Market Structure: Duopoly and Oligopoly

Overview

An oligopoly is an industry comprising a “few” firms. A duopoly, which is a special case of oligopoly, is an industry consisting of two firms. The distinguishing feature of oligopolistic or duopolistic market structures is the degree to which the output, pricing and other decisions of one firm affect, and are affected by, the similar decision made by other firms in the industry. What is important is the interdependence of the managerial decisions among the various firms in the industry. The analysis of oligopolistic behavior may be modeled as a non-cooperative game in which the actions of one firm to increase market share will, unless countered, result in a reduction of the market share of other firms in the industry. Thus, action will be followed by reaction. This interdependence is the essence of an analysis of duopolistic or oligopolistic market structures.

The characteristics of oligopoly are relatively few sellers, either standardized or differentiated products, price interdependence, and relatively difficult entry into and exit from the industry. A duopoly is an industry comprising two firms producing homogeneous or differentiated products in which entry and exit into and from the industry is difficult.

Two common measures for determining the degree of industrial concentration are the concentration ratio and the Herfindahl-Hirschman index. Concentration ratios measure the percentage of total industry revenue or market share accounted for by the industry’s largest firms. The Herfindahl-Hirschman index is a measure of the size distribution of firms in an industry but assigns greater weight to larger firms.
Mutual interdependence in pricing decisions, which is characteristic of industries with high concentration ratios, makes it difficult to determine the optimal price for a firm's product. Collusion occurs when firms coordinate their output and pricing decisions in order to maximize the output of the entire industry. Collusion may take the form of explicit price fixing agreements, through so-called price leadership, or other practices that ameliorate competitive pressures. Perhaps the best known example of collusive behavior is a cartel, which is a formal agreement among producers to allocate market share and/or industry profits.

Four popular models of firm behavior in oligopolistic industries are the Sweezy ("kinked" demand curve) model, the Cournot model, the Bertrand model, and the Stackelberg model. The Sweezy model, which provides insights into the pricing dynamics of oligopolistic firms, assumes that firms will follow a price decrease by other firms in the industry, but will not follow a price increase. In the Cournot model, each firm decides how much to produce assuming that its rival will not alter its level of production in response. The Bertrand model argues that each firm sets the price of its product to maximize profits and ignores the price charged by its rival. Finally, the Stackelberg model assumes that one firm will behave as in the Cournot model by taking the output of its rival as constant, but that the rival incorporate this behavior into its production decisions.

Game theory is perhaps the most important tool in the economists analytical kit for analyzing the strategic behavior. Strategic behavior is concerned with how individuals make decisions when they recognize that their actions affect, and are affected by, the actions of other individuals or groups. The Prisoner's Dilemma is an example of a two-person, non-cooperative, simultaneous-move, one-shot game in which both players have a strictly-dominant strategy. A player has a strictly-dominant strategy if it results in the largest payoff regardless of the strategy adopted by other players. A Nash equilibrium occurs in a non-cooperative game when each player adopts a strategy that is the best response to what is believed to be the strategy adopted by the other players. When a game is in Nash equilibrium, neither player can improve their payoff by unilaterally changing strategies.

### Multiple Choice Questions

10.1 Products produced by oligopolistic firms are:

A. Homogeneous.

B. Heterogeneous.

C. Differentiated.

D. Heterogeneous or differentiated.
10.2 Oligopoly is the only market structure characterized by:
A. Interdependence in pricing and output decisions.
B. Differentiated products.
C. Barriers to entry.
D. Profit-maximizing behavior.

10.3 Oligopoly is characterized by all of the following except:
A. A few large firms.
B. Differentiated product.
C. Difficult entry into the industry.
D. Price competition.

10.4 A duopoly is a market structure in which:
A. There are two very large firms and several smaller firms.
B. Each firm in the industry produces two complementary goods.
C. There are only two firms in the industry.
D. Each firm in the industry produces two very similar goods.

10.5 A measure of the degree of industrial concentration is the:
A. Concentration ratio.
B. Lerner index.
C. Herfindahl-Hirschman index.
D. A and C are correct.

10.6 The Herfindahl-Hirschman index:
A. Is a measure of market share that is accounted for by the largest firms in an industry.
B. Is a measure of industrial concentration that gives a larger than proportional weight to larger firms.
C. Is a measure of the monopoly power of the largest firms in an industry.
D. Was initially by the U.S. Justice Department to measure industrial concentration under the terms of the Clayton Act.

10.7 The Herfindahl-Hirschman Index is calculated as the:
A. Sum of the squared market shares of each firm in the industry.
B. Sum of the squared concentration ratios of each firm in the industry.
C. Sum of the squared market shares of the top four firms in the industry.
D. Sum of the squared concentration ratios of the top four firms in the industry.
10.8 Which of the following was an attempt by the U.S. Congress to encourage competition and limit market power?
A. The Interstate Commerce Commission.
B. The Sherman Act.
C. The Clayton Act.
D. The Celler-Kefauver Act.
E. All of the above.

10.9 The Sweezy model is an attempt explain:
A. Stable prices and volatile marginal cost.
B. Volatile prices and stable marginal cost.
C. Stable prices and stable marginal cost.
D. Volatile prices and volatile marginal cost.

10.10 The kinked demand curve model is an attempt to explain:
A. Interdependent pricing and output behavior.
B. The role of advertising in oligopolistic market structures.
C. Non-price competition.
D. Why the output of oligopolistic firms is sensitive to cost changes.

10.11 The kinked demand curve model suggests that if a firm in an oligopolistic industry raises its price, then:
A. Other firms in the industry will raise their prices as well.
B. Other firms in the industry will not raise their prices.
C. Other firms in the industry will lower their prices.
D. That firm will lose market share.
E. Both B and D are correct.

10.12 The kinked demand curve model suggests that if a firm in an oligopolistic industry lowers its price, then:
A. Other firms in the industry will raise their prices.
B. Other firms in the industry will not raise their prices.
C. Other firms in the industry will lower their prices as well.
D. That firm will gain market share.
E. Both C and D are correct.

10.13 The kinked demand curve model suggests that:
A. The firm’s marginal revenue curve is “discontinuous” at the kink.
B. There is a tendency towards price rigidity.
C. A firm’s competitors will match a price decrease but not match a price increase.
D. All of the above are correct.
E. None of the above are correct.
10.14 An profit-maximizing, oligopolistic firm produces at an output level where:
A. \( P = ATC \).
B. \( MR = MC \).
C. \( MR = ATC \).
D. \( AVC > MR \).
E. \( P = MR \).

10.15 Consider Figure 1. Section \( D_1 \) of the demand curve suggests that competing firms in an oligopolistic industry will:
A. Raise price in response to a price increase.
B. Lower price in response to a price decrease.
C. Will not raise price in response to a price increase.
D. Will not lower price in response to a price decrease.

10.16 Consider Figure 1. Section \( D_2 \) of the demand curve assumes that competing firms will:
A. Raise price in response to a price increase.
B. Lower price in response to a price decrease.
C. Will not raise price in response to a price increase.
D. Will not lower price in response to a price decrease.

10.17 Consider Figure 1. If marginal cost is equal to \( MC_1 \), the profit-maximizing oligopolist will produce an output level of:
A. G.
B. H.
C. I.
D. J.
E. K.

10.18 Consider Figure 1. If marginal cost is equal to \( MC_1 \), the profit-maximizing oligopolist will charge a price of:
A. A.
B. B.
C. C.
D. D.
E. F.
10.19 Consider Figure 1. If marginal cost equal to $MC_2$, the profit-maximizing oligopolist will produce at an output level:
A. G.  
B. H.  
C. I.  
D. J.  
E. K.

10.20 Consider Figure 1. If marginal cost equal to $MC_2$, the profit-maximizing oligopolist will charge a price of:
A. A.  
B. B.  
C. C.  
D. D.  
E. E.

10.21 Consider Figure 1. If marginal cost equal to $MC_4$, the profit-maximizing oligopolist will produce at an output level:
A. G.  
B. H.  
C. I.  
D. J.  
E. K.

10.22 Consider Figure 1. If the marginal cost curve shifts from $MC_2$ to $MC_4$, there will be:
A. A decrease in price and an increase in output.  
B. A decrease in price and no change in output.  
C. No change in price and an increase in output.  
D. No change in price or output.

10.23 The Cournot duopoly model assumes that:
A. Each firm decides what price to charge and that their rivals will not respond.  
B. Each firm decides what price to charge and that their rivals will respond.  
C. Each firm decides how much to produce and that their rivals will not respond.  
D. Each firm decides how much to produce and that their rivals will respond.
10.24 The Bertrand duopoly model assumes that:
A. Each firm decides what price to charge and that their rivals will not respond.
B. Each firm decides what price to charge and that their rivals will respond.
C. Each firm decides how much to produce and that their rivals will not respond.
D. Each firm decides how much to produce and that their rivals will respond.

10.25 Other things being equal, the Cournot duopoly model suggests that profit-maximizing firms in an oligopolistic industry:
I. Charge a higher price than monopolies.
II. Charge a higher price than perfectly-competitive firms.
III. Charge a lower price than monopolies.
IV. Charge a lower price than perfectly-competitive firms.
Which of the following is correct?
A. I only.
B. II only.
C. I and II only.
D. II and III only.
E. I and IV only.

10.26 According to the Cournot duopoly model, if both firms in the industry face identical demands for their product then:
A. Both firms will end up with an equal share of the market.
B. The first firm to enter the industry will end up producing $2/3$ of industry output while the second firm will end up producing $1/3$ of industry output.
C. The first firm to enter the industry will end up producing $1/3$ of industry output while the second firm will end up producing $2/3$ of industry output.
D. The first firm to enter the industry will end up producing $3/8$ of industry output while the second firm will end up producing $1/4$ of industry output.

10.27 According to the Bertrand duopoly model, if both firms in the industry face identical demands and marginal production costs, then:
A. The first firm to enter the industry will charge a higher price than the second firm.
B. The first firm to enter the industry will charge a lower price than the second firm.
C. Both firm’s will end up charging the same price.
D. The first firm to enter the industry will earn higher profits than the second firm.
E. The first firm to enter the industry will earn lower profits than the second firm.

10.28 According to the Bertrand duopoly model, if both firms in the industry face identical demands but different marginal production costs, then:
A. The first firm to enter the industry will charge a higher price than the second firm.
B. The first firm to enter the industry will charge a lower price than the second firm.
C. Both firm’s will end up charging the same price.
D. The firm with the lowest marginal cost will become a monopoly.

10.29 According to the Stackelberg duopoly model, if both firms in the industry face identical demands and identical total production costs, then:
A. The Stackelberg leader will produce more than the Stackelberg follower.
B. The Stackelberg follower will produce more than the Stackelberg leader.
C. The Stackelberg leader and Stackelberg follower will produce identical output.
D. The outcome is identical with that predicted by the Cournot model.

10.30 If both firms in a duopolistic industry face identical demands and identical total production costs, then:
A. Total industry output predicted by the Stackelberg duopoly model will less than total industry output predicted by the Cournot duopoly model.
B. Total industry output predicted by the Stackelberg duopoly model will greater than total industry output predicted by the Cournot duopoly model.
C. Total industry output predicted by the Stackelberg duopoly model will be the same as total industry output predicted by the Cournot duopoly model.
D. None of the above. It is more appropriate to compare the Stackelberg duopoly model with the Bertrand duopoly model.
10.31 Cartels are most likely to arise in which of the following market structures?
   A. Perfect competition.
   B. Monopolistic Competition.
   C. Oligopoly.
   D. Monopoly.

10.32 The main reason for forming a cartel is:
   A. To maximize total revenues of the collective membership of the cartel.
   B. To maximize the total market share of the collective membership of the cartel.
   C. To reduce the total output of the collective membership of the cartel.
   D. To maximize the total profit of the collective membership of the cartel.

10.33 Cartels usually break down because:
   A. Cartel members are less efficient than non-cartel members.
   B. Easy entry conditions tend to undermine the market power of the cartel.
   C. There is a tendency for individual members to violate the production terms of the cartel agreement.
   D. The products of individual members tend to become differentiated over time.

10.34 Cartels are undesirable:
   A. For the same reasons that monopolies are undesirable.
   B. Because they threaten the national security of the United States.
   C. Because they are illegal in the United States.
   D. Each member is forced to produce at the same per unit cost.

10.35 Game theory is used to explain:
   A. The strategic behavior of firms in oligopolistic industries.
   B. Interdependency of pricing and output decisions of firms in oligopolistic industries.
   C. Both A and B are correct.
   D. Neither A nor B are correct.
10.36 In a two-person, non-cooperative game, a Nash equilibrium:
A. May also be a strictly-dominant strategy equilibrium.
B. Occurs when each player adopts a strategy that is believe to be
the best response to the strategy adopted by the other players.
C. Implies that the player’s cannot improve their payoffs by
switching strategies.
D. All of the above.
E. None of the above.

10.37 Consider Figure 2, which illustrates the payoff matrix summarizing
the expected profits of two firms in an oligopolistic industry from
different combinations of pricing strategies. If this is a
simultaneous-move, non-cooperative game then:
A. Only Firm 1 has a dominant strategy.
B. Only Firm 2 has a dominant strategy.
C. Neither firm has a dominant strategy.
D. Both firms have a dominant strategy.

Firm 2

<table>
<thead>
<tr>
<th>Firm 1</th>
<th>High Price</th>
<th>Low Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Price</td>
<td>($250, $250)</td>
<td>($500, $150)</td>
</tr>
<tr>
<td>Low Price</td>
<td>($150, $500)</td>
<td>($400, $400)</td>
</tr>
</tbody>
</table>

Payoffs: (Firm 1, Firm 2)

10.38 Consider Figure 2, which illustrates the payoff matrix summarizing
the expected profits of two firms in an oligopolistic industry from
different combinations of pricing strategies. If this is a
simultaneous-move, *non-cooperative* game then the solution to this
game is:
A. {High Price, Low Price}
B. {Low Price, High Price}
C. {High Price, High Price}
D. {Low Price, Low Price}

10.39 Consider Figure 2, which illustrates the payoff matrix summarizing
the expected profits of two firms in an oligopolistic industry from
different combinations of pricing strategies. If this is a
simultaneous-move, *cooperative* game then the solution to this
game is:
A. {High Price, Low Price}
B. {Low Price, High Price}
C. {High Price, High Price}
D. {Low Price, Low Price}

10.40 Consider Figure 2, which illustrates the payoff matrix summarizing the expected profits of two firms in an oligopolistic industry from different combinations of pricing strategies. If this is a simultaneous-move, cooperative game then the solution to this game is:
A. A Nash equilibrium.
B. Not a Nash equilibrium.
C. A Prisoner’s dilemma.
D. A dominant strategy equilibrium.

SHORTER PROBLEMS

10.1 Suppose that an oligopolist is charging a price of $100 and selling 20 units of output per day. If the oligopolist were to increase price above $100, then quantity demanded would decline by 4 units for every $1 increase in price. On the other hand, if the oligopolist were to lower the price below $100, then quantity demanded would increase by only 2 units for every $1 decrease in price. If the marginal cost of producing the output is constant, within what range may marginal cost vary without the profit-maximizing oligopolist changing either the price of the product or the level of output?

10.2 Suppose that a duopolistic firm faces the following kinked demand for its product:

\[ Q_1 = 50 - 0.5P \]
\[ Q_2 = 90 - P \]

where \( Q \) represents units of output and \( P \) the price of the product.

A. What are the price and quantity demanded at the “kink”?
B. Between what values may marginal cost vary without causing a change in the equilibrium price and quantity?
D. Diagram your answer.
10.3 Suppose that an industry consists of two firms that produces a homogeneous product. Suppose that each firm decides how much to produce and assumes that its rival will not alter its level of production in response. The industry demand equation is:

\[ P = 145 - 5(Q_1 + Q_2) \]

where \( Q_1 \) and \( Q_2 \) represents the output of Firm 1 and Firm 2, respectively. The total cost equations of the two firms are:

\[ TC_1 = 3Q_1 \]
\[ TC_2 = 5Q_2 \]

A. Calculate each firm’s reaction function.

B. Calculate the equilibrium price, profit maximizing output levels, and profits for each firm. Assume that each duopolist maximizes its profit and that each firm’s output decision is invariant with respect to the output decision of each rival.

10.4 Suppose that an industry consisting of two firms produces a homogeneous product. The demand equation for the output of the industry is:

\[ P = 145 - 5(Q_1 + Q_2) \]

where \( Q_1 \) and \( Q_2 \) represents the output of Firm 1 and Firm 2, respectively. The total cost equations of the two firms are:

\[ TC_1 = 3Q_1 \]
\[ TC_2 = 5Q_2 \]

Suppose that Firm 2 believes that Firm 1 will take the output of Firm 2 as constant. By contrast, Firm 2 will attempt to exploit the behavior of Firm 1 by incorporating Firm 1’s reaction of the follower into its own production decisions. Calculate the equilibrium price, output levels, and profits of each firm.

10.5 Suppose that the demand equations for the products of two profit-maximizing firms in a duopolistic industry are:

\[ Q_1 = 250 - 5P_1 + 2P_2 \]
\[ Q_2 = 250 - 5P_2 + 2P_1 \]

Suppose, further, that the firms’ total cost functions are

\[ TC_1 = 25 + 5Q_1 \]
\[ TC_2 = 25 + 5Q_2 \]
where \( P_1 \) and \( P_2 \) represent the prices charged by each firm producing \( Q_1 \) and \( Q_2 \) units of output.

A. What is the equilibrium price charged by both firms?
B. What is the equilibrium quantity of each firm?
C. What are the profits of each firm?

10.6 Suppose that the demand equations for the products of two profit-maximizing firms in a duopolistic industry are:

\[
Q_1 = 250 - 5P_1 + 2P_2 \\
Q_2 = 250 - 5P_2 + 2P_1 
\]

Suppose, further, that the firms’ total cost functions are

\[
TC_1 = 25 + 5Q_1 \\
TC_2 = 10 + 2Q_2 
\]

where \( P_1 \) and \( P_2 \) are the prices charged by each firm producing \( Q_1 \) and \( Q_2 \) units of output.

A. What is the equilibrium price charged by both firms?
B. What is the equilibrium quantity of each firm?
C. What are the profits of each firm?

LONGER PROBLEMS

10.1 Suppose that a firm in an oligopolistic industry faces the following kinked demand for its product:

\[
Q_1 = 100 - P \\
Q_2 = 160 - 2P 
\]

where \( Q \) represents units of output and \( P \) the price of the product.

Suppose further that the firm’s total cost equation is given by the equation:

\[
TC = 100 + 30Q 
\]

A. What are the profit-maximizing price and output?
B. What is the firm’s marginal revenue equation?
C. Between what values may marginal cost vary without causing a change in the equilibrium price and quantity?
D. Calculate the maximum profit for this firm.
E. Suppose that the firm’s total cost of production were to increase to:
TC = 120 + 40Q

Will this alter the firm’s profit-maximizing price and output level? Calculate the maximum profit for this firm.

F. Suppose that the firm’s total cost of production were to increase to:

\[ TC = 140 + 50Q \]

Will this alter the firm’s profit-maximizing price and output level? Calculate the maximum profit for this firm.

10.2 Consider the situation in which two owners petroleum-refining companies, Oxxon and Nonox, are considering opening gasoline stations at a busy intersection in the middle of town or near the exit ramp of an interstate highway. The payoff matrix of monthly profits from any combination of strategies for this simultaneous-move, non-cooperative, one-shot game is illustrated in the following figure. The first entry in each cell of the payoff matrix refers to the monthly profits to Oxxon and the second entry refers to the monthly profits to Nonox.

<table>
<thead>
<tr>
<th>Nonox</th>
<th>Intersection</th>
<th>Interstate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oxxon</td>
<td>($150,000, $150,000)</td>
<td>($50,000, $200,000)</td>
</tr>
<tr>
<td>Interstate</td>
<td>($200,000, $50,000)</td>
<td>($75,000, $75,000)</td>
</tr>
</tbody>
</table>

Payoffs: (Oxxon, Nonox)

A. Does either company have a dominant strategy? Explain.
B. Is the solution to this game a Nash equilibrium?

10.3 Two competing firm’s are trying to decide whether on an advertising strategy. Firm A is considering whether to advertise on television or in magazines. Firm B, on the other hand, is considering whether to advertise on radio or in newspapers. The payoff matrix of annual profits from any combination of strategies for this simultaneous-move, non-cooperative, one-shot game is illustrated in the following figure. The first entry in each cell of the payoff matrix refers to the annual profits in millions of dollars to Firm A and the second entry refers to annual profits in millions of dollars to Firm B.
**Firm B**

<table>
<thead>
<tr>
<th>Firm A</th>
<th>Television</th>
<th>Newspapers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radio</td>
<td>($9, $7)</td>
<td>($14, $6)</td>
</tr>
<tr>
<td>Magazines</td>
<td>($7, $11)</td>
<td>($8, $7)</td>
</tr>
</tbody>
</table>

Payoffs: (Firm A, Firm B)

A. Does either firm have a dominant strategy? Explain.
B. Is the solution to this game a Nash equilibrium?

**ANSWERS TO MULTIPLE CHOICE QUESTIONS**

10.1 D. 10.21 B.
10.2 A. 10.22 D.
10.3 D. 10.23 C.
10.4 C. 10.24 A.
10.5 D. 10.25 D.
10.6 B. 10.26 A.
10.7 A. 10.27 C.
10.8 E. 10.28 D.
10.9 C. 10.29 A.
10.10 A. 10.30 B.
10.11 E. 10.31 C.
10.12 C. 10.32 D.
10.13 D. 10.33 C.
10.14 B. 10.34 A.
10.15 B. 10.35 C.
10.16 A. 10.36 D.
10.17 E. 10.37 D.
10.18 A. 10.38 C.
10.19 B. 10.39 D.
10.20 B. 10.40 B.
10.1  \[ P = 105 - 0.25Q \text{ for } P > 100 \]
\[ P = 110 - 0.5Q \text{ for } P < 500 \]
\[ TR_1 = 105Q - 0.25Q_1^2 \]
\[ MR_1 = 105 - 0.5Q_1 \]
\[ TR_2 = 110Q - 0.5Q_1^2 \]
\[ MR_2 = 110 - Q_1 \]

The firm maximizes profit by producing at an output level where \( MR = MC \). At \( Q = 20 \),
\[ MC_1 = 105 - 0.5(20) = 95 \]
\[ MR_2 = 110 - 20 = 90 \]
Thus, marginal cost may vary between \$90\) and \$95\) without the profit-maximizing oligopolist changing either the price of the product or the level of output. This situation is illustrated in the following figure.

10.2  A. The price and output level may be determined by equating the demand equations for this product of this firm at the kink, i.e.,
\[ 50 - 0.5P = 80 - P \]
\[ P^* = 80 \]
\[ Q^* = 50 - 0.5(80) = 90 - 80 = 10 \text{ units} \]

B. \[ P = 100 - 2Q \]
\[ TR_1 = 100Q - 2Q^2 \]
\[ MR_1 = 100 - 4Q \text{ for } Q \leq 10 \]
\[ P = 90 - Q \]
\[ TR_2 = 90Q - Q^2 \]
\[ MR_2 = 90 - 2Q \text{ for } Q \geq 10 \]
C. MR₁ = 100 - 4(10) = $60
MR₂ = 90 - 2(10) = $70
At an output level of 10 units, the marginal cost of production may vary between $60 and $70 without causing a change in the equilibrium price and quantity.

D. $80
$70
$60

10.3 A. TR₁ = 145Q₁ - 5Q₁² - 5Q₁Q₂
TR₂ = 145Q₂ - 5Q₁Q₂ - 5Q₂²
π₁ = 145Q₁ - 5Q₁² - 5Q₁Q₂ - 3Q₁ = 142Q₁ - 5Q₁² - 5Q₁Q₂
π₂ = 145Q₂ - 5Q₁Q₂ - 5Q₂² - 5Q₂ = 140Q₂ - 5Q₁Q₂ - 5Q₂²
∂π₁/∂Q₁ = 142 - 10Q₁ - 5Q₂ = 0
∂π₂/∂Q₂ = 140 - 5Q₁ - 10Q₂ = 0
Q₁ = 14.2 - 0.5Q₂
Q₂ = 14 - 0.5Q₁
B. Q₁* = 9.6 units
Q₂* = 14 - 0.5(9.6) = 9.2 units
Substituting these results into the profit functions yields
π₁* = 142(9.6) - 5(9.6)² - 5(9.6)(9.2) = $460.80
π₂* = 140(9.2) - 5(9.6)(9.2) - 5(9.2)² = $423.20
P* = 145 - 5(9.6 + 9.2) = $51

10.4 TR₁ = 145Q₁ - 5Q₁² - 5Q₁Q₂
TR₂ = 145Q₂ - 5Q₁Q₂ - 5Q₂²
π₁ = 145Q₁ - 5Q₁² - 5Q₁Q₂ - 3Q₁ = 142Q₁ - 5Q₁² - 5Q₁Q₂
∂π₁/∂Q₁ = 142 - 10Q₁ - 5Q₂ = 0
Thus, the reaction function of Firm 1, known as a Stackelberg follower, is:
Q₁ = 14.2 - 0.5Q₂
Firm 2, known as the Stackelberg leader, will incorporate the reaction function of Firm 1 into its own profit equation, i.e.,
π₂ = 140Q₂ - 5Q₁Q₂ - 5Q₂² = 140Q₂ - 5(14.2 - 0.5Q₂)Q₂ - 5Q₂²
= 69Q₂ - 5Q₂²
\[
d\pi_2/dQ_2 = 69 - 10Q_2 = 0, \text{ i.e., the first-order condition for } \pi_2 \\
d^2\pi_2/dQ_2^2 = -10 < 0, \text{ i.e., the second-order condition for } \pi_2 \\
\text{maximization is satisfied.}
\]

Solving the first-order condition for \( Q_2 \) we obtain
\[
Q_2^* = 6.9 \text{ units}
\]
\[
Q_1^* = 14.2 - 0.5(6.9) = 10.75 \text{ units}
\]
\[
P_1^* = 145 - 5(10.57 + 6.9) = $57.65
\]
\[
P_2^* = 140(6.9) - 5(10.75)(6.9) + 2P_1P_2 = $357.08
\]
\[
10.5 \text{ A. TR}_1 = P_1Q_1 = P_1(250 - 5P_1 + 2P_2) = 250P_1 - 5P_1^2 + 2P_1P_2
\]
\[
\text{TC}_1 = 25 + 5Q_1 = 25 + 5(250 - 5P_1 + 2P_2) = 1,275 - 25P_1 + 10P_2
\]
\[
\pi_1 = \text{TR}_1 - \text{TC}_1
\]
\[
\pi_2 = \text{TR}_2 - \text{TC}_2
\]
\[
\partial\pi_1/\partial P_1 = 275 - 10P_1 + 2P_2 = 0
\]
\[
\partial\pi_2/\partial P_2 = 275 - 10P_2 + 2P_1 = 0
\]

The firm’s reaction functions are:
\[
P_1 = 27.5 + 0.2P_2
\]
\[
P_2 = 27.5 + 0.2P_1
\]

Solving the reaction functions simultaneously we obtain
\[
P_1^* = $55
\]
\[
P_2^* = $55
\]
\[
10.6 \text{ A. TR}_1 = P_1Q_1 = P_1(250 - 5P_1 + 2P_2) = 250P_1 - 5P_1^2 + 2P_1P_2
\]
\[
\text{TC}_1 = 25 + 5Q_1 = 25 + 5(250 - 5P_1 + 2P_2) = 1,275 - 25P_1 + 10P_2
\]
\[
\pi_1 = \text{TR}_1 - \text{TC}_1
\]
\[
\pi_2 = \text{TR}_2 - \text{TC}_2
\]
\[
\partial\pi_1/\partial P_1 = 275 - 10P_1 + 2P_2 = 0
\]
\[
\partial\pi_2/\partial P_2 = 275 - 10P_2 + 2P_1 = 0
\]
\[ \frac{\partial \pi_1}{\partial P_1} = 275 - 10P_1 + 2P_2 = 0 \]
\[ \frac{\partial \pi_2}{\partial P_2} = 260 - 10P_2 + 4P_1 = 0 \]

The firm’s reaction functions are:

\[ P_1 = 27.5 + 0.2P_2 \]
\[ P_2 = 26 + 0.2P_1 \]

Solving the reaction functions simultaneously we obtain

\[ P_1^* = 54.50 \]
\[ P_2^* = 52.50 \]

B. \[ Q_1^* = 250 - 5(54.5) + 2(52.5) = 82.5 \text{ units} \]
\[ Q_2^* = 250 - 5(53.5) + 2(54.5) = 91.5 \text{ units} \]

C. \[ \pi_1^* = -1.275 + 275(54.5) - 5(54.5)^2 - 10(52.5) + 2(54.5)(52.5) = $4,083.75 \]
\[ \pi_2^* = -510 + 260(52.5) - 5(52.5)^2 - 4(54.5) + 2(54.5)(52.5) = $4,863.25 \]

**SOLUTIONS TO LONGER PROBLEMS**

10.1 A. The price and output level may be determined by equating the demand equations for this product of this firm at the kink, i.e.,

\[ 160 - 2P = 100 - P \]
\[ P^* = 60 \]
\[ Q^* = 160 - 2(60) = 100 - (60) = 40 \text{ units} \]

B. \[ P = 100 - Q \]
\[ \text{TR}_1 = 100Q - Q^2 \]
\[ P = 80 - 0.5Q \]
\[ \text{TR}_2 = 80Q - 0.5Q^2 \]

C. \[ \text{MR}_1 = 100 - 2(40) = 20 \]
\[ \text{MR}_2 = 80 - (40) = 40 \]

At an output level of 40 units, the marginal cost of production may vary between $20 and $40 without causing a change in the equilibrium price and quantity.

D. The firm’s marginal cost is:

\[ \text{MC} = \frac{d\text{TC}}{dQ} = 30 \]

which lies in the interval $20 and $40. Thus, the profit-maximizing price and output level remains $60 and 40, respectively. Thus,

\[ \pi^* = \text{TR} - \text{TC} = P^*Q^* - (100 + 30Q^*) = 60(40) - 100 - 30(40) = $1,100 \]
E. The firm’s new marginal cost of production is:
\[ MC = dTC/dQ = \$40 \]
Since marginal cost still lies in the interval $20 and $40, then there is no change in the firm’s profit-maximizing price and quantity.
\[ \pi^* = TR - TC = P^*Q^* - (120 + 40Q^*) \]
\[ = 60(40) - 120 - 40(40) = \$700 \]

F. The firm’s new marginal cost of production is:
\[ MC = dTC/dQ = \$50 \]
Since the firm’s marginal cost lies outside the interval $20 and $60, then the new profit-maximizing output occurs where:
\[ MC = MR_2 \]
\[ 50 = 80 - Q \]
\[ Q^* = 30 \]
The profit-maximizing price is:
\[ P^* = 80 - 0.5(30) = \$65 \]
\[ \pi^* = TR - TC = P^*Q^* - (120 + 40Q^*) \]
\[ = 65(30) - 140 - 50(30) = \$310 \]

10.2 A. Both companies have a dominant strategy to locate near the interstate highway. To see this, consider the problem from the perspective of Nonox. If Oxxon locates at the intersection, then it will be in Nonox’s best interest to relocate near the interstate highway since it will earn monthly profits of $200,000 compared with $150,000 by locating at the busy intersection. If Oxxon decides to locate near the interstate highway, then it will again be in Nonox’s best interest to locate near the interstate highway since it will earn monthly profits of $75,000 compared with $50,000 by locating at the busy intersection. Thus, regardless of what Oxxon does, Nonox’s will locate near the interstate highway. Locating near the interstate highway is Nonox’s dominant strategy. Since the payoff matrix is symmetrical the same conclusion must also be true for Oxxon. Thus, this game has the dominant-strategy equilibrium \([\text{Interstate, Interstate}]\).

B. Note that the optimal solution for both companies to remain to locate at the busy intersection since this will result in greater monthly profits. But, this outcome would require that both companies cooperate. If such collusive behavior is ruled out, then the dominant-strategy equilibrium \([\text{Interstate, Interstate}]\) is also a Nash equilibrium since neither company can unilaterally improve their monthly profits by switching strategies.
10.3  A. Both companies have a dominant strategy to locate near the interstate highway. To see this, consider the problem from Firm B’s perspective. If Firm A advertises on television, then it will be in Firm B’s best interest to advertise on radio because it will earn annual profits of $7 million compared with $6 million by advertising in newspapers. If Firm A advertises in magazines, then it will still be Firm B’s best interest to advertise on radio since it will earn annual profits of $11 million compared with $7 million by advertising in newspapers. Regardless of the advertising strategy adopted by Firm A, Firm B will advertise on radio. Advertising on radio is Firm B’s dominant strategy.

Now, consider the problem from Firm A’s perspective. If Firm B advertises on radio, then it will be in Firm A’s best interest to advertise on television because it will earn annual profits of $9 million compared with $7 million by advertising in magazines. If Firm B advertises in newspapers, then it will still be Firm A’s best interest to advertise on television since it will earn annual profits of $14 million compared with $8 million by advertising in magazines. Regardless of the advertising strategy adopted by Firm B, Firm A will advertise on television. Advertising on television is Firm A’s dominant strategy. Thus, this game has the dominant-strategy equilibrium \([\text{Television, Radio}]\).

B. The dominant-strategy equilibrium \([\text{Television, Radio}]\) is a Nash equilibrium since neither firm can improve their annual profits by switching advertising strategies.