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The Wave-Particle Duality - Solutions

1. The energy of photons in terms of the wavelength of light is given by Eq. (1.5). Substituting the given wavelength, following Example 1.1, gives:

$$E_{photon} = \frac{hc}{\lambda} = \frac{1240\ eV \cdot nm}{200\ nm} = 6.2\ eV$$

2. The energy of the beam each second is:

$$E = \frac{power}{time} = \frac{100\ W}{1\ s} = 100\ J$$

The number of photons can be calculated from the total energy above divided by the energy of each photon, which was calculated in Problem 1 and can be converted to SI units using $1.6 \times 10^{-19}\ J/eV$. The result is:

$$N_{photons} = \frac{100\ J}{9.9 \times 10^{-19}} = 1.01 \times 10^{20}$$

for the number of photons striking the surface each second.

3. Considering that we are given the power of the laser in units of milliwatts, the power may be expressed as: $1\ mW = 1 \times 10^{-3} J/s$. The energy of a single photon can be calculated as in Problem 1 by:

$$E_{photon} = \frac{hc}{\lambda} = \frac{1240\ eV \cdot nm}{632.8\ nm} = 1.960\ eV$$

We now convert to SI units:

$$1.960\ eV \times \frac{1\ J}{1.6 \times 10^{-19}\ eV} = 3.14 \times 10^{-19}\ J$$

Following the same procedure as Problem 2:

$$Rate\ of\ emission = \frac{1 \times 10^{-3}\ J/s}{3.14 \times 10^{-19}\ J/photon} = 3.18 \times 10^{15}\ \frac{photons}{s}$$

4. The maximum kinetic energy of photoelectrons is found using Eq. (1.6) and the work functions of the metals are given in Table 1.1. In each case, the energy of the photon is the same as in Problem 1, $E_{photon} = 6.20\ eV$. For Na metal in part (a):

$$(KE)_{max} = 6.20\ eV - 2.28\ eV = 2.92\ eV$$

Similarly, for Al metal in part (b), $W = 4.08\ eV$ giving $(KE)_{max} = 2.12\ eV$ and for Ag metal in part (c) with $W = 4.73\ eV$, then $(KE)_{max} = 1.47\ eV$.

5. This problem again concerns the photoelectric effect. As in Problem 4, we use Eq. (1.6):

$$(KE)_{max} = \frac{hc}{\lambda} - W$$

where W is the work function of the material and the term hc/λ describes the energy of the incoming photons. Solving for the latter:

$$\frac{hc}{\lambda} = (KE)_{max} + W = 2.3\ eV + 0.9\ eV = 3.2\ eV$$

Solving Eq. (1.5) for the wavelength:

$$\lambda = \frac{1240\ eV \cdot nm}{3.2\ eV} = 387.5\ nm$$

6. A potential energy of 0.72 eV is needed to stop the flow of electrons. Hence, $(KE)_{max}$ of the photoelectrons can be no more than 0.72 eV. Solving Eq. (1.6) for the work function:

$$W = \frac{hc}{\lambda} - (KE)_{max} = \frac{1240\ eV \cdot nm}{460\ nm} - 0.72\ eV = 1.98\ eV$$

7. Reversing the procedure from Problem 6, we start with Eq. (1.6):

$$(KE)_{max} = \frac{hc}{\lambda} - W = \frac{1240\ eV \cdot nm}{240\ nm} - 1.98\ eV = 3.19\ eV$$

Hence, a stopping potential of 3.19 eV prohibits the electrons from reaching the anode.

8. Following the equations in section 1.1.1, when the wavelength is at the threshold,

$$W = \frac{hc}{\lambda_0} = \frac{1240\ eV \cdot nm}{360\ nm} = 3.44\ eV$$

9. A frequency of 1200 THz is equal to 1200×10^{12} Hz. Using Eq. (1.10),

$$E_{photon} = hf = 4.136 \times 10^{-15}\ eV \cdot s \times 1.2 \times 10^{15}\ Hz = 4.96\ eV$$

Next, using the work function for sodium (Na) metal and Eq. (1.6),

$$(KE)_{max} = E_{photon} - W = 4.96\ ev - 2.28\ eV = 2.68\ eV$$

10. We start from Eq. (1.8) for the case of $m = 2$:

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

Now invert the equation and plug in for R:

$$\lambda = \frac{1}{1.0971 \times 10^5\ cm^{-1}}\left(\frac{1}{4} - \frac{1}{n^2}\right)^{-1}$$

Subtract the fractions by taking a common denominator:

$$\lambda = \frac{1\ cm}{1.0971 \times 10^5}\left(\frac{n^2 - 4}{4n^2}\right)^{-1}$$

Invert the term in the parenthesis and factor out the common factor of 4

$$\lambda = \frac{4\ cm}{1.0971 \times 10^5}\left(\frac{n^2}{n^2 - 4}\right)$$

Doing the division, we get Eq. (1.17):

$$\lambda = (3645.6 \times 10^{-8}\ cm)\left(\frac{n^2}{n^2 - 4}\right)$$

11. Following Example 1.2,

$$\Delta E = -\frac{13.6\ eV}{5^2} - \left(-\frac{13.6\ eV}{2^2}\right) = 2.86\ eV$$

Using Eq. (1.12):

$$\lambda = \frac{hc}{\Delta E} = \frac{1240\ eV \cdot nm}{2.86\ eV} = 434.2\ nm$$

12. Since the initial state has $m = 2$, we can use Eq. (1.7) with $n = 4$:

$$\lambda = (364.56\ nm)\left(\frac{4^2}{4^2 - 4}\right) = 486.1\ nm$$

To get the energy of the photon, use Eq. (1.2):

$$E_{photon} = \frac{hc}{\lambda} = \frac{1240\ eV \cdot nm}{486.1\ nm} = 2.551\ eV$$

13. Again, the initial state has $m = 2$, so we use Eq. (1.7) with $n = 3$:

$$\lambda = (364.56\ nm)\left(\frac{3^2}{3^2 - 4}\right) = 656.2\ nm$$

14. From Figure 1.6, the ionization energy of a hydrogen atom in the $n = 2$ state is $-3.4\ eV$. So it takes a photon of $3.4\ eV$ to just ionize this atom. To get the wavelength of light, just invert Eq. (1.5):

$$\lambda = \frac{hc}{E_{photon}} = \frac{1240\ eV \cdot nm}{3.40\ eV} = 364.7\ nm$$

15. Starting with Eq. (1.5) with a wavelength of 45 nm:

$$E_{photon} = \frac{hc}{\lambda} = \frac{1240\ eV \cdot nm}{45\ nm} = 27.6\ eV$$

Since the hydrogen atoms are in the ground state, the electron is in a bound state with energy $E = -13.6\ eV$. To conserve energy, the total energy is equal to the kinetic plus potential energy. Solving for the KE:

$$KE = 27.6\ eV - 13.6\ eV = 14.0\ eV$$

Next, to find the electron's velocity, first convert to SI units:

$$KE = 14.0\ eV \cdot \frac{1.6 \times 10^{-19} J}{1\ eV} = 2.24 \times 10^{-18}\ J$$

From Appendix A, an electron has mass $m = 9.11 \times 10^{-31}\ kg$. Using the well known formula $KE = (1/2)mv^2$, and solving for v:

$$v = \sqrt{\frac{2(KE)}{m}} = \sqrt{\frac{4.48 \times 10^{-18}\ J}{9.11 \times 10^{-31}\ kg}} = 2.22 \times 10^6\ \frac{m}{s}$$

16. From Figure 1.6, we see that the first transition for the Lyman series is between $n = 2$ to $n = 1$, and similarly we can get the transitions for the Balmer and Paschen series. Using Eq. (1.8):

$$\frac{1}{\lambda} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$

with $R = 1.0972 \times 10^5\ cm^{-1}$, then plugging in for each case: (a) $n = 2$ to $m = 1$ gives $\lambda = 1.215 \times 10^{-5}\ cm$, (b) $n = 5$ to $m = 2$ gives $\lambda = 4.340 \times 10^{-5}\ cm$, (c) $n = 5$ to $m = 3$ gives $\lambda = 1.282 \times 10^{-4}\ cm$.

17. We want to find the maximum wavelength possible for a hydrogen atom transition starting from the E_3 state. Using Eq. (1.8) for *absorption* of a photon, starting at $m = 3$:

$$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$$

we see that the maximum wavelength occurs when the RHS is at a minimum. Since the photon can only take the atom to a higher state, this requires $n > 3$, and so the minimum of the RHS occurs when $n = 4$. Inverting the above equation with $n = 4$,

$$\lambda = \frac{1}{R}\left(\frac{1}{3^2} - \frac{1}{4^2}\right)^{-1} = \frac{1}{R}\left(\frac{9 \cdot 16}{16 - 9}\right)$$

Plugging in $R = 1.0972 \times 10^5\ cm^{-1}$ gives:

$$\lambda = (9.114 \times 10^{-6}\ cm)(20.57) = 187.5 \times 10^{-6}\ cm$$

Converting this to standard units gives $\lambda = 187.5 \times 10^{-8}\ m = 1875\ nm$. Hence, light of wavelength greater than 1875 nm would not be absorbed by a hydrogen atom starting in the E_3 state.

18. Starting with Eq. (1.26),

$$p = \frac{h}{\lambda} = \frac{6.626 \times 10^{-34}\ J \cdot s}{0.2 \times 10^{-9} m} = 3.31 \times 10^{-24}\ J(s/m)$$

The kinetic energy is given by the standard formula:

$$KE = \frac{p^2}{2m_e} = 6.024 \times 10^{-18}\ J$$

where $m_e = 9.11 \times 10^{-31}\ kg$ is the mass of an electron.

19. Following Example 1.5, the de Broglie wavelength is given by:

$$\lambda = \frac{h}{\sqrt{2m(KE)}}$$

where m is the mass of the particle. It is more convenient to convert the KE to SI units:

$$KE = 20\ eV \times \frac{1.6 \times 10^{-19}\ J}{1\ eV} = 3.2 \times 10^{-18}\ J$$

From Appendix A, the mass of the electron is $m_e = 9.11 \times 10^{-31}\ kg$, so plugging this into the above gives $\lambda_e = 2.74 \times 10^{-10}\ m$. Similarly, the mass of the proton is $m_p = 1.67 \times 10^{-27}\ kg$, giving $\lambda_p = 6.41 \times 10^{-12}\ m$. The α-particle is made from two protons and two neutrons and its mass, to good approximation, can be found by adding up its parts: $m = 4(1.673 \times 10^{-27} kg) = 6.69 \times 10^{-27} kg$. Plugging this in gives $\lambda_\alpha = 3.20 \times 10^{-12}\ m$.

20. To get the wavelength of 40 keV photons, invert Eq.(1.5):

$$\lambda = \frac{hc}{E_{photon}} = \frac{1240\ eV \cdot nm}{40 \times 10^3\ eV} = 0.031\ nm$$

To get the electron's kinetic energy, first find its momentum using Eq. (1.26):

$$p = \frac{h}{\lambda} = \frac{6.626 \times 10^{-34}\ J \cdot s}{0.031 \times 10^{-9} m} = 2.14 \times 10^{-23}\ J(s/m)$$

Finally, the KE can be calculated in the usual way:

$$KE = \frac{p^2}{2m_e} = \frac{4.57 \times 10^{-46}}{2(9.11 \times 10^{-31})} = 2.51 \times 10^{-16}\ J$$

21. For all three cases, we can use Eqs. (1.26) for the momentum:

$$p = \frac{h}{\lambda} = \frac{6.626 \times 10^{-34}\ J \cdot s}{0.01 \times 10^{-9} m} = 6.626 \times 10^{-23}\ J(s/m)$$

and calculate the KE as in the problem above, using the appropriate mass from Appendix A. The results are:

Particle	mass (kg)	KE (J)	KE (eV)
electron	9.11×10^{-31}	2.41×10^{-15}	15060
proton	1.673×10^{-27}	1.312×10^{-18}	8.20
neutron	1.675×10^{-27}	1.311×10^{-18}	8.19

22. From Fig. 1.3, we see that visible light extends from about 400 nm to 700 nm. Using Eq. (1.26) as above and $v = p/m_e$ for the velocity:

Wavelength (nm)	momentum (kg·m/s)	velocity (m/s)
400	1.657×10^{-27}	1818
700	9.466×10^{-28}	1039

23. First convert the energy to SI units:

$$E = 40 \times 10^3\ eV \times \frac{1.6 \times 10^{-19}\ J}{1\ eV} = 6.4 \times 10^{-15}\ J$$

Using $p = \sqrt{2m(KE)} = 1.08 \times 10^{-22}\ kg(m/s)$ in Eq. (1.25):

$$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34}\ J \cdot s}{1.08 \times 10^{-22}\ kg(m/s)} = 6.14 \times 10^{-12}\ m$$

which is the de Broglie wavelength. Next, we observe that Eq. (1.25) does not depend on the mass, so a proton with the same de Broglie wavelength has the same momentum, $p = 1.08 \times 10^{-22}\ kg(m/s)$. Using the proton mass:

$$(KE)_p = \frac{p^2}{2m_p} = \frac{(1.08 \times 10^{-22}\ kg(m/s))^2}{2(1.673 \times 10^{-27}\ kg)} = 3.49 \times 10^{-18} J$$

If desired, the units can be converted giving $(KE)_p = 21.8\ eV$.

24. Using Eq. (1.5) we get the initial wavelength of the X-ray:

$$\lambda = \frac{hc}{E_{photon}} = \frac{1240\ eV \cdot nm}{15 \times 10^3\ eV} = 0.0827\ nm$$

Using Eq. (1.23) with an angle of 30° gives:

$$\lambda' = \lambda + \frac{h}{mc}(1 - \cos 30^\circ) = 0.0827\ nm + (0.0024\ nm)(1 - 0.866) = 0.0830\ nm$$

where $h/(mc) = 0.0024\ nm$ is the electron's Compton wavelength. Converting this back to energy, $E' = (hc)/\lambda' = 14.94\ keV$. By conservation of energy, the electron takes up the difference of

$$(KE)_e = E - E' = 15.00 - 14.94 = 0.06\ keV$$

25. Using Eq. (1.5) we get the wavelength of the 20 keV X-ray:

$$\lambda = \frac{hc}{E_{photon}} = \frac{1240\ eV \cdot nm}{20 \times 10^3\ eV} = 0.062\ nm$$

Solving Eq. (1.24) for θ with the spacing $d = 0.282\ nm$ and $n = 1$:

$$\theta = \sin^{-1}\left(\frac{0.062\ nm}{2(0.282\ nm)}\right) = 6.3^\circ$$

Continuing in this way for other $n > 1$ we get:

n	Angle
2	12.7°
3	19.3°
4	26.1°
5	33.3°
6	41.3°
7	50.3°
8	61.6°
9	81.6°