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Deterministic optimization of short-term scheduling for hydroelectric power generation

Marian G. Marcovecchio[a,b], Ricardo M. Lima[a], Augusto Q. Novais[a], Ignacio E. Grossmann[c]

[a]*Unidade de Modelação e Optimização de Sistemas Energéticos, LNEG, Estrada do Paço do Lumiar, 22, 1649-038 Lisboa, Portugal*
[b]*INGAR/CONICET, Instituto de Desarrollo y Diseño, Santa Fe, Argentina, UNL, Universidad Nacional del Litoral, Santa Fe, Argentina*
[c]*Chemical Engineering Department, Carnegie Mllon University, Pittsburgh, USA*

Abstract

This work focuses on the deterministic optimization of short-term scheduling for hydroelectric power generation. The proposed methodology builds on a detailed mathematical programming model by considering the topological, reservoir, and plant constraints, and by representing the hydro production by an accurate function. The formulation results in a non-convex MINLP problem. A deterministic optimization strategy to solve the problem to optimality is proposed. The goal of this technique is overcoming the limitations of the heuristics approaches proposed in the literature that cannot provide optimality guarantees. The performance of the proposed methodology is illustrated for a real case study.

Keywords: hydroelectric power generation, deterministic optimization, mixed-integer nonlinear programming problem.

1. Introduction

The optimization of the production schedule of electricity has been receiving a good deal of attention due to a) the need of meeting increasing demand; b) the integration of renewable sources into the production system; and c) the modifications on the electricity market. Previous work of the authors addressed the thermal Unit Commitment (UC) problem and led to the development of a new and efficient Branch and Cut method, which tackled a 100-units system with a much lower CPU time than current solvers. With a view to future incorporation into a comprehensive and diversified UC setup, the present work centers now on hydroelectric power generation. This is one of the most widely used forms of renewable energy, presenting several advantages as compared to other sources: no fuel consumption, long life, and no direct waste or CO2 emissions.

In this work the short term scheduling of a cascade of hydro plants for hydroelectric power generation is addressed using a detailed mathematical programming model, and a deterministic global optimization approach to deal with the non-convexities.

This type of system usually includes hydro plants with a reservoir and plants without capacity to store water, called run-of-the-river plants, which are interconnected by the flow of water. The total discharged outflow of a plant modifies the reservoir of the downstream plant, and may have an impact on the operations of the other plants in the cascade. Each hydro plant may have more than one turbine in parallel with different specifications. The scheduling of the operations involves the determination of the start and duration of each turbine, the discharged flow and the respective power output in order to maximize the operational profit. The power generated by a turbine is in general a nonlinear function of four variables: 1) the turbined flow; 2) the net water head; 3) the efficiency of the generation; and 4) the efficiency of the turbine (Diniz et al. 2008):

$$p_{ijt} = K\eta_{ij}\zeta_{ij}q_{ijt}H_{it}, \quad \forall i,j,t, \tag{1}$$

where p_{ijt} denotes the output power of turbine *j* from plant *i* in the time period *t*, *K* is a constant, η_{ij} is the efficiency of the generator, ζ_{ij} is the efficiency of the turbine, q_{ijt} is the flow discharged from turbine *i*, from plant *j* in the time period *t*, and H_{it} is the net water head. The efficiency of the turbines is a function of the turbined flow and net water head. For the hydro plants where the net water head is not considered constant, the net water head is equal to the forebay level, hup_{it}, minus the tailrace level, hdn_{it}, as given by

$$H_{it} = (hup_{it} - hdn_{it})(1 - \xi_{ij}), \quad \forall i,t,$$

where ξ_{ij} is the penstock efficiency lost.

The forebay level is a function of the volume of water stored, while the tailrace level is a function of the total flow discharged from the hydro plant. The volume stored on each period of time results from the water mass balances between the inflows of the reservoir and the total discharged flow:

$$v_{it} = v_{it-1} + n_{it} - qt_{i,t} - s_{it} + \sum_{k\in M_i}\left(qt_{k,t-\tau_{ki}} + s_{k,t-\tau_{ki}}\right), \quad \forall i,t,$$

where v_{it} denotes the volume of the reservoir of plant *i* in the time period *t*, n_{it} represents the deterministic natural inflow of water, $qt_{i,t}$ is the sum of the turbined flow of each turbine of plant j, s_{it} is the spillage flow from plant *i*, and the set M_i defines the plants upstream of plant i. The mathematical expressions relating the forebay level and volume, and tailrace level and total flow discharged, depend on the characteristics of the hydro plant and reservoir size and shape. Detailed expressions for calculation of p_{ijt}, η_{ij} and H_{it} may be complex and nonlinear, and therefore their utilization in hydro scheduling models has been limited. Most of the approaches in the literature use simplified models based on piecewise linear approximations in order to keep the models linear. Conejo et al. (2002) and Borghetti et al. (2008) use a simplified approach to relate the power output with the net water head and the water discharged, and therefore their models result in Mixed-Integer Linear Programming (MILP) models. Conejo et al. (2002) decouples the three variables by building three piecewise linear approximations to relate the power output with the water discharged, for three levels of the net water head. Borghetti et al. (2008) enhances the previous approach by interpolating the values between the piecewise linear approximations. Basically, these authors replace the nonlinearities in the power output function by piecewise linear approximations, with the cost of adding extra equations and binary variables. Catalão et al. (2009) linearize the relation between the efficiency of the turbine and the net water head, and also the relation between the forebay level and volume, and therefore the power output is a nonlinear function of the turbined flow and water stored. Diniz et al. (2008) developed a detailed four dimension piecewise approximation model to relate the power output with the head variation, water stored, turbined flow, and spillage. Recently, Díaz et al. (2011) proposed a detailed equation to calculate the efficiency of a specific turbine as a function of the net water head and discharged flow, and linearized the equation that relates the net water head with the volume of the reservoir. Their problem results in a Mixed-Integer Nonlinear Programming (MINLP) problem, which is solved without guarantee of global optimality. In this work, the power output calculation relies on detailed expressions to estimate the forebay and tailrace levels, while the efficiencies of the generator and turbines are considered constant. The forebay and tailrace levels are calculated using fourth degree polynomials as a function of the volume of water stored and total discharged flow (Diniz, 2010):

$$hup_{it} = a_0 + a_1 v_{it} + a_2 v_{it}^2 + a_3 v_{it}^3 + a_4 v_{it}^4, \tag{2}$$

$$hdn_{it} = b_0 + b_1 D_{it} + b_2 D_{it}^2 + b_3 D_{it}^3 + b_4 D_{it}^4. \tag{3}$$

Therefore, the power generated in turbine *j* in plant *i* in the time period *t* is defined by the following equation:

$$p_{ijt} = K' q_{ijt} H_{it}, \quad \forall i, j, t. \tag{4}$$

The level of detail given by the above equations introduces nonlinearities in the formulation of the hydro scheduling model, resulting in a non-convex model, which may lead local optimization solvers to local solutions. In this work, a deterministic optimization strategy to solve the problem to optimality is proposed. This strategy is based on an appropriate underestimation of the involved non-convex terms.

2. Problem Statement

Given is a set of hydro plants in cascade that produce electricity for the day-ahead market, see Figure 1. Some plants can store water in a reservoir, while others are run-of-the-river plants. Each hydro plant has a set of turbines with a maximum turbined flow and power generated, linked to the same reservoir. The problem is to determine the start and duration of operation of each turbine, and the respective power output that maximize the operating profit, subject to a) the limits of the reservoirs; b) the mass balances of water; and c) the limits on the performance of the turbines. Each reservoir has as inputs a deterministic natural inflow and the discharge from upstream plants, and as outputs the flow discharged by each turbine linked to the reservoir. The profit is calculated as the difference between the revenues of selling electricity minus the start-up costs of the turbines. The value of the water is not considered in the profit, since the volume of each reservoir at the end of the time horizon must be greater than or equal to the initial volume. The system is considered as an electricity price taker, with the price of electricity following a given hourly profile. The time horizon is equal to one day, discretized in periods of one hour. For each pair of plants (*i*,*i'*) there is a time delay between the total flow discharged from plant *i* to plant *i'*. The system does not have to match a specific demand pattern, since all energy produced is delivered, without considering electrical network constraints.

3. Solution approach

The mathematical formulation adopted for the short-term scheduling for hydroelectric power generation is highly non-linear. The constraints presenting non-convexities are the ones involved in the computing of the power output for each turbine. As (2) and (3) show, the forebay and tailrace levels for each plant at each time period are estimated by fourth degree polynomials, which are neither convex nor concave, since the coefficients may have alternative signs. Moreover, the calculation of the power generated by each turbine at each time period involves the product of the net water head by the flow discharged. Consequently, the resulting mathematical programming model is a MINLP problem with large numbers of univariate non-convex functions and bilinear terms included in the constraints. The objective function that calculates the total profit is a linear function. With the aim of finding the global optimal solution for the short-term hydroelectric scheduling within a pre-specified tolerance, a Branch-and-Bound framework is adopted. For this purpose, a valid over-estimator problem is formulated, henceforth denominated as OEP. The feasible region of the OEP problem consists of a linear over estimation of the region defined by the constraints in the MINLP problem. The OEP problem has the following features:

- Each non-linear term in the forebay and tailrace equations is tackled separately. That implies 6 univariate non-linear functions for each plant and time period. Each univariate non-linear function is over-estimated by six planes that constitute a good approximation of the convex hull.
- The non-linearities present in (4) consist of one bilinear term for each turbine and time period. The McCormick planes are used to over-estimate these terms. However, the relaxation error introduced by over-estimating the bilinear terms by these planes is still large. With the aim of reducing these errors, the planes are also applied over subintervals of the variables. To do that, the disjunction for choosing the subinterval for each variable and its corresponding overestimating planes are modeled by using "either-or" constraints which imply adding a binary variable for each new sub-interval. The number of sub-intervals for each variable in the bilinear terms is determined so that each of them has a length lower than 100 units.
- The binary variables present in the MINLP problem for representing the on/off status of each turbine at each time period are not relaxed for the OEP problem.

Hence, the OEP is a Mixed-Integer Linear Programming (MILP) problem that provides a tight linear over-estimation for the non-convex feasible region of the MINLP model. It is solved at each node of the Branch and Bound tree and its solution provides an upper bound for the objective function of the MINLP model in the current node. The global lower bound is the objective function of the best known solution of the MINLP model. At each node, an attempt to obtain a better MINLP solution is done by fixing the binary variables at the value of the optimal solution of OEP problem, and carrying out a local search. The branching variable is chosen as the one corresponding to the greater difference between the non-linear term and its approximation in the OEP problem. If the greatest error occurs in a bilinear term, the branching is performed over the associated variable having a larger range. Once the pre-specified tolerance for global optimality has been reached the search stops.

4. Results and Conclusions

As a case study, a hydro system involving a cascade with 12 hydro plants is used to assess the utilization of a deterministic approach to solve non-convex hydro scheduling problems. The system has eight hydro plants that can store water in a reservoir, represented by a triangle, four run-of-the-river plants, represented by a circle, and 56 turbines distributed by the hydro plants, see Figure 1. This case study and the topological, reservoir and hydro data are based on the first test case proposed by Diniz A.L. (2010).

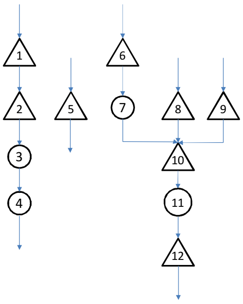

Figure 1- Hydro system configuration.

The deterministic optimization strategy is implemented in GAMS (Booke et al., 1998), and the case study solved on a machine with an Intel Core i7 Q740 CPU @1.73GHz, and 8 GB RAM memory. The OEP problem is solved with CPLEX 12.2, and the relaxed Mixed Integer Non Linear Programming model for the local search is solved with CONOPT. The optimal short-term scheduling for the case study is obtained within 1% of tolerance for global optimality, in 3478 seconds, and after analyzing 56 nodes. The objective value of the optimal solution is $3,759,637, for a time horizon of 24 hours. Fig. 2 shows the power production profile for the optimal schedule.

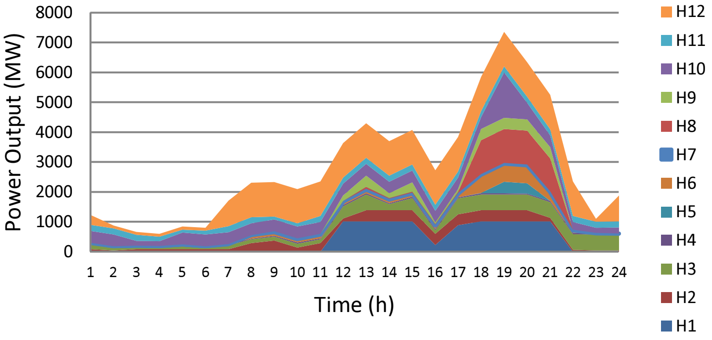

Figure 2- Power Production profile.

The technique could achieve 1% of tolerance for global optimality within reasonable computational times. Therefore, this case study illustrates the performance of the deterministic optimization strategy proposed to solve the short-term scheduling of hydroelectric power generation, overcoming the limitations on optimality of the heuristic approaches proposed in the literature.

References

A. Borghetti, C. D’Ambrosio, A. Lodi, S. Martello, 2008, An MILP approach for short-term hydro scheduling and unit commitment with head-dependent reservoir, IEEE Trans. Power Syst., 23,3,1115-1124.

J.P.S. Catalão, S.J.P.S. Mariano, V.M.F. Mendes, and L.A.F.M. Ferreira, 2009, Scheduling of head-sensitive cascade hydro systems: A nonlimear approach, IEEE Trans. Power Syst., 24,1, 336-346.

A.J. Conejo, J.M. Arroyo, J. Contreras, F.A. Villamor, 2002, Self-Scheduling of a Hydro Producer in a Pool-Based Electricity Market. IEEE Trans. Power Syst., 17,4,1265-1272.

F.J. Díaz, J. Contreras, J. I. Munoz, D. Pozo, 2011, Optimal scheduling of a price taker cascade reservoir system in a pool-based electricity market, IEEE Trans. Power Syst., 26,2,604-615.

A.L. Diniz, 2010, Test Cases for unit commitment and hydrothermal scheduling problems, IEEE Power and Energy Society General Meeting 2010, 1-8.

A.L. Diniz, M.E. Piñero Maceira, 2008, A Four-Dimensional Model of Hydro Generation for the Short-Term Hydrothermal Dispatch Problem Considering Head and Spillage Effects, IEEE Trans. Power Syst., 23,3, 1298-1308.

M.G. Marcovecchio, A.Q. Novais, I.E. Grossmann, 2011, A deterministic optimization approach for the Unit Commitment problem”. 21st European Symposium on Computer Aided Process Engineering – ESCAPE 21, 29th May-1st June, Chalkidiki, Greece.

A. Brooke, D. Kendrick, A. Meeraus and R. Raman, *GAMS — A User Guide*, GAMS Development Corporation, Washington, DC, USA. GAMS Development Corporation