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Particle Physics - Solutions

1.a) This process can not occur. It violates the conservation of baryon number and lepton number. The baryon number is one of the LHS but zero on the RHS. The electron lepton number is zero on the LHS but -1 on the RHS.

b) This process can not occur due to violation of energy conservation. The Σ° has a rest energy of 1192 MeV while the Λ and the π° have a total rest energy of 1250 MeV.

c) This process is allowed. Conservations of charge, energy, baryon number, and lepton number are all obeyed. This process conserves strangeness and proceeds by the strong interaction.

d) This process can not occur. The conservation of muon lepton number is violated. The total number is zero on the LHS, but is -1 on the RHS.

e) This process is allowed. K^- has a strange quantum number S= -1 where the $\pi^-\pi^\circ$ system has S=0. Since charge, energy, lepton number, and baryon number are all conserved, so this process proceeds via the weak interaction.

f) This process can not occur. The ρ^0, being a boson, with total spin S=1, must have a symmetric wave function. The ρ^0 has a total angular momentum of J=1. By the conservation of angular momentum, the spinless pions would have an antisymmetric total wave function with L=1. Pions are bosons, so this is forbidden.

g) This process is not seen experimentally. Strangeness is not conserved while the other conservation laws are, so the process can only occur via the weak interaction. But since no neutrinos or lepton pairs are present, indications of a weak decay are lacking in the final state. The photon does not couple directly to the neutral weak current (the photon couples only to charged particles).

2.a) This reaction is allowed, assuming that the K^- and proton collision has enough energy.

b) This reaction is allowed only if the neutrino and antineutrino are of the electron type.

c) This reaction can not occur. On the LHS, the lepton number is $L_\mu = +1$, but on the RHS it is $L_\mu = -1$.

d) This reaction is allowed.

e) This reaction is allowed. Both L_e and L_μ are conserved separately.

f) This reaction can not occur. On the LHS, the lepton number is $L_e = +1$, but on the RHS it is $L_e = -1$. (Strangeness is conserved.)

g) This reaction is not seen experimentally. Strangeness is not conserved. (This can not be a weak decay, since the RHS has more mass than the LHS. The weak process is too weak to be seen in typical beam collision experiments.)

3. The rest energies may be found in Tables 13.9 through 13.11.
a) $\pi^- + p \to n + \pi^0$
Initial state: $140 + 938 = 1078$ MeV
Final state: $940 + 135 = 1074$ Mev

b) $\Lambda \to p + \pi^-$
Initial state: 1116 MeV
Final state: $938 + 140 = 1078$ Mev

c) $\pi^+ + p \to K^+ + \Sigma^+$
Initial state: $140 + 938 = 1078$ MeV
Final state: $494 + 1189 = 1683$ Mev
The kinetic energy of this initial state must be greater than that of the final state to ensure conservation of energy.

4.a) The decay is allowed.

b) This reaction violates electron lepton number. It has $L_e = +1$ on the LHS, but $L_e = -1$ on the RHS.

c) This decay violates electron lepton number. It has $L_e = 0$ on the LHS, but $L_e = 2$ on the RHS.

d) This decay is allowed.

5. See Tables 13.9-13.11 to find the quark content, then use Eq. (13.26).
a) The Ω^- has a quark content of sss, therefore $S(\Omega) = -3$. For the particles on the RHS, $S(\Lambda) = -1$ and $S(K^-) = -1$. Since $\Delta S = 1$, the process must occur via the weak interaction.

b) On the LHS, $S(\Sigma^\circ) = -1$ and on the RHS, $S(\Lambda) = -1$, and $S(\gamma) = 0$. There is no change in strangeness. The presence of the photon signals that this is decay via the electromagnetic interaction.

c) On the LHS, $S(\Lambda) = -1$ and on the RHS, $S(p) = 0$, and $S(\pi^-) = 0$. Since $\Delta S = 1$, the decay must occur via the weak interaction.

6.a) Here, $\Delta S = 1$, so this process occurs via the weak interaction.

b) $\Delta S = 0$. The presence of the antineutrino tells us this must be a weak interaction.

c) Again, $\Delta S = 1$, so this process occurs via the weak interaction.

7.a) This process occurs via the strong interaction. All conservation laws are met.

b) This process occurs via the weak interaction. The strangeness of the initial state is zero, but has strangeness of -1 in the final state. The other conservation laws are met.

c) This process is strictly forbidden. The baryon number of the initial state is $+1$ while that of the final is zero. Note that the antiparticle has a baryon number of -1.

d) This process is strictly forbidden. The muon lepton number and the electron lepton number must each be conserved separately. In this case, both lepton numbers are violated. The muon lepton number is -1 on the LHS but zero on the RHS. Similarly, the electron lepton number is zero on the LHS but -1 on the RHS.

e) This process is strictly forbidden. Here again, lepton number conservation is violated. The electron neutrino on the left-hand side has an electron lepton number of $+1$ whereas the positron on the right-hand side has an electron lepton number of -1.

8. The quark compositions for most particles are given in Tables 13.3-13.4 and 13.9-13.11.

a) The Lambda is uds, the proton is uud and the π^0 is a mixture of $u\bar{u}$ and $d\bar{d}$.

$$\Lambda(uds) \to p(uud) \ + \ \pi^0(u\bar{u} \text{ or } d\bar{d})$$

The weak interaction has changed one s quark to a d quark and produced a $u\bar{u}$ (or $d\bar{d}$) pair in the final state.

b) The π^- is $d\bar{u}$, the proton is uud, the neutron is udd, and the π^0 is a mixture of $u\bar{u}$ and $d\bar{d}$.

$$\pi^-(d\bar{u}) \ + \ p(uud) \to n(udd) \ + \ \pi^0(u\bar{u} \text{ or } d\bar{d})$$

All quark numbers are conserved. This is a strong interaction.

c) The π^+ is $u\bar{d}$, the proton is uud, the K^+ is $u\bar{s}$ and the Σ^+ is uus.

$$\pi^+(u\bar{d}) \ + \ p(uud) \to K^+(u\bar{s}) \ + \ \Sigma^+(uus)$$

All quark numbers are conserved. This is a strong interaction, where a $u\bar{u}$ pair annihilated in the initial state and a $s\bar{s}$ pair created in the final state from the kinetic energy of the collision.

9. The quark composition may be found in Tables 13.3-13.4 and 13.9-13.11:

a)

$$\Omega^-(sss) \to \Lambda(uds) \ + \ K^-(s\bar{u})$$

There are only strange quarks in the initial state. The weak interaction has changed one s quark to a d quark, and produced a $u\bar{u}$ pair in the final state.

b)

$$\pi^-(d\bar{u}) \ + \ p(uud) \to \Lambda \ (uds) \ + \ K^0 \ (s\bar{d})$$

There are no strange quarks in the initial state, and a $s\bar{s}$ pair in the final state. All quark numbers are conserved. This is a strong interaction.

c)

$$p(uud) \ + \ K^-(s\bar{u}) \to \Xi^-(dss) \ + \ K^+(u\bar{s})$$

There is no change in strangeness from initial to final state. All quark numbers are conserved. This is a strong interaction.

10. The isospin quantum numbers for various particle are given in Tables 13.9-13.11.

a) The isospin of the π^- is $I = 1$, $I_3 = -1$. For the proton, $I = 1/2$, $I_3 = +1/2$. The possible values of the combined isospin range from $|I_\pi - I_p|$ to $|I_\pi + I_p|$, or $I = 1/2,\ 3/2$. The isospin projections add linearly: $I_3 = -1/2$. Hence, the possible values are:

$$I = 1/2,\ I_3 = -1/2$$
$$I = 3/2,\ I_3 = -1/2$$

b) The isospin of the π^- is $I = 1$, $I_3 = -1$. For the neutron, $I = 1/2$, $I_3 = -1/2$. The possible values of the combined isospin range from $|I_\pi - I_p|$ to $|I_\pi + I_p|$, or $I = 1/2,\ 3/2$. The isospin projections add linearly: $I_3 = -3/2$. Since the isospin I cannot be less than $|I_3|$, the only possible values are:

$$I = 3/2,\ I_3 = -3/2$$

c) Similarly to the above two cases, there is only one possible value of isospin numbers:

$$I = 3/2,\ I_3 = +3/2$$

d) Here, we have only one type of particle, with $I = 1/2$ and $I_3 = -1/2$. The total isospin projection is just the sum of the two, $I_3 = -1$. Now I can range from $I = 0$ to $I = 1$, but only the latter is possible for the given total I_3. The possible values are:

$$I = 1,\ I_3 = -1$$

e) Similarly to the above problems, the total $I_3 = 0$ and $I = 0,\ 1$. The allowed values are:

$$I = 0,\ I_3 = 0$$
$$I = 1,\ I_3 = 0$$

11. Given that the wave has periodic boundary conditions, then the length L must equal an integer multiple of the wavelength:

$$L = n\lambda$$

Using the de Broglie relation:

$$p = \frac{h}{\lambda} = \frac{hn}{L} = \frac{2\pi\hbar n}{L}$$

In momentum space, each state has a "length" of $2\pi\hbar/L$. The number of states in a differential length dp will be:

$$ \tag{1} $$

$$frac dp 2\pi\hbar/L = \frac{L}{2\pi\hbar}dp \tag{2}$$

Expanding the region to a cubic volume, $x = L,\ y = L,\ z = L$ from the origin, a volume element in a momentum space is:

$$\begin{aligned} dN &= \frac{L}{2\pi\hbar}d\mathbf{p}_x \frac{L}{2\pi\hbar}d\mathbf{p}_y \frac{L}{2\pi\hbar}d\mathbf{p}_x \\ &= \frac{V}{(2\pi\hbar)^3}d^3\mathbf{p} \end{aligned}$$

This is exactly Eq. (13.51) as desired.

12. The quark contents of various particles is given in Tables 13.9-13.11. The relation for the Cabibbo angles are given by Eq. (13.58) and Fig. 13.23.
a)

$$\Lambda(uds) \ \rightarrow \ \pi^-(d\bar{u}) \ + \ p(uud)$$

The weak interaction has changed a s quark to a d quark, via Z^0 exchange, creating a $u\bar{u}$ in the final state. This has no Cabibbo angle.
b)

$$n(udd) \ \rightarrow \ p(uud) \ + \ e^- \ + \ \bar{\nu}_e$$

The weak interaction has changed a d quark to a u quark, via W^- exchange, creating a $e^-\bar{\nu}_e$ lepton pair in the final state. The Cabibbo angle is $\cos\theta_c$ at the quark vertex.
c)

$$\Xi^0(uss) \ \rightarrow \ \Sigma^-(dds) \ + \ e^+ \ + \ \nu_e$$

This process is difficult to draw as a Feynman diagram, as it involves the change of two quarks, a $u \rightarrow d$ and a $s \rightarrow d$, at the same time. The $u \rightarrow d$ goes via W^+ exchange, since the quark charge changes by one unit, whereas the $s \rightarrow d$ goes via Z^0 exchange, since the quark charge is the same for both quarks. In other words, two weak interactions must occur, and this has a very low probability. This process has not been seen experimentally.
d)

$$\Omega^-(sss) \ \rightarrow \ \Sigma^0(uds) \ + \ e^- \ + \ \bar{\nu}_e$$

As in the previous problem, this is difficult to draw as a Feynman diagram, as it involves the change of two quarks, $s \rightarrow u$ and $s \rightarrow d$. The former goes via W^- exchange and the latter goes via Z^0 exchange. This process changes strangeness by two units, and has a very small probability to occur.

13. Figure 13.29 serves as a guide to obtain the remaining weights from the highest weight. In Figure 13.29(b), the highest weight is $3\mu_1$, where μ_1 is given by Eq. (13.84). Along a diagonal, such as the one with the weights labeled in Fig. 13.29(b), just subtract an additional α_1. Going right-to-left along a row, just subtract $(\alpha_1 + \alpha_2)$ for each step. Below, the results go right-to-left across a row, and after completing rows, from the top down to the last element, $3\mu_1 - 3\alpha_1$. Matrices for α_1 and α_2 are given by Eq. (13.78):

$$3\mu_1 = 3\begin{bmatrix} 1/2 \\ \sqrt{3}/6 \end{bmatrix} = \begin{bmatrix} 3/2 \\ \sqrt{3}/2 \end{bmatrix}$$

$$3\mu_1 - \alpha_1 - \alpha_2 = \begin{bmatrix} 1/2 \\ \sqrt{3}/2 \end{bmatrix}$$

$$3\mu_1 - 2\alpha_1 - 2\alpha_2 = \begin{bmatrix} -1/2 \\ \sqrt{3}/2 \end{bmatrix}$$

$$3\mu_1 - 3\alpha_1 - 3\alpha_2 = \begin{bmatrix} -3/2 \\ \sqrt{3}/2 \end{bmatrix}$$

$$3\mu_1 - \alpha_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$3\mu_1 - 2\alpha_1 - \alpha_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$3\mu_1 - 3\alpha_1 - 2\alpha_2 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$3\mu_1 - 2\alpha_1 = \begin{bmatrix} 1/2 \\ -\sqrt{3}/2 \end{bmatrix}$$

$$3\mu_1 - 3\alpha_1 - \alpha_2 = \begin{bmatrix} -1/2 \\ -\sqrt{3}/2 \end{bmatrix}$$

$$3\mu_1 - 3\alpha_1 = \begin{bmatrix} 0 \\ -\sqrt{3} \end{bmatrix}$$