 14         CONTINUE
 11      CONTINUE
 10   CONTINUE
C
      DO 20 I=2,L2
         DO 21 L=2,K2
            DO 22 M=2,MPHI3
                 F(I,M1,L,M)=F(I,M2,L,M)
 22          CONTINUE
            DO 23 M=MP3P1,J2
               F(I,1,L,M)=F(I,2,L,M)
 23         CONTINUE
 21      CONTINUE
 20   CONTINUE
C
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE WLSUM(I,J,K2,J2,INDX,DC,FC,SUM)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      INCLUDE 'PARAM.FOR'
      DIMENSION DC(NT,NP),FC(NI,NJ,NT,NP)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      SUM=0.
```

```
      IF(INDX.GT.0) THEN
         DO 10 L=2,K2
            DO 20 M=2,J2
               SUM=SUM+FC(I,J,L,M)*AMAX1(DC(L,M),0.0)
 20         CONTINUE
 10      CONTINUE
      ELSE
         DO 30 L=2,K2
            DO 40 M=2,J2
               SUM=SUM+FC(I,J,L,M)*AMAX1(-DC(L,M),0.0)
 40         CONTINUE
 30      CONTINUE
      ENDIF
C
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE PHASEF
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      INCLUDE 'PARAM.FOR'
      INCLUDE 'COMMON.FOR'
      DIMENSION PSUB(NP,NPSUB),TSUB(NT,NPSUB),
     1          PSUBI(NP,NPSUB),TSUBI(NT,NPSUB)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      ENTRY ISOTRP
C
      DO 10 L=2,K2
         DO 20 M=2,J2
            DO 30 LL=2,K2
               DO 40 MM=2,J2
                  PHASE(LL,MM,L,M)=1.0
 40            CONTINUE
 30         CONTINUE
 20      CONTINUE
 10   CONTINUE
C
      RETURN
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      ENTRY ANISO
C
      nsub=5
      CALL PCOEF
C
      DO 2000 M=2,J2
         PLOW=PHII(M)
         PHIGH=PHII(M+1)
         CALL PHISUB(PLOW,PHIGH,NSUB,PSUB,PSUBI,M)
         DPHI=(PHIGH-PLOW)/NSUB
         DO 2100 L=2,K2
            TLOW=THETAI(L)
            THIGH=THETAI(L+1)
            CALL TETASUB(TLOW,THIGH,DPHI,NSUB,TSUB,TSUBI,L)
 2100     CONTINUE
 2000  CONTINUE
C
      DO 2200 M=2,J2
         PHIL=PHI(M)
         DO 2210 L=2,K2
            TETAK=THETA(L)
            DO 2300 MM=2,J2
               DO 2310 LL=2,K2
                  SUMLL=0.0
                  DO 2220 MS=2,NSUB+1
```

```
                     PLSSUB=PSUB(M,MS)
                     TERMA=PSUBI(M,MS+1)-PSUBI(M,MS)
                     DO 2230 LS=2,NSUB+1
                        TKSSUB=TSUB(L,LS)
                        TERMB=COS(TSUBI(L,LS+1))-COS(TSUBI(L,LS))
                        DOMA=-TERMA*TERMB
                        XMU=SIN(TKSSUB)*COS(PLSSUB)
                        PSI=SIN(TKSSUB)*SIN(PLSSUB)
                        ETA=COS(TKSSUB)
                        DO 2410 MMS=2,NSUB+1
                           PLSUB=PSUB(MM,MMS)
                           TERM1=PSUBI(MM,MMS+1)-PSUBI(MM,MMS)
                           DO 2420 LLS=2,NSUB+1
                              TKSUB=TSUB(LL,LLS)
                              XMUL=SIN(TKSUB)*COS(PLSUB)
                              PSIL=SIN(TKSUB)*SIN(PLSUB)
                              ETAL=COS(TKSUB)
                              ANG=XMU*XMUL+PSI*PSIL+ETA*ETAL
                              SUM=1.0
                              DO 2500 MO=1,MORDER
                                 SUM=SUM+A(MO)*PLCOS(MO,ANG,SMALL)
 2500                          CONTINUE
                              TERM2=COS(TSUBI(LL,LLS+1))-
     1                              COS(TSUBI(LL,LLS))
                              DOMS=-TERM1*TERM2
                              SUMLL=SUMLL+SUM*DOMS*DOMA
 2420                      CONTINUE
 2410                   CONTINUE
 2230                CONTINUE
 2220             CONTINUE
                  PHASE(LL,MM,L,M)=SUMLL/DOM(LL,MM)/DOM(L,M)
 2310          CONTINUE
 2300        CONTINUE
 2210     CONTINUE
 2200  CONTINUE
C
       CALL PNORM
C
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE PHISUB(PLOW,PHIGH,NSUB,PSUB,PSUBI,L)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      INCLUDE 'PARAM.FOR'
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DIMENSION PSUBI(NP,NPSUB),PSUB(NP,NPSUB)
C
      NSUBK=NSUB+2
      PSUBI(L,2)=PLOW
      PSUBI(L,NSUBK)=PHIGH
      DPHI=(PHIGH-PLOW)/FLOAT(NSUB)
C
      DO 10 LL=3,NSUBK-1
         PSUBI(L,LL)=PSUBI(L,LL-1)+DPHI
 10   CONTINUE
C
      PSUB(L,1)=PSUBI(L,2)
      DO 20 LL=2,NSUBK-1
         PSUB(L,LL)=0.5*(PSUBI(L,LL)+PSUBI(L,LL+1))
 20   CONTINUE
      PSUB(L,NSUBK)=PSUBI(L,NSUBK)
C
      RETURN
      END
```

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE TETASUB(TLOW,THIGH,DPHI,NSUB,TSUB,TSUBI,K)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      INCLUDE 'PARAM.FOR'
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DIMENSION TSUBI(NT,NPSUB),TSUB(NT,NPSUB)
C
      NSUBL=NSUB+2
      TSUBI(K,2)=TLOW
      TSUBI(K,NSUBL)=THIGH
      DPHI=(THIGH-TLOW)/FLOAT(NSUB)
      DO 10 KK=3,NSUBL-1
         TSUBI(K,KK)=TSUBI(K,KK-1)+DPHI
 10   CONTINUE
C
      TSUB(K,1)=TSUBI(K,2)
      DO 20 KK=2,NSUBL-1
         TSUB(K,KK)=0.5*(TSUBI(K,KK)+TSUBI(K,KK+1))
 20   CONTINUE
      TSUB(K,NSUBL)=TSUBI(K,NSUBL)
C
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE PCOEF

C*****************************************************************
      INCLUDE 'PARAM.FOR'
      INCLUDE 'COMMON.FOR'
      COMPLEX IOR
C**********************************************************************
C
C   KPHASE     PHASE FUNCTION
C ------------------------------
C
C     2             F1
C     3             F2
C     6             B1
C     7             B2
C
C
         IF(KPHASE.EQ.1) IPH=0
         IF(KPHASE.EQ.2) IPH=2
         IF(KPHASE.EQ.3) IPH=4
         IF(KPHASE.EQ.4) IPH=5
         IF(KPHASE.EQ.5) IPH=6
         IF(KPHASE.EQ.6) IPH=10
         IF(KPHASE.EQ.7) IPH=20

C--- FORWARD SCATTERING PHASE FUNCTIONS.
      IF(IPH.EQ.100)THEN
      MORDER =1
c      A(0)  =1.0
      A(1)  =3.*0.30
      ENDIF
C-----
      IF(IPH.EQ.0) THEN
      XSIZ  = 999.
      IOR   = (999.,999.)
      QSCA  = 999.
      QABS  = 999.
      MORDER = 26
c      A(0)  = 1.0
      A(1)  = 2.78197
```

```
      A(2)  = 4.25856
      A(3)  = 5.38653
      A(4)  = 6.19015
      A(5)  = 6.74492
      A(6)  = 7.06711
      A(7)  = 7.20999
      A(8)  = 7.20063
      A(9)  = 7.03629
      A(10) = 6.76587
      A(11) = 6.35881
      A(12) = 5.83351
      A(13) = 5.22997
      A(14) = 4.47918
      A(15) = 3.69000
      A(16) = 2.81577
      A(17) = 1.92305
      A(18) = 1.11502
      A(19) = 0.50766
      A(20) = 0.20927
      A(21) = 0.07138
      A(22) = 0.02090
      A(23) = 0.00535
      A(24) = 0.00120
      A(25) = 0.00024
      A(26) = 0.00004
      ENDIF
C -----
      IF(IPH.EQ.1) THEN
      XSIZE =10.0
      IOR =(1.33,0.0)
      QSCA=2.20654869
      QABS=0.0
      GFAC=0.71245915
      MORDER=20
c      A(0)=1.0000000
      A(1)=2.1373777
      A(2)=2.9336057
      A(3)=2.8347003
      A(4)=2.7405264
      A(5)=2.6281443
      A(6)=2.5408404
      A(7)=2.6078286
      A(8)=2.7392752
      A(9)=2.9413168
      A(10)=3.2081311
      A(11)=3.4706223
      A(12)=3.7823384
      A(13)=4.1312394
      A(14)=4.2848716
      A(15)=4.7362566

      A(16)=4.4471574
      A(17)=4.3439150
      A(18)=3.3834202
      A(19)=2.2265594
      A(20)=1.3012373
      ENDIF
C -----
      IF(IPH.EQ.2) THEN
      XSIZE =   5.0
      IOR   =  (1.33,0.0)
      QSCA  =   3.59103251
      QABS  =   0.0
      GFAC  =   0.84534043
```

```
      MORDER =   12
c      A(0)  =   1.0
      A(1)  =   2.5360217
      A(2)  =   3.5654900
      A(3)  =   3.9797626
      A(4)  =   4.0029206
      A(5)  =   3.6640084
      A(6)  =   3.0160117
      A(7)  =   2.2330437
      A(8)  =   1.3025078
      A(9)  =   0.5346286
      A(10) =   0.2013563
      A(11) =   0.0547964
      A(12) =   0.0109929
      ENDIF
C-----
      IF(IPH.EQ.3) THEN
      XSIZE =3.0
      IOR =(1.33,0.0)
      QSCA=1.75339794
      QABS=0.0
      GFAC=0.78320068
      MORDER=9
c      A(0)  =   1.0000000
      A(1)  =   2.3496020
      A(2)  =   2.7382560
      A(3)  =   2.3145776
      A(4)  =   1.3907945
      A(5)  =   0.5534959
      A(6)  =   0.1744258
      A(7)  =   0.0401137
      A(8)  =   0.0069153
      A(9)  =   0.0008899
      ENDIF
C-----
      IF(IPH.EQ.4) THEN
      XSIZE =  2.0
      IOR   = (1.33,0.0)
      QSCA  =  0.71294856
      QABS  =  0.0
      MORDER =  8
c      A(0)  =  1.0
      A(1)  =  2.0091653
      A(2)  =  1.5633900
      A(3)  =  0.6740690
      A(4)  =  0.2221484
      A(5)  =  0.0472529
      A(6)  =  0.0067132
      A(7)  =  0.0006743
      A(8)  =  0.0000494
      ENDIF
C-----
      IF(IPH.EQ.5) THEN
      XSIZE =1.0
      IOR =(1.33,0.0)
      QSCA=9.39240903E-02
      QABS=0.0
      GFAC=0.18451715
      MORDER=6
c      A(0)  =   1.0000000
      A(1)  =   0.5535514
      A(2)  =   0.5600496
      A(3)  =   0.1157242
      A(4)  =   0.0107823
```

```
      A(5)  =   0.0005812
      A(6)  =   0.0000230
      ENDIF
C-----
      IF(IPH.EQ.6) THEN
      XSIZE = 999.
      IOR   =(999.,999.)
      QSCA  = 999.
      QABS  = 999.
      MORDER = 2
c      A(0)  = 1.0
      A(1)  = 1.2
      A(2)  = 0.5
      ENDIF
C
C-----
      IF(IPH.EQ.7) THEN
      XSIZE = 999.
      IOR   =(999.,999.)
      QSCA  = 999.
      QABS  = 999.
      MORDER = 1
c      A(0)  = 1.0
      A(1)  = 1.0
      ENDIF
C
C-------------------------------------------------------------
C--- BACKWARD SCATTERING PHASE FUNCTIONS.
      IF(IPH.EQ.10) THEN
         XSIZE = 999.
         IOR   =(999.,999.)
         QSCA  = 999.
         QABS  = 999.
         MORDER = 5
C        A(0)  = 1.0
         A(1)  =-0.56524
         A(2)  = 0.29783
         A(3)  = 0.08571
         A(4)  = 0.01003
         A(5)  = 0.00063
      ENDIF
C-----
      IF(IPH.EQ.20) THEN
         XSIZE = 999.
         IOR   =(999.,999.)
         QSCA  = 999.
         QABS  = 999.
         MORDER = 2
C        A(0)  = 1.0
         A(1)  =-1.2
         A(2)  = 0.5
      ENDIF
C-----
      IF(IPH.EQ.30) THEN
         XSIZE = 999.
         IOR   =(999.,999.)
         QSCA  = 999.
         QABS  = 999.
         MORDER = 1
C        A(0)  = 1.0
         A(1)  = -1.0
      ENDIF
C
      RETURN
```

```
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE PNORM
C*******************************************************************
      INCLUDE 'PARAM.FOR'
      INCLUDE 'COMMON.FOR'
      DIMENSION PHOLD(NT,NP,NT,NP)
C************************************************************************
      DO 10 L=2,K2
         DO 20 M=2,J2
            SUM=0.0
            DO 30 LL=2,K2
               DO 40 MM=2,J2
                  SUM=SUM+PHASE(LL,MM,L,M)*DOM(LL,MM)
 40            CONTINUE
 30         CONTINUE
C
            FACT=SUM/(4.*PI)
C
            DO 50 LL=2,K2
               DO 60 MM=2,J2
                  PHOLD(LL,MM,L,M)=PHASE(LL,MM,L,M)
                  PHASE(LL,MM,L,M)=PHASE(LL,MM,L,M)/(FACT+SMALL)
 60            CONTINUE
 50         CONTINUE
 20      CONTINUE
 10   CONTINUE
C
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      FUNCTION PLCOS(L,X,SMALL)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C     COMPUTES THE LEGENDRE POLYNOMIAL P-SUB-L.
C     X IS IN THE RANGE FROM -1 TO +1.
C     MODIFIED FROM "NUMERICAL RECIPES, THE ART OF SCIENTIFIC COMPUTING' BY
C     W. H. PRESS, B. P. FLANNERY, S. A. TEUKOLSKY, W. T. VETTERLING
C     ORIGINAL FUNCTION NAME = PLGNDR

cc      IF(L.LT.1.OR.ABS(X).GT.1.+SMALL) PRINT*,'CHECK ANG',X
C
      PMM=1.
      PMMP1=X
      IF(L.EQ.1) THEN
         PLCOS=PMMP1
C
      ELSE
         DO 12 LL=2,L
            PLL=(X*(2*LL-1)*PMMP1-(LL-1)*PMM)/(LL)
            PMM=PMMP1
            PMMP1=PLL
 12      CONTINUE
         PLCOS=PLL
      ENDIF
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE SUPPLY
C*******************************************************************
      INCLUDE 'PARAM.FOR'
      INCLUDE 'COMMON.FOR'
C*******************************************************************
    7 FORMAT(/1X,6(1H*),3X,A18,3X,6(1H*)/9X,20(1H-))
    8 FORMAT(/'  I =',I6,6I9)
```

```
    9 FORMAT('   J')
   10 FORMAT(26(1H*),3X,A10,3X,26(1H*))
   20 FORMAT(4H I =,I6,6I9)
   30 FORMAT(1HJ)
   40 FORMAT(1X,I2,3X,1P7E9.2)
   50 FORMAT(1H )
   51 FORMAT(1X,'I =',2X,7(I4,5X))
   52 FORMAT(1X,'X =',1P7E9.2)
   53 FORMAT(1X,'TH =',1P7E9.2)
   54 FORMAT(1X,'J =',2X,7(I4,5X))
   55 FORMAT(1X,'Y =',1P7E9.2)
   62 FORMAT(1X,'XU=',1P7E9.2)
   65 FORMAT(1X,'YV=',1P7E9.2)
 56   FORMAT(1X,'K =',2X,7(I4,5X))
 57   FORMAT(1X,'Z =',1P7E9.2)
 58   FORMAT(1X,'L =',2X,7(I4,5X))
 59   FORMAT(1X,'TH =',1P7E9.2)
 60   FORMAT(1X,'M =',2X,7(I4,5X))
 61   FORMAT(1X,'PH =',1P7E9.2)
C*****************************************************************
      ENTRY EZGRID
C
CONSTRUCT THE X-DIRECTION GRID
      L1=NCVLX+2
      XU(1)=XU(2)
      XU(L1)=XL+XU(2)
      L2=L1-1
      FCVLX=FLOAT(NCVLX)
      DO 21 I=3,L2
      DD=FLOAT(I-2)/FCVLX
      IF(POWERX.GT.0.) THEN
      XU(I)=XL*DD**POWERX+XU(2)
      ELSE
      XU(I)=XL*(1.-(1.-DD)**(-POWERX))+XU(2)
      ENDIF
 21   CONTINUE
CONSTRUCT THE Y-DIRECTION GRID
      M1=NCVLY+2
      YV(2)=0.
      YV(M1)=YL
      M2=M1-1
      FCVLY=FLOAT(NCVLY)
      DO 31 J=3,M2
      DD=FLOAT(J-2)/FCVLY
      IF(POWERY.GT.0.) THEN
      YV(J)=YL*DD**POWERY
      ELSE
      YV(J)=YL*(1.-(1.-DD)**(-POWERY))
      ENDIF
 31   CONTINUE
C
      RETURN
C*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
      ENTRY ZGRID
CONSTRUCT THE GRID ZONE-BY-ZONE
C
CONSIDER THE X DIRECTION
      XU(1)=XU(2)
      I2=2
      DO 1101 NZ=1,NZX
      FCVLX=FLOAT(NCVX(NZ))
      ILAST=I2
      I1=ILAST+1
      I2=ILAST+NCVX(NZ)
```

```
      DO 1101 I=I1,I2
      DD=FLOAT(I-ILAST)/FCVLX
      IF(POWRX(NZ).GT.0.) THEN
      XU(I)=XU(ILAST)+XZONE(NZ)*DD**POWRX(NZ)
      ELSE
      XU(I)=XU(ILAST)+XZONE(NZ)*(1.-(1.-DD)**(-POWRX(NZ)))
      ENDIF
 1101 CONTINUE
      L1=I2
C
CONSIDER THE Y DIRECTION
C
      YV(2)=0.
      JJ2=2
      DO 1100 NZ=1,NZY
      FCVLY=FLOAT(NCVY(NZ))
      JLAST=JJ2
      JJ1=JLAST+1
      JJ2=JLAST+NCVY(NZ)
      DO 1100 J=JJ1,JJ2
      DD=FLOAT(J-JLAST)/FCVLY
      IF(POWRY(NZ).GT.0.) THEN
      YV(J)=YV(JLAST)+YZONE(NZ)*DD**POWRY(NZ)
      ELSE
      YV(J)=YV(JLAST)+YZONE(NZ)*(1.-(1.-DD)**(-POWRY(NZ)))
      ENDIF
 1100 CONTINUE
      M1=JJ2
      RETURN
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      ENTRY QUAD
C
CONSTRUCT THE PHI-DIRECTION GRID
C
      J1=NCVLP+2
      PHII(2)=0.
      PHII(J1)=PL
      J2=J1-1
      FCVLP=FLOAT(NCVLP)
      DO 4 M=3,J2
      DD=FLOAT(M-2)/FCVLP
      IF(POWERP.GT.0.) THEN
      PHII(M)=PL*DD**POWERP
      ELSE
      PHII(M)=PL*(1.-(1.-DD)**(-POWERP))
      ENDIF
 4    CONTINUE
C
CONSTRUCT THE THETA-DIRECTION GRID
C
      K1=NCVLT+2
      THETAI(2)=0.
      THETAI(K1)=TL
      K2=K1-1
      FCVLT=FLOAT(NCVLT)
      DO 5 L=3,K2
         DD=FLOAT(L-2)/FCVLT
         IF(POWERT.GT.0.) THEN
            THETAI(L)=TL*DD**POWERT
         ELSE
            THETAI(L)=TL*(1.-(1.-DD)**(-POWERT))
         ENDIF
 5    CONTINUE
C
```

```
      RETURN
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      ENTRY PRINT
C
      PRINT 50
      WRITE(7,50)
      IEND=0
 301  IF(IEND.EQ.L1) GO TO 310
      IBEG=IEND+1
      IEND=IEND+7
      IEND=MIN0(IEND,L1)
      PRINT 50
    WRITE(7,50)
      PRINT 51,(I,I=IBEG,IEND)

      WRITE(7,51)(I,I=IBEG,IEND)
      IF(MODE.EQ.3) GO TO 302
      PRINT 52,(X(I),I=IBEG,IEND)
      PRINT 62,(XU(I),I=IBEG,IEND)
      WRITE(7,52)(X(I),I=IBEG,IEND)
      WRITE(7,62)(XU(I),I=IBEG,IEND)
      GO TO 303
 302  PRINT 53,(X(I),I=IBEG,IEND)
    WRITE(7,53)(X(I),I=IBEG,IEND)
 303  GO TO 301
 310  JEND=0
      PRINT 50
    WRITE(7,50)
 311  IF(JEND.EQ.M1) GO TO 320
      JBEG=JEND+1
      JEND=JEND+7
      JEND=MIN0(JEND,M1)
      PRINT 50
      PRINT 54,(J,J=JBEG,JEND)
      PRINT 55,(Y(J),J=JBEG,JEND)
      PRINT 55,(YV(J),J=JBEG,JEND)
      WRITE(7,50)
      WRITE(7,54)(J,J=JBEG,JEND)
      WRITE(7,55)(Y(J),J=JBEG,JEND)
      WRITE(7,65)(YV(J),J=JBEG,JEND)
      GO TO 311
 320  JEND=0
      PRINT 50
      WRITE(7,50)
 331  IF(JEND.EQ.K1) GO TO 340
      JBEG=JEND+1
      JEND=JEND+7
      JEND=MIN0(JEND,K1)
      PRINT 50
      PRINT 58,(J,J=JBEG,JEND)
      PRINT 59,(THETA(J),J=JBEG,JEND)
      WRITE(7,50)
      WRITE(7,58)(J,J=JBEG,JEND)
      WRITE(7,59)(THETA(J),J=JBEG,JEND)
      GO TO 331
 340  JEND=0
      PRINT 50
      WRITE(7,50)
 341  IF(JEND.EQ.J1) GO TO 350
      JBEG=JEND+1
      JEND=JEND+7
      JEND=MIN0(JEND,J1)
      PRINT 50
      PRINT 60,(J,J=JBEG,JEND)
```

```
      PRINT 61,(PHI(J),J=JBEG,JEND)
      WRITE(7,50)
      WRITE(7,60)(J,J=JBEG,JEND)
      WRITE(7,61)(PHI(J),J=JBEG,JEND)
      GO TO 341
 350  CONTINUE
C
      WRITE(6,7) TITLE(1)
      WRITE(7,7) TITLE(1)
      IBEG=1
      JBEG=1
      IEND=l1
      JEND=m1
      IREP=(IEND-IBEG+7)/7
      DO 551 KP=1,IREP
         INCR=MIN(6,IEND-IBEG)
         ISTOP=IBEG+INCR
         WRITE(6,8) (I,I=IBEG,ISTOP)
         WRITE(6,9)
         WRITE(7,8) (I,I=IBEG,ISTOP)
         WRITE(7,9)
         DO 552 J=JEND,JBEG,-1
            WRITE(6,40) J,(G(I,j),I=IBEG,ISTOP)
            WRITE(7,40) J,(G(I,j),I=IBEG,ISTOP)
 552      CONTINUE
         IBEG=ISTOP+1
 551   CONTINUE
C
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE HFLUX
C*********************************************************************
      INCLUDE 'PARAM.FOR'
      INCLUDE 'COMMON.FOR'
C*********************************************************************
      DO 500 I=1,L1
         DO 510 J=1,M1
            SQPY=0.
            SQMY=0.
            SQPX=0.
            SQMX=0.
            DO 520 L=2,K2
               DO 530 M=2,J2
                  ADCY=ABS(DCY(L,M))
                  ADCX=ABS(DCX(L,M))
                  IF(I.GT.1.AND.I.LT.L1) THEN
                     IF(DCY(L,M).GT.0.0) THEN
                        SQPY=SQPY+ADCY*F(I,J,L,M)
                     ELSE
                        SQMY=SQMY+ADCY*F(I,J,L,M)
                     ENDIF
                  ENDIF
                  IF(J.GT.1.AND.J.LT.M1) THEN
                     IF(DCX(L,M).GT.0.0) THEN
                        SQPX=SQPX+ADCX*F(I,J,L,M)
                     ELSE
                        SQMX=SQMX+ADCX*F(I,J,L,M)
                     ENDIF
                  ENDIF
 530           CONTINUE
 520        CONTINUE
            QPY(I,J)=SQPY
            QMY(I,J)=SQMY
```

```
          QPX(I,J)=SQPX
          QMX(I,J)=SQMX
 511    CONTINUE
 510    CONTINUE
 500  CONTINUE
C
      DO 540 I=1,L1
        DO 550 J=1,M1
          GSUM=0.
          DO 560 L=2,K2
            DO 570 M=2,J2
              GSUM=GSUM+F(I,J,L,M)*DOM(L,M)
 570        CONTINUE
 560      CONTINUE
          G(I,J)=GSUM
 551    CONTINUE
 550    CONTINUE
 540  CONTINUE
C
      DO 580 J=1,M1
        IF(KBCI1(J).EQ.2) THEN
          QPY(1,J)=QPY(2,J)
          QMY(1,J)=QMY(2,J)
          G(1,J)  =G(2,J)
        ENDIF
        IF(KBCL1(J).EQ.2) THEN
          QPY(L1,J)=QPY(L2,J)
          QMY(L1,J)=QMY(L2,J)
          G(L1,J)  =G(L2,J)
        ENDIF
 580  CONTINUE
C
      DO 600 I=1,L1
        IF(KBCJ1(I).EQ.2) THEN
          QPX(I,1)=QPX(I,2)
          QMX(I,1)=QMX(I,2)
          G(I,1)  =G(I,2)
        ENDIF
        IF(KBCM1(I).EQ.2) THEN
          QPX(I,M1)=QPX(I,M2)
          QMX(I,M1)=QMX(I,M2)
          G(I,M1)  =G(I,M2)
        ENDIF
 600  CONTINUE
C
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
```

APPENDIX

B

FUNCTIONS OF VARIOUS SUBROUTINES AND ENTRIES

DEFLT	:	default values are set here.
GRID*	:	geometry of the problem is specified here.
SETUP1	:	initial setup.
START*	:	properties and temperatures are set here.
SETUP2	:	final setup.
LC*	:	inhomogeneous medium is set here.
ISOTRP	:	set the phase function to the isotropic phase function.
ANISO	:	calculates anisotropic phase function.
OUTPUT*	:	output routine.
GAMSOR*	:	irregular geometries and radiative equilibrium conditions are set here.
HEART	:	main solution loop.
PRINT	:	print grid related variables and incident radiation energy.
EZGRID	:	usually called in entry grid to construct the spatial grids.
QUAD	:	usually called in entry grid to construct the angular grids.
HFLUX	:	heat fluxes and incident radiation energy are calculated here.

* Denotes entries in the ADAPT subroutine.

APPENDIX

C

FORTRAN VARIABLES

LAST	:	maximum number of iterations.
MODE	:	indicator for co-ordinate system.
EROR	:	convergence criteria.
SMALL	:	a small number.
BIG	:	a big number.
PI	:	π
PIBY2	:	$\pi/2$
PI32	:	$3\pi/2$
PI4	:	4π
TL	:	θ-direction length of the calculation domain = π
PL	:	ϕ-direction length of the calculation domain = 2π
XL	:	x-direction length of the calculation domain
YL	:	y-direction length of the calculation domain
POWERT	:	non-uniformity index for the theta-direction grid.
POWERP	:	non-uniformity index for the phi-direction grid.
POWERX	:	non-uniformity index for the x-direction grid.
POWERY	:	non-uniformity index for the y-direction grid.
NCVLP	:	number of phi-direction control volume widths in the domain. Presently, please use NCVLP that is divisible by 4.
NCVLT	:	number of theta-direction control volume widths in the domain. Presently, please use NCVLP that is divisible by 2.
NCVLX	:	number of x-direction control volume widths in the domain.
NCVLY	:	number of y-direction control volume widths in the domain.
XZONE(NZ)	:	x-direction length of a zone.
YZONE(NZ)	:	y-direction length of a zone.
NCVX(NZ)	:	number of x-direction control volume widths in a zone.
NCVY(NZ)	:	number of y-direction control volume widths in a zone.
POWRX(NZ)	:	non-uniformity index for the x-direction grid in a zone.
POWRY(NZ)	:	non-uniformity index for the y-direction grid in a zone.
EPSJ1	:	emissivity of the south wall.
EPSM1	:	emissivity of the north wall.
EPSL1	:	emissivity of the east wall.
EPSI1	:	emissivity of the west wall.
RHOJ1	:	1.- EPSJ1.
RHOM1	:	1.- EPSM1.

RHOL1	: 1.- EPSL1.
RHOI1	: 1.- EPSI1.
ALPHA	: absorption coefficient, α.
SIG	: scattering coefficient.
STFAN	: Stefan-Boltzmann constant, σ.
KISO	: = 1; isotropic scattering.
	: = 0; anisotropic scattering.
L1	: maximum number of x-direction grid locations = NCVLX + 2.
M1	: maximum number of y-direction grid locations = NCVLY + 2.
K1	: maximum number of theta-direction grid locations = NCVLT + 2.
K1	: maximum number of phi-direction grid locations = NCVLP + 2.
FCOLJ1()	: intensity of the collimated beam at the south wall.
F()	: actual nodal intensity.
FOLD()	: actual nodal intensity from the previous iteration.
RIBI1()	: $\sigma T^4/\pi$ at the west wall.
RIBL1()	: $\sigma T^4/\pi$ at the east wall.
RIBJ1()	: $\sigma T^4/\pi$ at the south wall.
RIBM1()	: $\sigma T^4/\pi$ at the north wall.
T(I,J)	: temperature.
RIB(I,J)	: $\sigma T_g^4/\pi$ of the medium.
KBCI1()†	: boundary condition indicator for the west boundary:
	: = 1; given temperature.
	: = 2; symmetry
	: = 3; periodic.
CAPPA(I,J)	: absorption coefficient, κ.
SIGMA(I,J)	: scattering coefficient, σ_s.
BETA(I,J)	: extinction coefficient, $\beta = \kappa + \sigma_s$.
BM()	: modified extinction coefficient.
SM()	: modified source coefficient.
PHASE()	: phase function.
L1	: value of I for the right-boundary grid line.
M1	: value of J for the top-boundary grid line.
K1	: value of L for the $\theta = 180^0$ grid line.
J1	: value of M for the $\phi = 360^0$ grid line.
L2	: L1 – 1
L3	: L2 – 1
M2	: M1 – 1
M3	: M2 – 1
K2	: K1 – 1
K3	: K2 – 1
J2	: J1 – 1

† KBCL1(), KBCJ1() and KBCM1() are similar to KBCI1().

J3	: J2 - 1
X(I)	: value of X at grid location I.
XU(I)	: value of X at the control-volume face. XU(1) is meaningless.
XCV(I)	: *x*-direction width of control-volume. XCV(1) and XCV(L1) are meaningless.
Y(J)	: value of Y at grid location J.
YV(J)	: value of Y at the control-volume face. YV(1) is meaningless.
YCV(J)	: *y*-direction width of control-volume. YCV(1) and YCV(M1) are meaningless.
AX(J)	: YCV(J)
AY(I)	: XCV(I)
VOL(I,J)	: XCV(I) * YCV(J)
THETA(L)	: value of θ at grid location L.
THETAI(L)	: value of θ at the control-volume face. THETAI(1) is meaningless.
PHI(M)	: value of ϕ at grid location M.
PHII(M)	: value of ϕ at the control-volume face. PHII(1) is meaningless.
DCX()	: D_{CX}^{L}
DCY()	: D_{CY}^{L}
DOM()	: $\Delta\Omega$
ISOLID()	: index denoting blockage.
QPY()	: q_y^+
QMY()	: $\left\|q_y^-\right\|$
QPX()	: q_x^+
QMX()	: $\left\|q_x^-\right\|$
G	: incident radiation energy.

APPENDIX

D

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