EXAMPLE 2.20 AN N-RESISTOR CURRENT DIVIDER Now consider the more general current divider having N resistors, as shown in Figure 2.38. It can be analyzed in the same manner as the two-resistor current divider. To begin, the element laws are

$$i_0 = -I \tag{2.101}$$

$$v_n = R_n i_n, \quad 1 \leq n \leq N. \tag{2.102}$$

Next, the application of KCL to either node yields

$$i_0 + i_1 + \cdots i_N = 0 \tag{2.103}$$

and the application of KVL to the $N - 1$ internal loops yields

$$v_n = v_{n-1}, \quad 1 \leq n \leq N. \tag{2.104}$$

Finally, Equations 2.101 through 2.104 can be solved to yield

$$i_0 = -I \tag{2.105}$$

$$i_n = \frac{G_n}{G_1 + G_2 + \cdots G_N} I, \quad 1 \leq n \leq N \tag{2.106}$$

$$v_n = \frac{1}{G_1 + G_2 + \cdots G_N} I, \quad 0 \leq n \leq N \tag{2.107}$$

where $G_n \equiv 1/R_n$. This completes the analysis.

As was the case for the two-resistor current divider, the preceding analysis shows that parallel resistors divide current in proportion to their conductances. This follows from the G_n in the numerator of the right-hand side of Equation 2.106. Additionally, the analysis again shows that parallel conductances add. To see this, let G_P be the equivalent conductance of the N parallel resistors. Then, from Equation 2.107 we see that

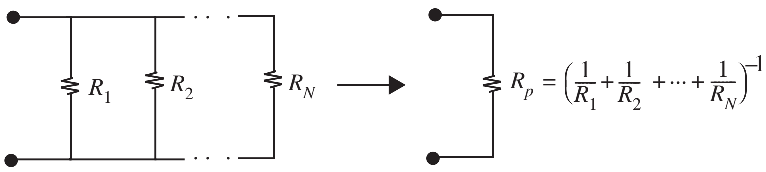
$$G_P = \frac{I}{v_n} = G_1 + G_2 + \cdots G_N \tag{2.108}$$

from which it also follows that

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots \frac{1}{R_N} \tag{2.109}$$

where $R_P \equiv 1/G_P$ is the equivalent resistance of the N parallel resistors. The latter result is summarized in Figure 2.40.

FIGURE 2.40 The equivalence of parallel resistors; for $N = 2$, $R_P = R_1R_2/(R_1 + R_2)$.



Finally, the two current-divider examples illustrate an important point, namely that parallel elements all have the same voltage across their terminals because their terminals are connected directly across one another. This results in the KVL seen in Equations 2.80, 2.81, and 2.104, which state the equivalence of the terminal voltages.