CHAPTER 12 PROBLEMS AND EXERCISES

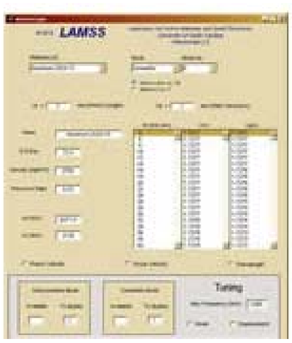
Problem 1: Consider a PWAS transducer of length $l = 7\text{ mm}$, width $b = 1.65\text{ mm}$, thickness $t = 0.2\text{ mm}$, and material properties as given in Table 12.2. The PWAS is bonded at one end of a 1-mm thick 1000-mm long aluminum strip with $E = 70\text{ GPa}$, $\rho = 2700\text{ kg/m}^3$, $\nu = 0.33$. The PWAS length is oriented along the strip. The PWAS is excited with a 3.5-counts tone burst. (i) Calculate the first frequency at which the S0 wave propagation is dominant. (ii) Assuming that the frequency is adjusted to the value at which the S0 wave propagation is predominant as calculated in part (i), calculate the time taken by the wave packet to travel to the other end of the strip specimen and come back. Sketch the wave pattern. (iii) Repeat part (ii) assuming that a through-the-thickness crack reflector is present at 400 mm from the PWAS. Sketch the wave pattern. (iv) Superpose the effects of (ii) and (iii) assuming that the energy is equally partitioned between the waves reflecting from the end and reflecting from the crack. Sketch the wave pattern. Discuss.

<u>Solution</u>

(i) To calculate the tuning frequencies, use the tuning theory developed in textbook Chapter 11. For rectangular PWAS, one refers to Section 10.4.5, specifically Eq. (10.110), i.e.,

$$\varepsilon_x(x,t) = -i\frac{a\tau_a}{\mu}\left(\sin\xi_0^S a\right)\frac{N_S(\xi_0^S)}{D_S'(\xi_0^S)}e^{i(\xi_0^S x-\omega t)} - i\frac{a\tau_a}{\mu}\left(\sin\xi_0^A a\right)\frac{N_A(\xi_0^A)}{D_A'(\xi_0^A)}e^{i(\xi_0^A x-\omega t)} \qquad (10.110)\ (1)$$

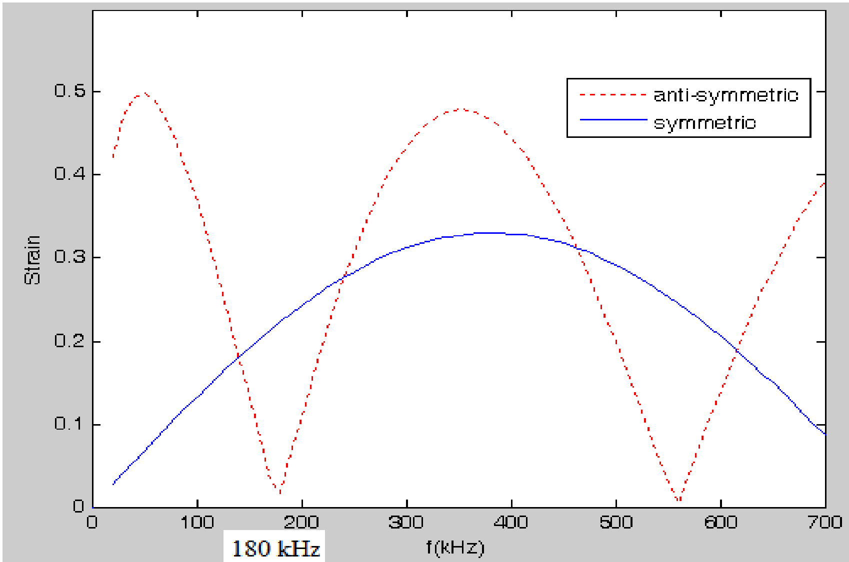
One could code Eq. (1) or could use the software programs posted on the LAMSS website http://www.me.sc.edu/research/lamss/html/software.html . In particular, download and activate the following program:



WAVESCOPE: DISPERSION CURVES, GROUP VELOCITIES, AND TUNING FOR METALLIC STRUCTURES

Download

After activating the program, one obtains the chart below.



Since the problem asks us to find the first tuning frequency, the chart is restricted to lower frequencies. In the considered frequency range, only two Lamb wave modes exist, S0 and A0. In the chart, the S0 mode is presented in solid line (‘symmetric”), whereas the A0 mode is presented in dashed line (“anti-symmetric”). Examination of this tuning chart indicates that the A0 mode goes through a minimum (mode rejection) at 180 kHz. When one mode is rejected, the other becomes dominant. Hence, the first frequency at which the S0 mode is dominant is f_1^{S0}=180 kHz .

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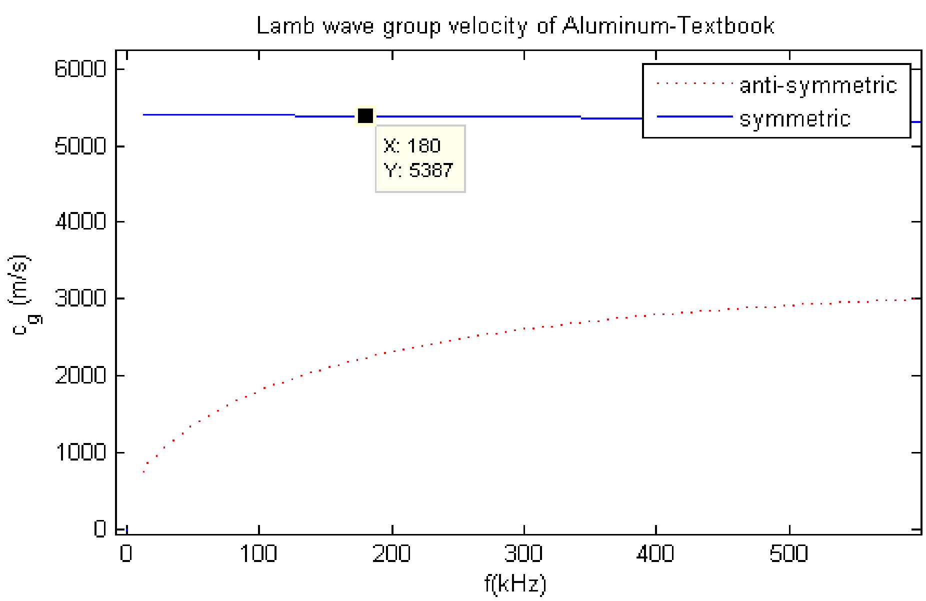
(ii) Assuming that the frequency is tuned at 180 kHz (i.e., when S0 is dominant), we calculate the group velocity for this mode at this frequency. By plotting the group velocity, we obtain the chart below



The chart shows that, in this frequency range, the S0 group velocity (solid line) varies very slowly with frequency. The exact value is calculate separately as $c_g^{S0}\Big|_{f=180\text{ kHz}} = 5387\text{ m/s}$. For convenience, we express the speed in $\text{mm/}\mu\text{s}$, i.e., $c_g^{S0}\Big|_{f=180\text{ kHz}} = 5.387\text{ mm/}\mu\text{s}$.

Using group velocity, one calculates the round-trip time of flight (TOF) to the end of the strip as

$$t_{end} = \frac{2L}{c_g^{S0}} = \frac{2\times 1000\text{ mm}}{5.387\text{ mm/}\mu\text{s}} = 371\ \mu\text{s} \tag{2}$$

To calculate the wave reflection process, define the incident forward wave and the reflected backward wave as the following continuous waves (CWs:

$$u_i(x,t) = Ae^{i(\xi^{S0}x-\omega t)} \text{ ......incident forward wave} \tag{3}$$

$$u_i(x,t) = Be^{-i\left(\xi^{S0}(x+2x_R)+\omega t\right)} = Be^{-i\xi^{S0}(x+2x_R)}e^{-i\omega t} \text{ ......reflected backward wave} \tag{4}$$

where x_R is the reflector position and ξ^{S0} is the frequency dependent wavenumber defined as

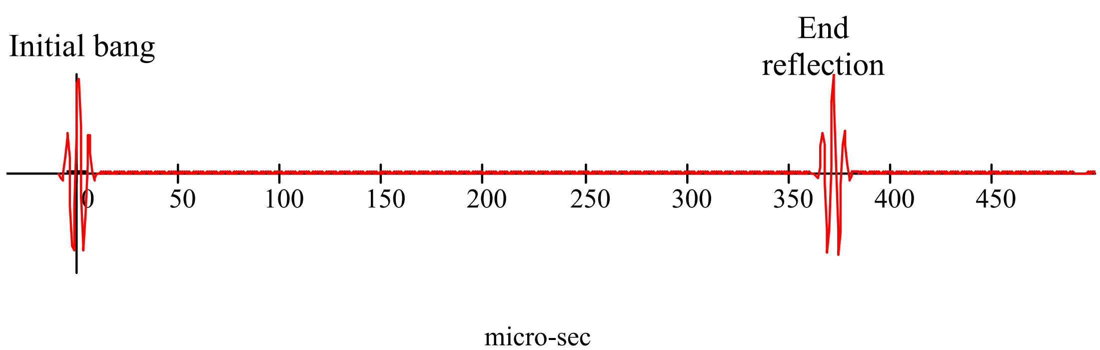
$$\xi^{S0} = \frac{\omega}{c^{S0}} \text{ ......frequency-dependent wavenumber} \tag{5}$$

where c^{S0} is the phase velocity of the S0 mode. At this relatively low frequencies, the S0 mode is very slowly dispersive, hence we will take a linear approximation of its variation over the frequency range of interest. The total wave is the summation of the incident and reflected waves, i.e.,

$$u(x,t) = u_i(x,t) + u_r(x,t) = \left(Ae^{i\xi^{S0}x} + Be^{-i\xi^{S0}(x+2x_R)}\right)e^{-i\omega t} \text{ ......complete wave} \tag{6}$$

For a generic situation, we take $A=1$. Assuming perfect reflection at the $x_R = L$ end of the strip, we take $B = a = 1$. For a given excitation signal $s(t)$, e.g., a tone burst, the solution is obtained by performing the convolution between the excitation signal and the CW definition of Eq. (6). The convolution is performed in the frequency domain as a multiplication.

The resulting wave pattern is shown below; the transmitted incident wave package and received reflected wave packet are clearly defined. The TOF can be easily estimated. The $t_{end} = 371\ \mu\text{s}$ value calculated with Eq. (2) matches well the estimation obtained from the chart.



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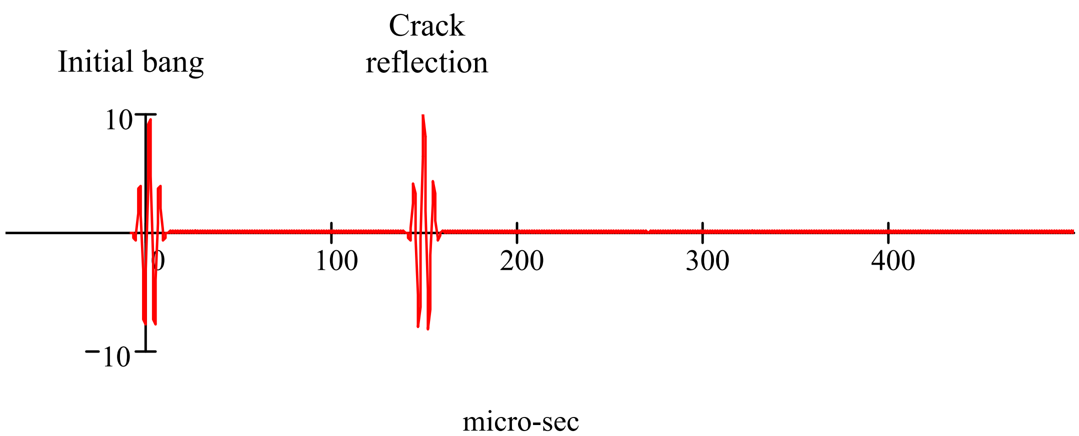
(iii) Assuming a through-the-thickness crack reflector at L_c=400 mm , we calculate the time of flight from the crack reflector as

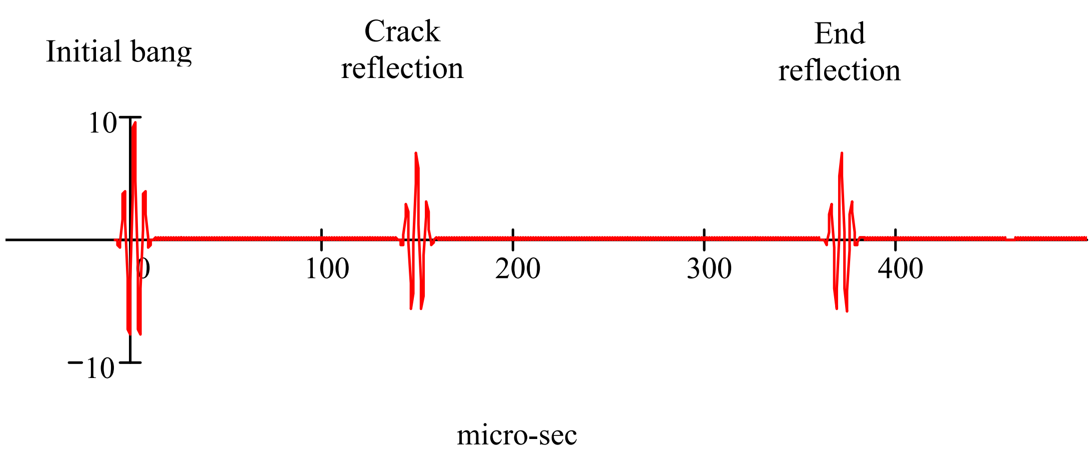
$$t_{crack} = \frac{2L_c}{c_g^{S0}} = \frac{2\times 400 \text{ mm}}{5.387 \text{ mm}/\mu\text{s}} = 149 \ \mu\text{s} \tag{7}$$

The wave pattern of the transmitted and received wave packets is shown below. The t_{crack} can be easily estimated. The value calculated with Eq. (7) matches well the estimations obtained from the chart.



(iv) Assuming equal partition of energy between the end reflection and crack reflection we take $B_{end} = B_{crack} = A/\sqrt{2}$. The superposed plot of the two reflections is shown below.

Discussion: One sees the reflections from the crack and the reflection from the end. The t_{end} and t_{crack} can be easily estimated. The values calculated with Eqs. (2) and (7) match well the estimations obtained from the chart.



PROBLEM 12.1 SOLUTION

Units

$kHz := 1000 \cdot Hz$ $\quad MHz := 10^{6} \cdot Hz$ $\quad krad := 10^{3} \cdot rad$

$ms := 10^{-3} \cdot s$ $\quad \mu s := 10^{-6} \cdot s$

$pF := 10^{-12} \cdot F$ $\quad nF := 10^{-9} \cdot F$

$\mu m := 10^{-6} \cdot m$ $\quad \mu\varepsilon := 10^{-6}$ $\quad MPa := 10^{6} \cdot Pa$ $\quad GPa := 10^{9} \cdot Pa$ $\quad TPa := 10^{12} \cdot Pa$

$MN := 10^{6} \cdot N$ $\quad GN := 10^{9} \cdot N$

$mS := 10^{-3} \cdot S$ $\quad \mu S := 10^{-6} \cdot S$ $\quad nS := 10^{-9} \cdot S$

$k\Omega := 10^{3} \cdot \Omega$ $\quad M\Omega := 10^{6} \cdot \Omega$ $\quad G\Omega := 10^{9} \cdot \Omega$

SETTINGS AND CONTROLS

$fs := 1.5 \cdot MHz$ $\quad dt := \frac{1}{fs}$ $\quad dt = 0.667\,\mu s$ $\quad$ s $\quad fNQ := \frac{fs}{2}$

$tmax := 5000 \cdot \mu s$

$$im := ceil\left(\frac{\log\left(\frac{tmax}{\mu s}\right) - \log\left(\frac{dt}{\mu s}\right)}{\log(2)}\right) \quad im = 13 \quad NN := 2^{im} \quad NN = 8192 \quad NQ := \frac{NN}{2}$$

$Nt := NN$ $\quad Nf := NQ$

$T := NN \cdot dt$ $\quad T = 5461\,\mu s$ $\quad df := \frac{1}{T}$ $\quad df = 183\,Hz$

Display controls: $\quad Tmax := 10^{-6} \cdot floor\left(\frac{10^{6} \cdot T}{1000}\right) \cdot 1000$ $\quad Tmax = 5000\,\mu s$ $\quad fmax := 200 \cdot kHz$

Frequency range: $\quad nf := 0 .. Nf$ $\quad f_{nf} := nf \cdot df$ $\quad f_{Nf} = 750\,kHz$

$\omega_{nf} := 2 \cdot \pi \cdot f_{nf}$ $\quad \omega_{Nf} = 4712\,\frac{krad}{s}$

definition of the space domain

$dx := 1.0 \cdot m$ $\quad Nx := 2$ $\quad nx := 0 .. Nx - 1$ $\quad x_{nx} := nx \cdot dx$

$x_1 = 1000\,mm$

Specimen properties

Aluminum strip $E := 70 \cdot GPa$ $\quad \rho := 2700 \cdot \frac{kg}{m^3}$ $\quad \nu := 0.33$ $\quad d := \frac{1}{2} \cdot mm$ $\quad L := 1000 \cdot mm$ $\quad Lc := 400 \cdot mm$

$L := L$ $\quad Lc := Lc$

Hanning smoothed tone burst $\quad Nc := 3.5 \quad fc := 180 \cdot kHz \quad \omega c := 2 \cdot \pi \cdot fc$

Signal amplitude $\quad p0 := 10$

Window size function of central frequency and Ncount $\quad a := \frac{1}{fc} \cdot Nc \cdot \frac{1}{2} \quad a = 9.722\ \mu s$

Define time range $\quad t0 := -a \quad nt := 0 .. Nt - 1 \quad t_{nt} := t0 + nt \cdot dt \quad t_{Nt-1} = 5450.9\ \mu s$

$$H(t) := \begin{vmatrix} p0 \ \text{ if } -0 < t - t0 < 2 \cdot a \\ 0 \ \text{ otherwise} \end{vmatrix}$$

square window

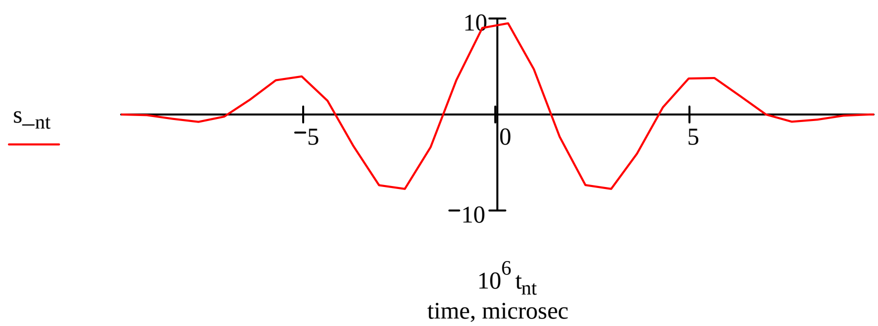
$$H(t) := \begin{vmatrix} p0 \cdot \sin\left(\pi \cdot \frac{t - t0}{2 \cdot a}\right)^2 \ \text{ if } 0 < t - t0 < 2 \cdot a \\ 0 \ \text{ otherwise} \end{vmatrix} \quad h_{nt} := H(t_{nt}) \quad HH := FFT(h)$$

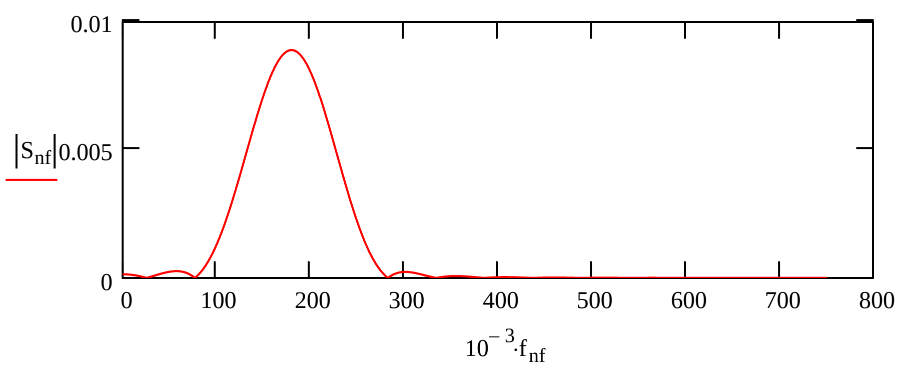
Hanning window and its FFT

$$HS_{nf} := HH_{nf}$$

$c(t) := -\sin[\omega c \cdot (t - t0)] \quad sc_{nt} := c(t_{nt}) \quad SC := FFT(sc)$ carrier signal and its FFT

$ss(t) := -\sin[\omega c \cdot (t - t0)] \cdot H(t) \quad s\_{nt} := ss(t_{nt}) \quad S := FFT(s\_)$ tone burst signal and its FFT



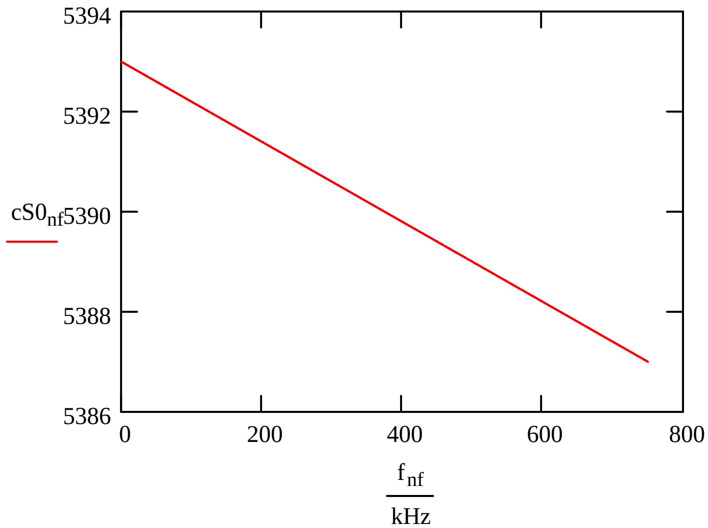


S0 and axial wave speed in aluminum strip

$c0 := \sqrt{\dfrac{E}{\rho \cdot \left(1 - \nu^2\right)}}$ $cS0g := 5387 \cdot \dfrac{m}{s}$ $cS0\_0kHz := 5393 \cdot \dfrac{m}{s}$ $cS0\_300kHz := 5387 \cdot \dfrac{m}{s}$

$Nf = 4096$

$c0 = 5394 \dfrac{m}{s}$ $cS0g = 5387 \dfrac{m}{s}$ $cS0_{nf} := cS0\_0kHz + \left(\dfrac{cS0\_300kHz - cS0\_0kHz}{Nf}\right) \cdot nf$



S0 waves analysis

Basic wave propagation analysis

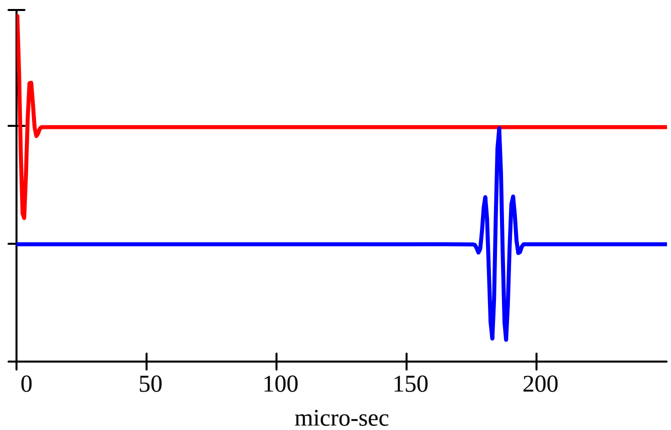
wave numbers $\xi_{nf} := \frac{\omega_{nf}}{cS0_{nf}}$

medium transfer function for the Nx locations x in space domain $G_{nf,nx} := e^{-i \cdot \xi_{nf} \cdot x_{nx}}$

excitation spectrum x medium transfer function at all x locations $U_{nf,nx} := S_{nf} \cdot G_{nf,nx}$

IFFT to return to the time domain $u^{\langle nx \rangle} := \mathrm{IFFT}\left(U^{\langle nx \rangle}\right)$

$\Delta := 10$ plotting offset control



End reflection analysis

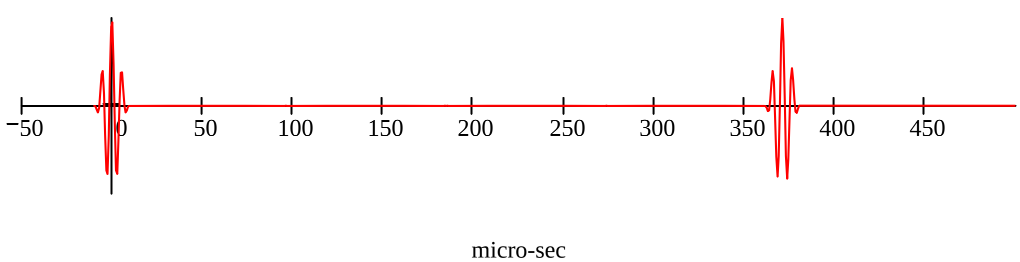
End reflection TOF estimate: $cS0g = 5.387 \frac{mm}{\mu s}$ $t\_end := \frac{2 \cdot L}{cS0g}$ $t\_end = 371\ \mu s$

End reflection mean the whole signal is returned, hence A=B

$A := 1$ $B := A$ $G_{nf,nx} := A \cdot e^{i \cdot \xi_{nf} \cdot x_{nx}} + B \cdot e^{-i \cdot \xi_{nf} \cdot \left(x_{nx} + 2 \cdot L\right)}$

$U_{nf,nx} := S_{nf} \cdot G_{nf,nx}$ $u^{\langle nx \rangle} := \mathrm{IFFT}\left(U^{\langle nx \rangle}\right)$

$\Delta := 0$



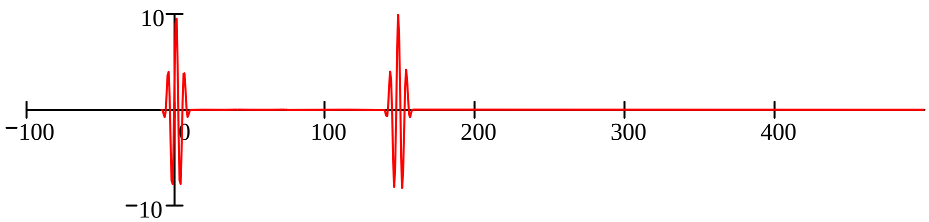
Crack reflection analysis

Crack reflection TOF estimate: $t\_crack := \frac{2 \cdot Lc}{cS0g}$ $t\_crack = 149\ \mu s$

$Ac := A$

$$G_{nf,nx} := A \cdot e^{i \cdot \xi_{nf} \cdot x_{nx}} + Ac \cdot e^{-i \cdot \xi_{nf} \cdot (x_{nx} + 2 \cdot Lc)}$$

$$U_{nf,nx} := S_{nf} \cdot G_{nf,nx} \qquad u^{\langle nx \rangle} := IFFT\left(U^{\langle nx \rangle}\right)$$



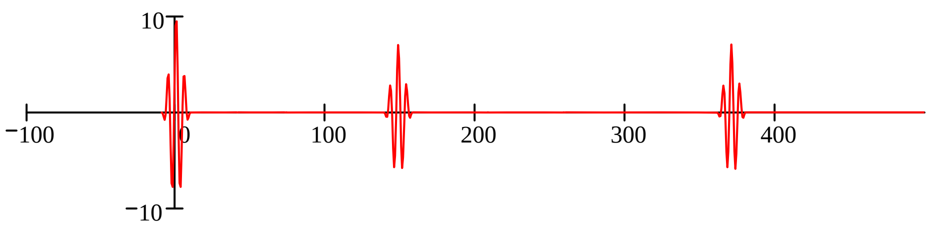
micro-sec

End reflection and crack reflection of axial wave (half-half energy split):

$$B := \frac{A}{\sqrt{2}} \qquad Ac := \frac{A}{\sqrt{2}}$$

$$G_{nf,nx} := A \cdot e^{i \cdot \xi_{nf} \cdot x_{nx}} + B \cdot e^{-i \cdot \xi_{nf} \cdot (x_{nx} + 2 \cdot L)} + Ac \cdot e^{-i \cdot \xi_{nf} \cdot (x_{nx} + 2 \cdot Lc)}$$

$$U_{nf,nx} := S_{nf} \cdot G_{nf,nx} \qquad u^{\langle nx \rangle} := IFFT\left(U^{\langle nx \rangle}\right)$$



micro-sec

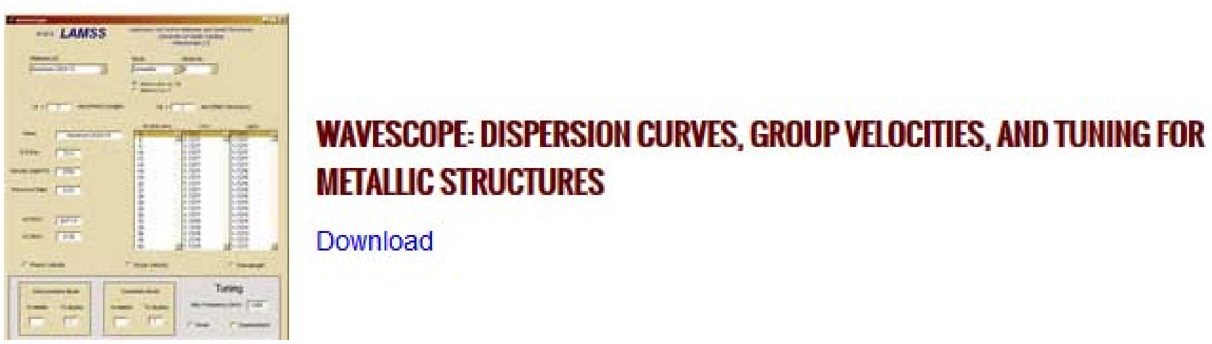
Problem 2: Consider a PWAS transducer of length $l = 7$ mm, width $b = 1.65$ mm, thickness $t = 0.2$ mm, and material properties as given in Table 12.2. The PWAS is bonded at one end of a 1-mm thick 1000-mm long aluminum strip with E = 70 GPa and ρ = 2.7 g/cc. The PWAS length is oriented along the strip. The PWAS is excited with a 3.5-counts tone burst. (i) Calculate the first frequency at which the A0 wave propagation is dominant. (ii) Assuming that the frequency is adjusted to the value at which the A0 wave propagation is dominant as calculated in part (i), calculate the time taken by the wave packet to travel to the other end of the strip specimen and come back. Sketch the wave pattern. (iii) Repeat part (ii) assuming that a through-the-thickness crack reflector is present at 400 mm from the PWAS. Sketch the wave pattern. (iii) Repeat part (ii) assuming that a through-the-thickness crack reflector is present at 400 mm from the PWAS. Sketch the wave pattern. (iv) Superpose the effects of (ii) and (iii) assuming that the energy is equally partitioned between the waves reflecting from the end and reflecting from the crack. Sketch the wave pattern. Discuss.

<u>Solution</u>

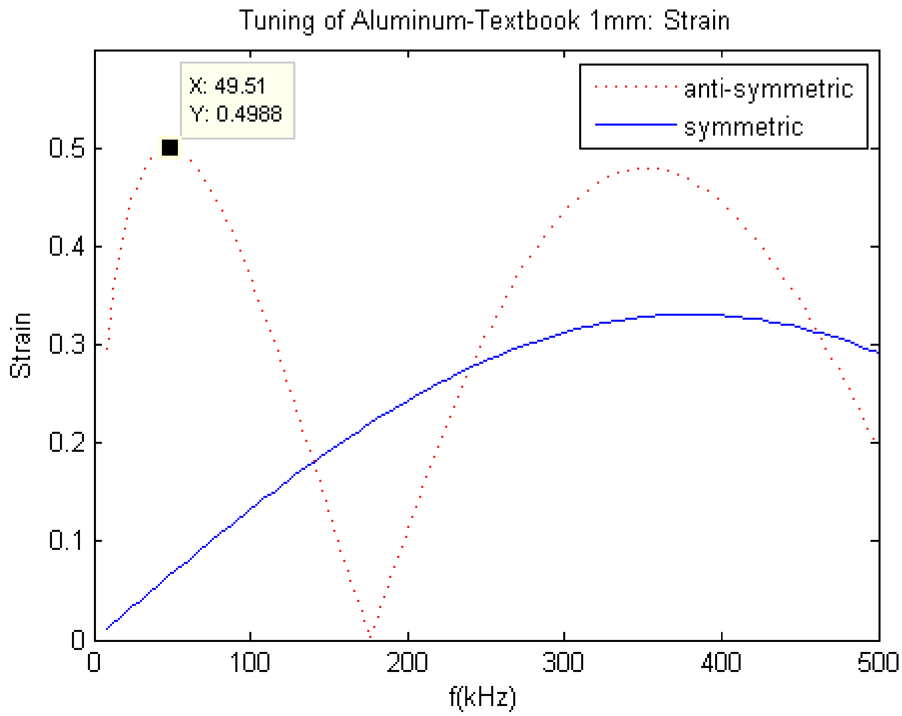
(i) To calculate the tuning frequencies, use the tuning theory developed in textbook Chapter 11. For rectangular PWAS, one refers to Section 10.4.5, specifically Eq. (10.110), i.e.,

$$\varepsilon_x(x,t) = -i\frac{a\tau_a}{\mu}\left(\sin\xi_0^S a\right)\frac{N_S(\xi_0^S)}{D_S'(\xi_0^S)}e^{i(\xi_0^S x-\omega t)} - i\frac{a\tau_a}{\mu}\left(\sin\xi_0^A a\right)\frac{N_A(\xi_0^A)}{D_A'(\xi_0^A)}e^{i(\xi_0^A x-\omega t)} \qquad (10.110)\ (1)$$

One could code Eq. (1) or could use the software programs posted on the LAMSS website http://www.me.sc.edu/research/lamss/html/software.html . In particular, download and activate the following program:

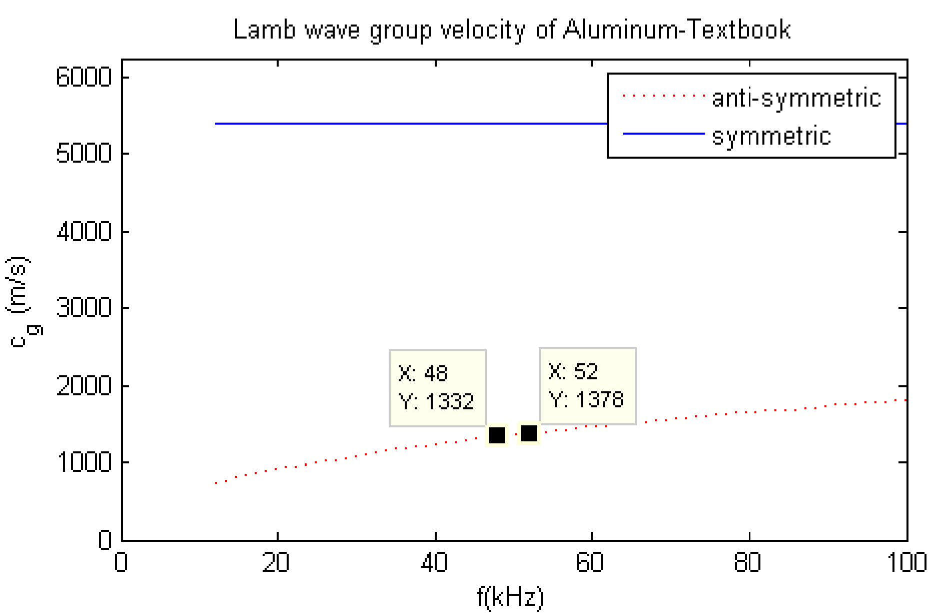


After activating the program, one obtains the chart below.



This chart indicates that the first maximum of the A0 is obtained around ~50 kHz. At this frequency, the S0 response is relatively weak. Hence, this is the first frequency at which the A0 mode is dominant, i.e., f_1^{A0}=50 kHz 50 kHz.

(ii) Assuming that the frequency is tuned at 50 kHz (i.e., when A0 is dominant), we calculate the group velocity for this mode at this frequency. By plotting the group velocity, we obtain the chart below



The chart shows that, in this frequency range, the A0 group velocity (dash line) varies very rapidly with frequency. The exact value is calculate as $c_g^{A0}\Big|_{f=50\text{ kHz}} = 1355 \text{ m/s} = 1355 \text{ mm/µs}$.

Using group velocity, one calculates the round-trip time of flight (TOF) to the end of the strip as

$$t_{end} = \frac{2L}{c_g^{A0}} = \frac{2 \times 1000 \text{ mm}}{1.355 \text{ mm}/\mu\text{s}} = 1476 \ \mu\text{s} \tag{2}$$

To calculate the wave reflection process, define the incident forward wave and the reflected backward wave as the following continuous waves (CWs:

$$u_i(x,t) = Ae^{i(\xi^{A0}x - \omega t)} \ \ldots\ldots\text{incident forward wave} \tag{3}$$

$$u_i(x,t) = Be^{-i\left(\xi^{A0}(x+2x_R)+\omega t\right)} = Be^{-i\xi^{A0}(x+2x_R)}\, e^{-i\omega t} \ \ldots\ldots\text{reflected backward wave} \tag{4}$$

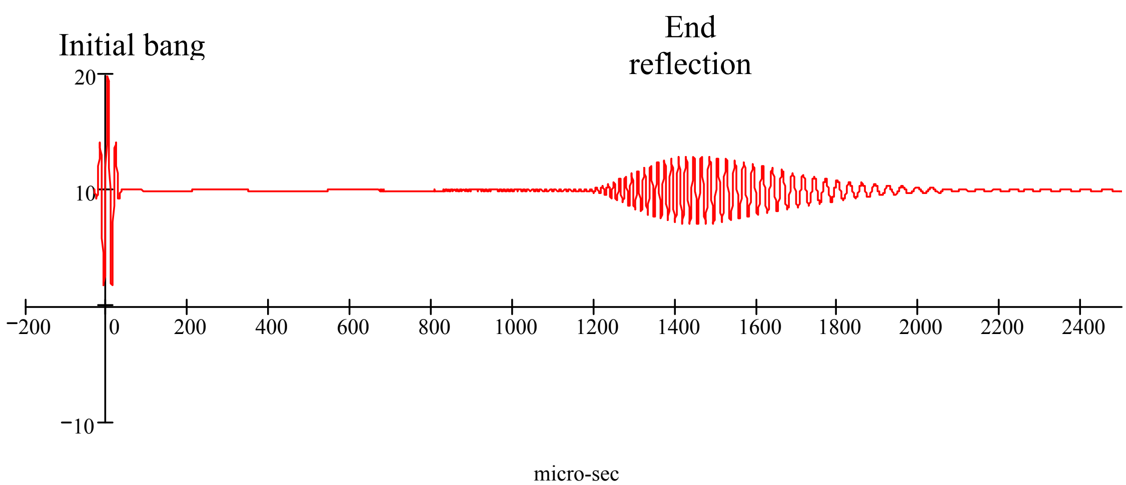
where x_R is the reflector position and ξ^{A0} is the frequency dependent wavenumber defined as

$$\xi^{A0} = \frac{\omega}{c^{A0}} \ \ldots\ldots\text{frequency-dependent wavenumber} \tag{5}$$

where c^{A0} is the phase velocity of the A0 mode. At this relatively low frequencies, the a0 mode is very dispersive and varies like $\sqrt{\omega}$; hence we will take a $\sqrt{\omega}$ approximation of its variation over the frequency range of interest. The total wave is the summation of the incident and reflected waves, i.e.,

$$u(x,t) = u_i(x,t) + u_r(x,t) = \left(Ae^{i\xi^{A0}x} + Be^{-i\xi^{A0}(x+2x_R)}\right)e^{-i\omega t} \ \ldots\ldots\text{complete wave} \tag{6}$$

For a generic situation, we take $A = 1$. Assuming perfect reflection at the $x_R = L$ end of the strip, we take $B = a = 1$. For a given excitation signal $s(t)$, e.g., a tone burst, the solution is obtained by performing the convolution between the excitation signal and the CW definition of Eq. (6). The convolution is performed in the frequency domain as a multiplication. The resulting wave pattern is shown below.

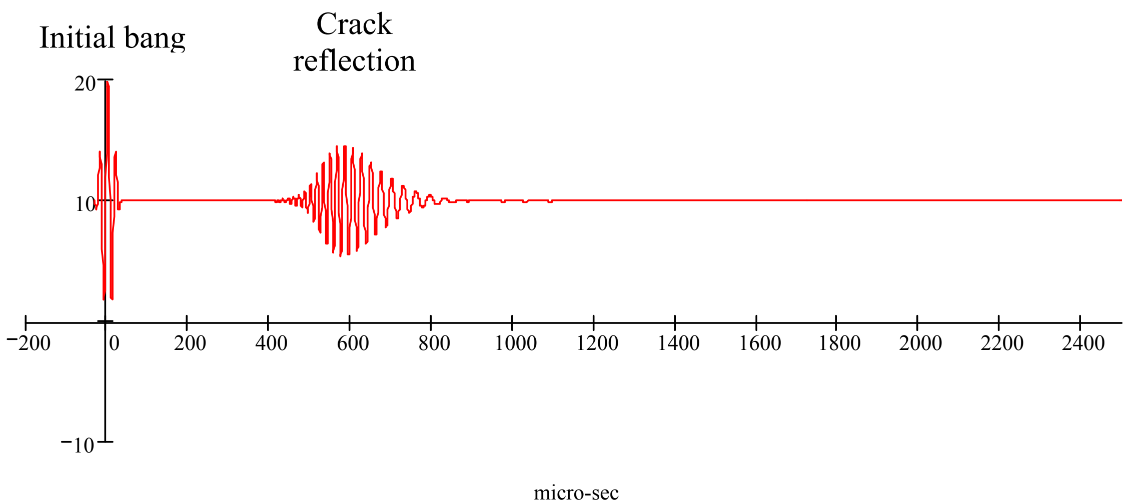


The transmitted incident wave package and received reflected wave packet are clearly defined. The TOF can be easily estimated. The $t_{end} = 1476$ µs value calculated in Eq. (2) matches with CG of the envelope of the plot of the squared dispersed A0 wave signal shown on the graph.

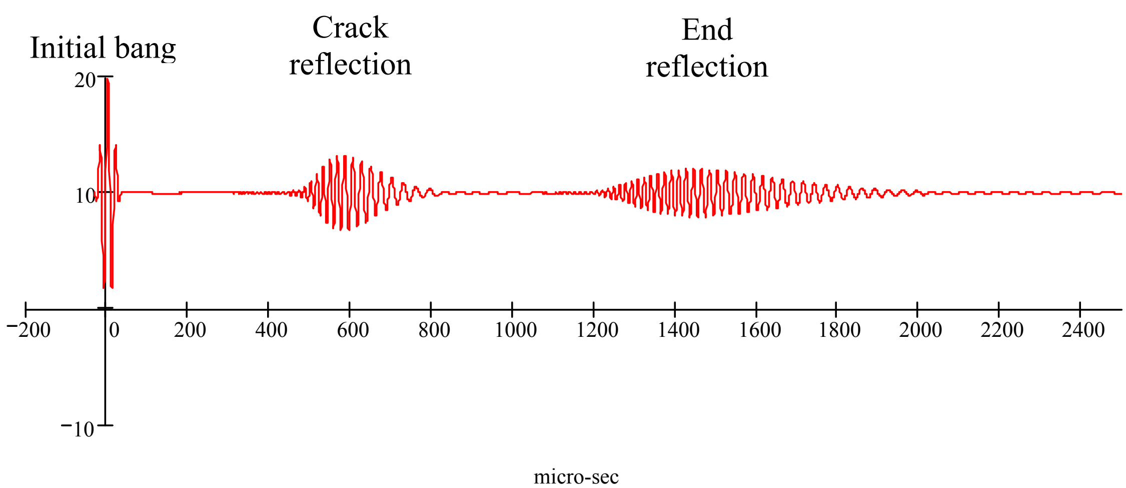
(iii) Assuming a through-the-thickness crack reflector at L_c=400 mm , we calculate the time of flight from the crack reflector as

$$t_{crack} = \frac{2L_c}{c_g^{A0}} = \frac{2 \times 400\ \text{mm}}{1.355\ \text{mm}/\mu\text{s}} = 590\ \mu\text{s} \tag{7}$$

The wave pattern of the transmitted and received wave packets is shown below. The t_{crack} can be easily estimated. The value calculated with Eq. (7) matches well with CG of the envelope of the plot of the squared dispersed A0 wave signal shown on the graph.



(iv) Assuming equal partition of energy between the end reflection and crack reflection we take $B_{end} = B_{crack} = A/\sqrt{2}$. The superposed plot of the two reflections is shown below.



Discussion: One sees the reflections from the crack and the reflection from the end. The t_{end} and t_{crack} can be easily estimated from the CG of the dispersed A0 wave packets shown on the graph. The values calculated with Eqs. (2) and (7) match well the estimations obtained from the chart.

PROBLEM 12.2 SOLUTION

Units

$kHz := 1000 \cdot Hz$ $\quad MHz := 10^{6} \cdot Hz$ $\quad krad := 10^{3} \cdot rad$

$ms := 10^{-3} \cdot s$ $\quad \mu s := 10^{-6} \cdot s$

$pF := 10^{-12} \cdot F$ $\quad nF := 10^{-9} \cdot F$

$\mu m := 10^{-6} \cdot m$ $\quad \mu\varepsilon := 10^{-6}$ $\quad MPa := 10^{6} \cdot Pa$ $\quad GPa := 10^{9} \cdot Pa$ $\quad TPa := 10^{12} \cdot Pa$

$MN := 10^{6} \cdot N$ $\quad GN := 10^{9} \cdot N$

$mS := 10^{-3} \cdot S$ $\quad \mu S := 10^{-6} \cdot S$ $\quad nS := 10^{-9} \cdot S$

$k\Omega := 10^{3} \cdot \Omega$ $\quad M\Omega := 10^{6} \cdot \Omega$ $\quad G\Omega := 10^{9} \cdot \Omega$

SETTINGS AND CONTROLS

$fs := 1.5 \cdot MHz$ $\quad dt := \frac{1}{fs}$ $\quad dt = 0.667\,\mu s$ $\quad fNQ := \frac{fs}{2}$

$tmax := 5000 \cdot \mu s$

$im := \mathrm{ceil}\left(\frac{\log\left(\frac{tmax}{\mu s}\right) - \log\left(\frac{dt}{\mu s}\right)}{\log(2)}\right)$ $\quad im = 13$ $\quad NN := 2^{im}$ $\quad NN = 8192$ $\quad NQ := \frac{NN}{2}$

$Nt := NN$ $\quad Nf := NQ$

$T := NN \cdot dt$ $\quad T = 5461\,\mu s$ $\quad df := \frac{1}{T}$ $\quad df = 183\,Hz$

Display controls: $\quad Tmax := 10^{-6} \cdot \mathrm{floor}\left(\frac{10^{6} \cdot T}{1000}\right) \cdot 1000$ $\quad Tmax = 5000\,\mu s$

$fmax := 200 \cdot kHz$

Frequency range: $\quad nf := 0 .. Nf$ $\quad f_{nf} := nf \cdot df$ $\quad f_{Nf} = 750\,kHz$

$\omega_{nf} := 2 \cdot \pi \cdot f_{nf}$ $\quad \omega_{Nf} = 4712\,\frac{krad}{s}$

Definition of the space domain

$dx := 1.0 \cdot m$ $\quad Nx := 2$ $\quad nx := 0 .. Nx - 1$ $\quad x_{nx} := nx \cdot dx$

$x_1 = 1000\,mm$

Specimen properties

Aluminum strip

$E := 70 \cdot GPa$ $\quad \rho := 2700 \cdot \frac{kg}{m^3}$ $\quad \nu := 0.33$ $\quad d := \frac{1}{2} \cdot mm$ $\quad L := 1000 \cdot mm$ $\quad Lc := 400 \cdot mm$

$L := L$ $\quad Lc := Lc$

Hanning smoothed tone burst $Nc := 3.5$ $fc := 50 \cdot kHz$ $\omega c := 2 \cdot \pi \cdot fc$

Signal amplitude $p0 := 10$

Window size function of central frequency and Ncount $a := \frac{1}{fc} \cdot Nc \cdot \frac{1}{2}$ $a = 35\,\mu s$

Define time range $t0 := -a$ $nt := 0 .. Nt - 1$ $t_{nt} := t0 + nt \cdot dt$ $t_{Nt-1} = 5425.7\,\mu s$

$$H(t) := \begin{cases} p0 & \text{if } -0 < t - t0 < 2 \cdot a \\ 0 & \text{otherwise} \end{cases}$$

square window

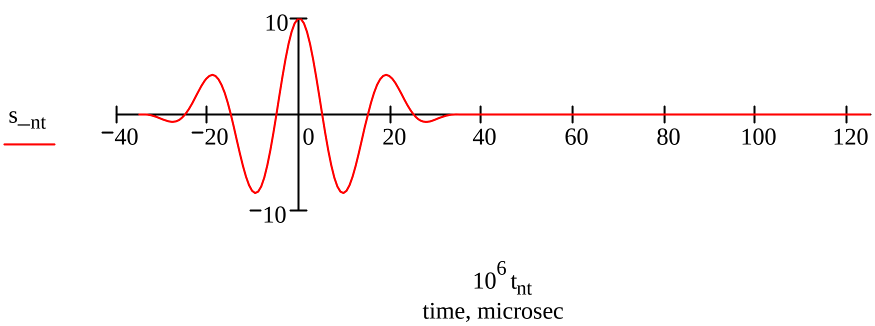
$$H(t) := \begin{cases} p0 \cdot \sin\left(\pi \cdot \frac{t - t0}{2 \cdot a}\right)^2 & \text{if } 0 < t - t0 < 2 \cdot a \\ 0 & \text{otherwise} \end{cases}$$

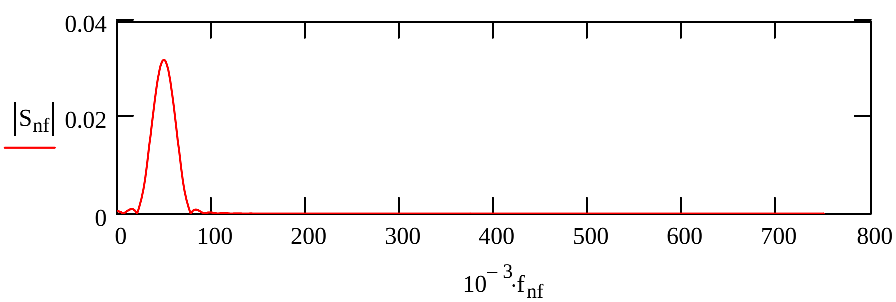
Hanning window and its FFT

$h_{nt} := H(t_{nt})$ $HH := FFT(h)$ $HS_{nf} := HH_{nf}$

$c(t) := -\sin[\omega c \cdot (t - t0)]$ $sc_{nt} := c(t_{nt})$ $SC := FFT(sc)$ carrier signal and its FFT

$ss(t) := -\sin[\omega c \cdot (t - t0)] \cdot H(t)$ $s\_{nt} := ss(t_{nt})$ $S := FFT(s\_)$ tone burst signal and its FFT





A0 and flexural wave speeds in aluminum strip

Group velocity at 50 kHz

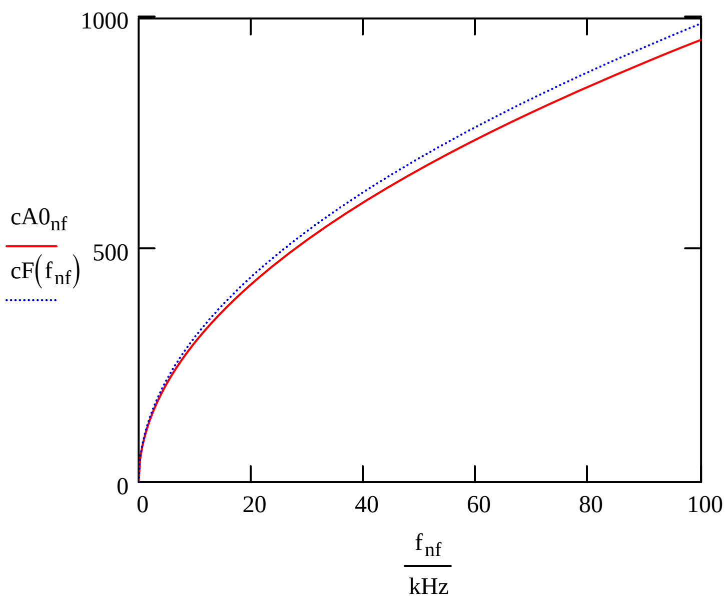
$$cA0g\_50kHz := \frac{1332 + 1378}{2} \cdot \frac{m}{s}$$

$$cA0g\_50kHz = 1355 \frac{m}{s}$$

Phase velocity interpolation using a square root rule

$$cA0\_100kHz := 954 \cdot \frac{m}{s}$$

$$cA0_{nf} := cA0\_100kHz \cdot \sqrt{\frac{f_{nf}}{100 \cdot kHz}}$$



Flexural wave approximation

$$cF(f) := \left[\frac{E \cdot d^2}{3 \cdot \rho \cdot \left(1 - \nu^2\right)}\right]^{\frac{1}{4}} \cdot \sqrt{2 \cdot \pi \cdot f}$$

$$fc := 50 \cdot kHz \qquad cgF(f) := 2 \cdot cF(f)$$

$$cF(fc) = 699 \frac{m}{s} \qquad cgF(fc) = 1399 \frac{m}{s}$$

A0 waves analysis

Basic wave propagation analysis

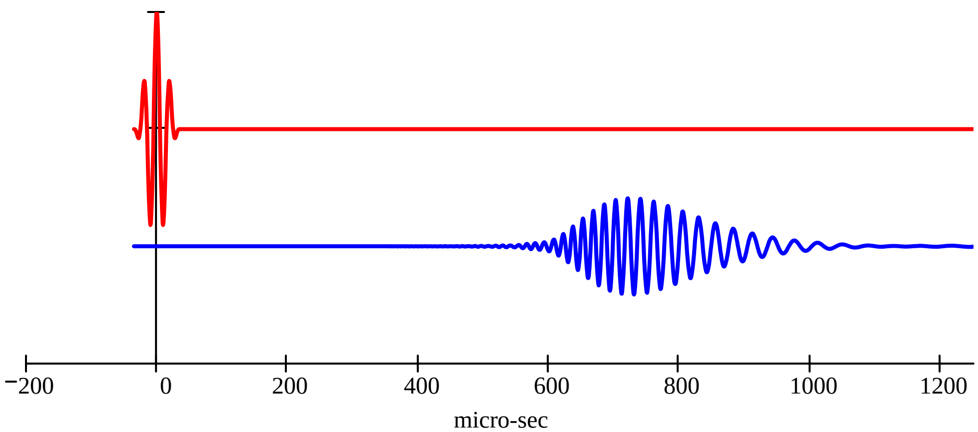
wave numbers $\xi_{nf} := \frac{\omega_{nf}}{cA0_{nf}}$

medium transfer function for the Nx locations x in space domain $G_{nf,nx} := e^{-i \cdot \xi_{nf} \cdot x_{nx}}$

excitation spectrum x medium transfer function at all x locations $U_{nf,nx} := S_{nf} \cdot G_{nf,nx}$

IFFT to return to the time domain $u^{\langle nx \rangle} := \mathrm{IFFT}\left(U^{\langle nx \rangle}\right)$

$\Delta := 10$ plotting offset control



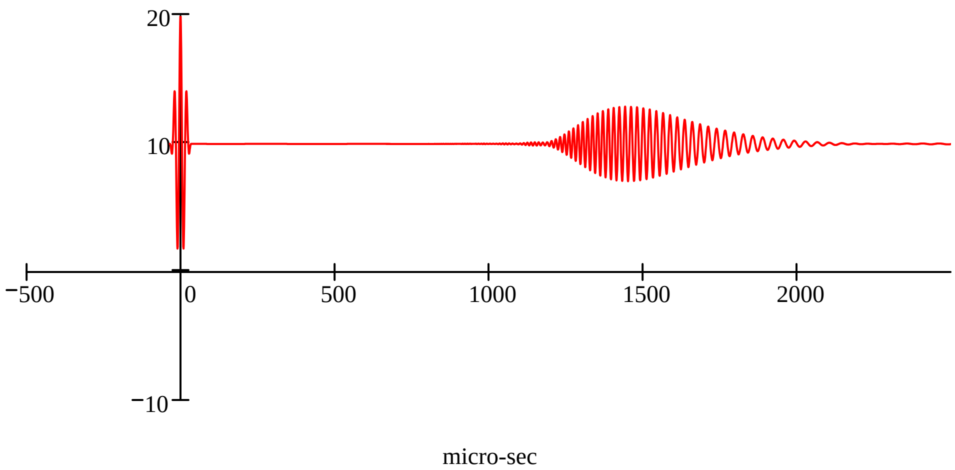
End reflection analysis

End reflection TOF estimate: $cA0g\_50kHz = 1.355\,\frac{mm}{\mu s}$ $\quad t\_end := \frac{2 \cdot L}{cA0g\_50kHz}$ $\quad t\_end = 1476\,\mu s$

End reflection mean the whole signal is returned, hence A=B

$A := 1 \qquad B := A \qquad G_{nf,nx} := A \cdot e^{i \cdot \xi_{nf} \cdot x_{nx}} + B \cdot e^{-i \cdot \xi_{nf} \cdot (x_{nx} + 2 \cdot L)}$

$U_{nf,nx} := S_{nf} \cdot G_{nf,nx} \qquad u^{\langle nx \rangle} := IFFT\left(U^{\langle nx \rangle}\right)$



Crack reflection analysis

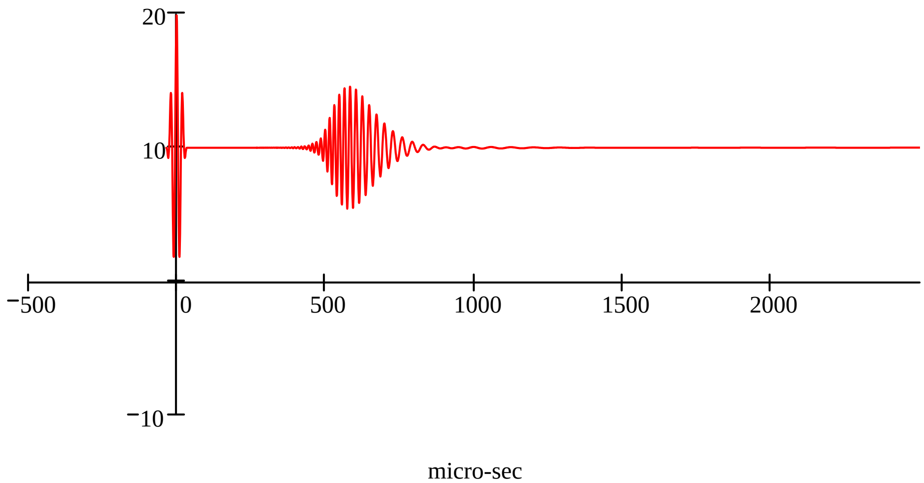
Crack reflection TOF estimate: $t\_crack := \dfrac{2 \cdot Lc}{cA0g\_50kHz}$ $t\_crack = 590\ \mu s$

$Ac := A$

$cA0g\_50kHz = 1.355\ \dfrac{mm}{\mu s}$

$$G_{nf,nx} := A \cdot e^{i \cdot \xi_{nf} \cdot x_{nx}} + Ac \cdot e^{-i \cdot \xi_{nf} \cdot \left(x_{nx} + 2 \cdot Lc\right)}$$

$$U_{nf,nx} := S_{nf} \cdot G_{nf,nx} \qquad u^{\langle nx \rangle} := \mathrm{IFFT}\left(U^{\langle nx \rangle}\right)$$

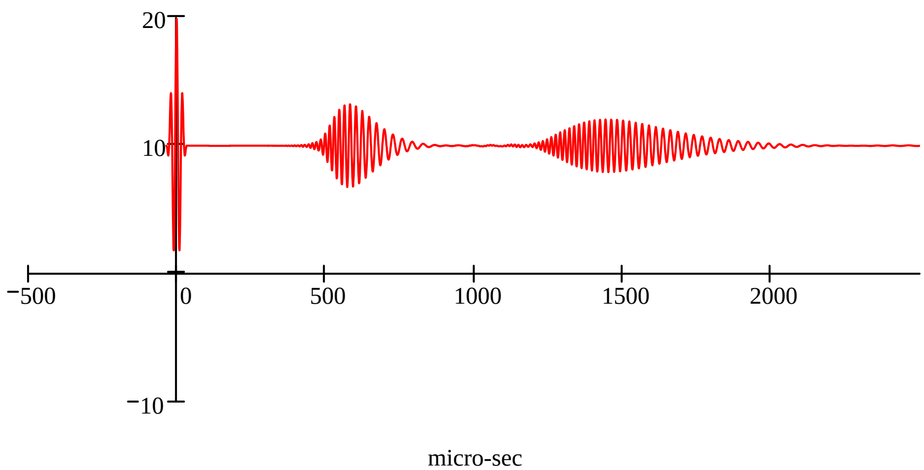


End reflection and crack reflection of axial wave (half-half energy split):

$$B := \frac{A}{\sqrt{2}} \qquad Ac := \frac{A}{\sqrt{2}}$$

$$G_{nf,nx} := A \cdot e^{i \cdot \xi_{nf} \cdot x_{nx}} + B \cdot e^{-i \cdot \xi_{nf} \cdot \left(x_{nx}+2 \cdot L\right)} + Ac \cdot e^{-i \cdot \xi_{nf} \cdot \left(x_{nx}+2 \cdot Lc\right)}$$

$$U_{nf,nx} := S_{nf} \cdot G_{nf,nx} \qquad u^{\langle nx \rangle} := \mathrm{IFFT}\left(U^{\langle nx \rangle}\right)$$



Problem 3: (i) Explain the principles of the time reversal methods. (ii) What difficulties are encountered when the time reversal method is applied to Lamb waves and how they might be alleviated.

<u>Solution</u>

(i) The principles of the time reversal method are covered in some detail in the textbook Section 12.6.1 and will not be repeated here.

(ii) As explained in the textbook Section 12.6.2.1, the difficulties that are encountered when the time reversal method is applied to Lamb waves are related to the multi-modal character of the Lamb waves. Hence, when time reversal is applied, the reconstructed signal presents other wave packet besides the original wave packet. For example, if S0 and A0 modes are simultaneously present in the Lamb wave, then the reconstructed signal will have three wave packets instead of the original one. These difficulties might be alleviated through Lamb wave tuning which may result in a single wave packet after time reversal reconstruction.

Problem 4: Describe how the wave propagation method and PWAS transducers can be used to detect the location of an impact on a plate. (i) Highlight the methodology. (ii) List the main difficulties to be overcome and explain how they might be alleviated.

<u>Solution</u>

(i) The principles of detecting an impact on a plate using the wave propagation method and PWAS transducers are covered in some detail in the textbook Section 12.8.1 and will not be repeated here.

(ii) The main difficulties to be overcome are related to the dispersive nature of the Lamb waves generated by the impact events. Being dispersive, the Lamb waves spread out and make difficult the correct time-of-flight (TOF) identification. These difficulties may be alleviated through the use of a different TOF criterion, e.g., the energy-peak arrival time.