10

Semiconductor Lasers - Solutions

1. Starting with Eq. (1.5) from Chapter 1:

$$E = \frac{hc}{\lambda}$$

and putting in the given energy needed to hop over the band gap:

$$\lambda = \frac{hc}{E} = \frac{1240\ eV \cdot nm}{1.519\ eV} = 816.3\ nm$$

2. Note: there are typos in the text for the given values of a and k. Given for GaAs the lattice constant $a = 5.65$Å and effective mass $m_0 = 0.067$, then Eq. (10.2) becomes:

$$\varepsilon(k) = \epsilon_c + \frac{\hbar^2 k^2}{2(0.067)m_e}$$

Multiplying the top and bottom of the fraction by c^2 and given $k = 0.1(\pi/a)$:

$$\varepsilon(k) - \epsilon_c = \frac{(hc)^2(0.1)^2}{0.134(0.556\ nm)^2(m_e c^2)}$$

From Appendix A, $hc = 1240\ eV \cdot nm$ and $m_e c^2 = 511 \times 10^3\ eV$, giving

$$\varepsilon(k) - \epsilon_c = 0.703\ eV$$

3. From Section 10.3.1, heterostructures are formed using semiconductors with similar lattice constants. The heterostructures grown on a substrate of InP must have the same lattice constant as InP. Using a linear interpolation procedure for each alloy:

$$a(\mathrm{In}_x\mathrm{Ga}_{1-x}\mathrm{As}) = a(\mathrm{InP}) = xa(\mathrm{InAs}) + (1-x)a(\mathrm{GaAs})$$

$$5.869 \;= 6.058x + 5.653(1-x)$$

Solving for x gives the value 0.53. The alloy has composition: $In_{.53}Al_{.48}As$

4. Referring back to Chapter 2, Figure 2.5 shows a finite well of depth 0.3 eV and width 10 nm. As shown below Figure 2.5, using an effective mass of $m_0 = 0.067$, the value of θ_0^2 is found to be 13.2 from the graphical solution shown in Figure 2.6. In that case, the equation to be solved was:

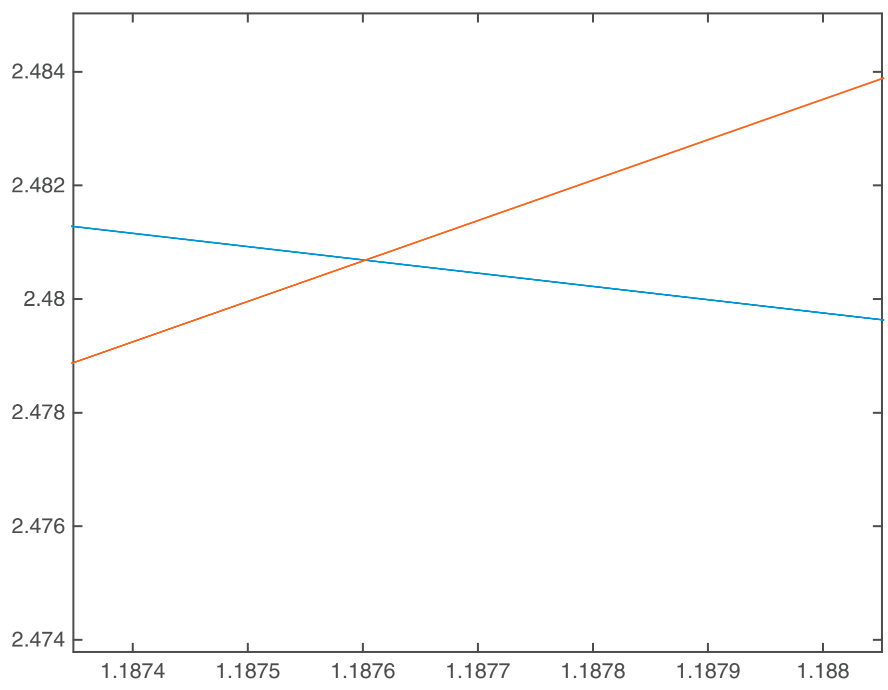
$$\tan(\theta) = \frac{\kappa}{k} = \sqrt{\frac{\theta_0^2}{\theta^2} - 1}$$

where $\theta = kL/2$. Now looking at Eq. (10.11), we want to solve:

$$\tan(\theta) = \frac{m_W \kappa}{m_B k} = \frac{0.067\kappa}{0.091k}$$

where m_W is the effective mass in the well and m_B is the effective mass in the barrier. Hence, we need to solve the following equality:

$$\tan(\theta) = 0.7363\frac{\sqrt{2(0.091)m_e(V_0 - E)}}{\sqrt{2(0.067)m_e(E)}}$$



where the numerator is $\hbar\kappa$, defined just above Eq. (10.9), and the denominator $\hbar k$. Using the definition of θ_0^2 from Chapter 2 gives:

$$\tan(\theta) = 0.7363\sqrt{\frac{0.091}{0.067}\left(\frac{\theta_0^2}{\theta^2} - 1\right)} = 0.858\sqrt{\frac{\theta_0^2}{\theta^2} - 1}$$

Following MATLAB Program 2.1, but inserting this new equation gives a value of $\theta = 1.1876 = kL/2$. Plugging in the definition of k above Eq. (10.9):

$$\frac{2(1.1876)}{10\ nm} = \sqrt{\frac{2(0.067m_e)(E - E_c^W)}{\hbar^2}}$$

Solving this gives

$$E - E_c^W = 0.032\ eV$$

This is the energy of the first bound state above the conduction band. This can be compared with the result from Chapter 2, Eq. (2.39), where $\theta = 1.2264$ giving $E = 0.0342\ eV$.

5. We use the following procedure for these matrix multiplication operations:

$$\begin{bmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} A_1D_1 + A_2D_2 + A_3D_3 \\ B_1D_1 + B_2D_2 + B_3D_3 \\ C_1D_1 + C_2D_2 + C_3D_3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 2 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ 3 \end{bmatrix}$$

6. Applying the standard methods of matrix multiplication, the first case is:

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = \begin{bmatrix} (0+i) & (0+0) \\ (0+0) & (-i+0) \end{bmatrix} = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$$

The second case gives:

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$$

The final case is:

$$\begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 2 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 1 & 3 \\ 5 & 3 & 2 \\ 6 & 2 & 4 \end{bmatrix}$$

7. Proceeding left to right, first consolidating the $1/2k_1$ and $1/2\ k_2$ terms:

$$\begin{bmatrix} k_1 + k_2 & k_1 - k_2 \\ k_1 - k_2 & k_1 + k_2 \end{bmatrix} \begin{bmatrix} e^{ik_2L} & 0 \\ 0 & e^{-ik_2L} \end{bmatrix} = \begin{bmatrix} e^{ik_2L}(k_1 + k_2) & e^{-ik_2L}(k_1 - k_2) \\ e^{ik_2L}(k_1 - k_2) & e^{-ik_2L}(k_1 + k_2) \end{bmatrix}$$

Using this result in Eq. (10.30):

$$T = \frac{1}{4k_1k_2} \begin{bmatrix} e^{ik_2L}(k_1 + k_2) & e^{-ik_2L}(k_1 - k_2) \\ e^{ik_2L}(k_1 - k_2) & e^{-ik_2L}(k_1 + k_2) \end{bmatrix} \begin{bmatrix} k_1 + k_2 & k_2 - k_1 \\ k_2 - k_1 & k_1 + k_2 \end{bmatrix}$$

Carrying this through:

$$T = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}$$

where each element is:

$$T_{11} = \frac{1}{4k_1k_2}[e^{ik_2L}(k_1 + k_2)^2 - e^{-ik_2L}(k_1 - k_2)^2]$$

$$T_{12} = \frac{-1}{4k_1k_2}[e^{ik_2L}(k_1^2 - k_2^2)^2 - e^{-ik_2L}(k_1^2 - k_2^2)]$$

$$T_{21} = \frac{1}{4k_1k_2}[e^{ik_2L}(k_1^2 - k_2^2) - e^{-ik_2L}(k_1^2 - k_2^2)]$$

$$T_{22} = \frac{-1}{4k_1k_2}[e^{ik_2L}(k_1 - k_2)^2 - e^{-ik_2L}(k_1 + k_2)^2]$$

The Euler formula can be used in the second part to further simplify:

$$\begin{aligned} T_{11} &= \frac{1}{4k_1k_2}[e^{ik_2L}(k_1 + k_2)^2 - e^{-ik_2L}(k_1 - k_2)^2] \\ &= \frac{1}{4k_1k_2}[(cos\ k_2L + i\ sin\ k_2L)(k_1^2 + 2k_1k_2 + k_2^2) \\ &\quad - (cos\ k_2L - i\ sin\ k_2L)(k_1^2 - 2k_1k_2 + k_2^2) \\ &= \frac{1}{2k_1k_2}[2k_1k_2\ cos\ k_2L + i(k_1^2 + k_2^2)sin\ k_2L] \end{aligned}$$

The same may be done for the remaining three elements.

8. This is a small variation on MATLAB Function 10.2 given in the text. It is only necessary to modify one line. For a separation of 5 nm,

```
D=[exp (i*k1*L*0.5) 0; 0 exp(-i*k1*L*0.5) ];
```

The rest is the same as Function 10.2. Also, MATLAB Program 10.2 is unchanged. For a separation of 20 nm, just replace `0.5` above with `2.0`.

9. Following MATLAB Function 10.2, but modifying it for the given values of the barrier widths and the electron effective mass:

```
function T = twobarrier (E)
L=5.0;
V0=0.3;
k1=sqrt (1.759*E) ;
k2=sqrt (2.389*(E-V0) ) ;
A=[(k1+k2) (k1-k2) ; (k1-k2) (k1+k2) ] ;
B=[(k1+k2) (k2-k1) ; (k2-k1) (k1+k2) ] ;
C=[exp(i*k2*L) 0; 0 exp(-i*k2*L) ] ;
D=[exp ( i *k1*L*2) 0; 0 exp(-i*k1*L*2) ] ;
M=A*C*B*D*A*C*B;
t=(4*k1*k2)^2/M(1 ,1);
T = abs ( t )^2;
```

where Eqs. (10.13) and (10.15) were used to calculate k_1 and k_2. Note that both L and matrix `D` have been changed, similar to the previous problem. To plot the transmission coefficient, MATLAB Program 10.2 can be used without any changes.

10. Again, this is a small modification of MATLAB Function 10.2. Only two lines need to be changed:

```
D=[exp (i*k1*L*0.5) 0; 0 exp(-i*k1*L*0.5) ];
M=A*C*B*D*A*C*B*D*A*C*B;
```

The rest of the function is unchanged. Again, MATLAB Program 10.2 can be used unchanged to calculated the transmission coefficient for this case.

11. For the downward step, just modify Eq. (10.15) for negative V_0:

$$k_2 = \sqrt{\frac{2m(E+V_0)}{\hbar^2}}$$

which, when substituted into the equations above (10.27), gives (after some simplification):

$$T_{11} = T_{22} = \frac{1}{2}\left(1 + \sqrt{1 + V_0/E}\right)$$
$$T_{12} = T_{21} = \frac{1}{2}\left(1 - \sqrt{1 + V_0/E}\right)$$

12. Eqs.(10.25) and (10.26) give the following:

$$A_1 = T_{11}A_2 + T_{12}B_2$$

$$B_1 = T_{21}A_2 + T_{22}B_2$$

The transmission amplitude of light incident from the right will be equal to B_1/B_2 and the corresponding reflection amplitude is equal to A_2/B_2. As directed, we set $A_1 = 0$.

$$0 = A_1 = T_{11}A_2 + T_{12}B_2 \tag{1}$$

$$B_1 = T_{21}A_2 + T_{22}B_2 \tag{2}$$

Solving (1) for A_2 and substituting into (2):

$$B_1 = \frac{-T_{21}T_{12}B_2}{T_{11}} + T_{22}B_2 = B_2\frac{1}{T_{11}}(T_{11}T_{22} - T_{21}T_{12})$$

The quantity in parentheses as the determinant of the T-matrix, therefore:

$$t_{21} = \frac{B_1}{B_2} = \frac{det\ T}{T_{11}}$$

This just Eq. (10.37). For Eq. (10.38), now solve (1) above for A_2/B_2:

$$r_{21} = \frac{A_2}{B_2} = \frac{-T_{12}}{T_{11}}$$

13. It is only necessary to make a small change to MATLAB Function 10.3. For a two-strip Fabry-Perot laser, we just need to extend Eq. (10.42) in a similar way as was done for the two-barrier case of MATLAB Function 10.2. Hence, we change only one line of Function 10.3:

```
M=A*C*B*A*C*B
```

This assumes that there is essentially no space between the two strips. In other words, the matrix `D` in Function 10.2 becomes the unity matrix for sufficiently small L.

14. MATLAB Program 10.3 can be used unchanged, along with the change given in Problem 13 above, to plot the transmission through the Fabry-Perot laser of the previous problem as a function of λ for the given wavelength reflection coefficients.

15.a) The total rate of change in the amount of water in the reservoir will equal the rate of water entering minus the rate in which it is leaving. Letting V equal the volume of water in the reservoir:

$$\frac{dV}{dt} = R_f - R_D$$

where R_D is the total rate at which water drains. Since the reservoir is draining from two ports:

$$R_D = R_{d1} + R_{d2}$$

Plugging this in, the total rate equation is:

$$\frac{dV}{dt} = R_f - h(C_1 + C_2)$$

b) To find the steady state height, set:

$$\frac{dV}{dt} = 0$$

And solve for h:

$$h = \frac{R_f}{c_1 + c_2}$$