EXAMPLE 3.12 A MORE COMPLEX DEPENDENT-CURRENT SOURCE PROBLEM As a more complex example of the node analysis of a circuit containing dependent sources, consider the analysis of the circuit shown in Figure 3.28. This circuit has two dependent sources: one VCCS and one CCVS. In addition, its resistors are labeled with their conductances for convenience.

To analyze the circuit in Figure 3.28, we redraw it as shown in Figure 3.29. Here, the VCCS is replaced by an independent current source having value $\tilde{I}$, and the CCVS is replaced by an independent voltage source having value $\tilde{V}$. Note that the new independent voltage source is not a floating voltage source because it is connected to ground through the known voltage V.

The circuit in Figure 3.29 can be analyzed by the node method presented earlier. Since ground is already defined in the figure at Node 5, Step 1 is already complete. To complete Step 2, the node voltages are labeled as shown. The voltages at Nodes 1 and 2 are the unknown node voltages e_1 and e_2. The voltage at Node 3 is set by the original independent voltage source, and is labeled accordingly. The voltage at Node 4 is also known since the new voltage source is an independent source, and it is labeled as such.

Next, we perform Step 3, writing KCL for Nodes 1 and 2 in the process. This yields

$$G_1(e_1 - V - \tilde{V}) + G_2(e_1 - e_2) - I = 0 \tag{3.64}$$

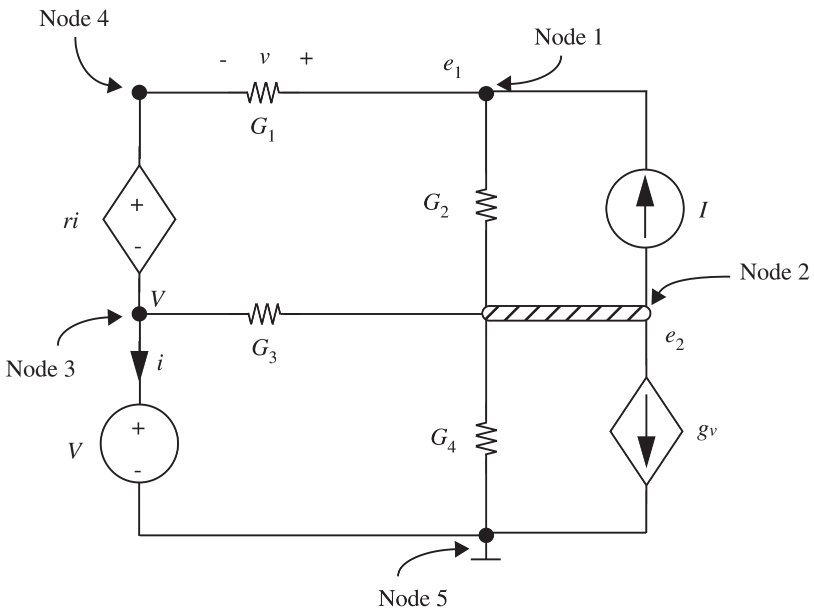

FIGURE 3.28 A circuit with two dependent sources.

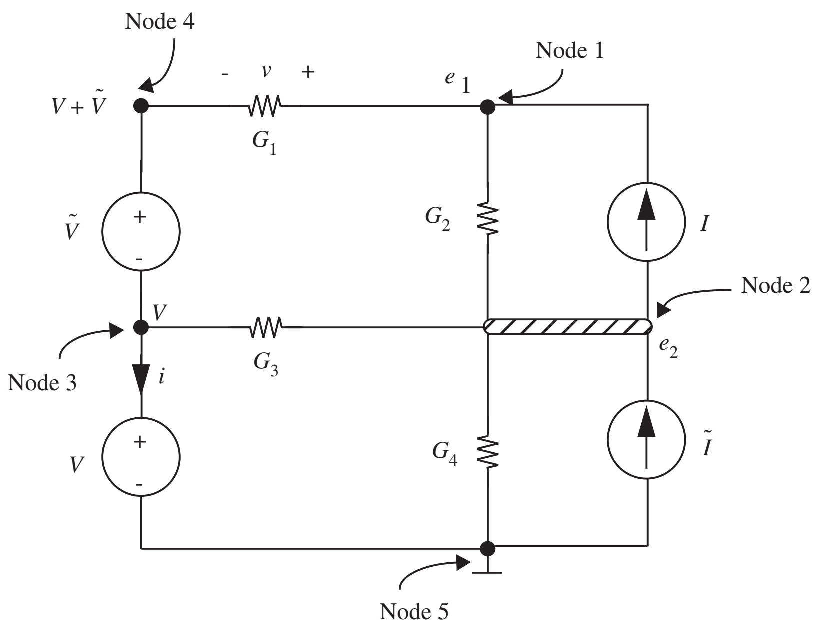

FIGURE 3.29 The circuit from Figure 3.28 redrawn with independent sources.

for Node 1, and

$$G_2(e_2 - e_1) + G_3(e_2 - V) + G_4 e_2 + I - \tilde{I} = 0 \tag{3.65}$$

for Node 2. Equations 3.64 and 3.65 can be restated as

$$\begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 + G_4 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} 1 & G_1 & 0 & G_1 \\ -1 & G_3 & 1 & 0 \end{bmatrix} \begin{bmatrix} I \\ V \\ \tilde{I} \\ \tilde{V} \end{bmatrix}. \tag{3.66}$$

Following Step 4, Equation 3.66 is solved for e_1 and e_2. This yields

$$\begin{aligned} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} &= \frac{1}{\Delta} \begin{bmatrix} G_2+G_3+G_4 & G_2 \\ G_2 & G_1+G_2 \end{bmatrix} \begin{bmatrix} 1 & G_1 & 0 & G_1 \\ -1 & G_3 & 1 & 0 \end{bmatrix} \begin{bmatrix} I \\ V \\ \tilde{I} \\ \tilde{V} \end{bmatrix} \\ &= \frac{1}{\Delta} \begin{bmatrix} (G_3+G_4)I + (G_1(G_2+G_3+G_4)+G_2G_3)V + G_2\tilde{I} + G_1(G_2+G_3+G_4)\tilde{V} \\ -G_1 I + (G_1G_2+G_1G_3+G_2G_3)V + (G_1+G_2)\tilde{I} + G_1G_2\tilde{V} \end{bmatrix} \end{aligned} \tag{3.67}$$

where

$$\Delta = (G_1 + G_2)(G_2 + G_3 + G_4) - G_2^2. \tag{3.68}$$

Finally, we use Equations 3.67 and 3.68 to solve for i and v, the branch variables that control the CCVS and the VCCS, respectively. This yields

$$\begin{aligned} i &= \tilde{I} - G_4 e_2 \\ &= \frac{1}{\Delta}\left[G_1 G_4 I - (G_1 G_2 + G_1 G_3 + G_2 G_3)(G_4 V - \tilde{I}) - G_1 G_2 G_4 \tilde{V}\right] \end{aligned} \tag{3.69}$$

$$\begin{aligned} v &= e_1 - V - \tilde{V} \\ &= \frac{1}{\Delta}\left[(G_3 + G_4)I - G_2 G_4 V + G_2 \tilde{I} - G_2(G_3 + G_4)\tilde{V}\right], \end{aligned} \tag{3.70}$$

which completes the node analysis of the circuit in Figure 3.29. Note that KCL was used at Node 5 to derive the first equality in Equation 3.69.

To find the actual values for $\tilde{I}$ and $\tilde{V}$, we now substitute Equations 3.69 and 3.70 into the element laws for the CCVS and the VCCS, respectively. This yields

$$\tilde{V} = ri = \frac{r}{\Delta}\left[G_1 G_4 I - (G_1 G_2 + G_1 G_3 + G_2 G_3)(G_4 V - \tilde{I}) - G_1 G_2 G_4 \tilde{V}\right] \tag{3.71}$$

for the CCVS, and

$$\tilde{I} = gv = \frac{g}{\Delta}\left[(G_3 + G_4)I - G_2 G_4 V + G_2 \tilde{I} - G_2(G_3 + G_4)\tilde{V}\right] \tag{3.72}$$

for the VCCS. Finally, Equations 3.71 and 3.72 are jointly written as

$$\begin{aligned} &\begin{bmatrix} \Delta - gG_2 & gG_2(G_3 + G_4) \\ -r(G_1 G_2 + G_1 G_3 + G_2 G_3) & \Delta + rG_1 G_2 G_4 \end{bmatrix} \begin{bmatrix} \tilde{I} \\ \tilde{V} \end{bmatrix} \\ &= \begin{bmatrix} g(G_3 + G_4) & -gG_2 G_4 \\ rG_1 G_4 & -rG_4(G_1 G_2 + G_1 G_3 + G_2 G_3) \end{bmatrix} \begin{bmatrix} I \\ V \end{bmatrix} \end{aligned} \tag{3.73}$$

and then solved simultaneously to yield

$$\begin{bmatrix} \tilde{I} \\ \tilde{V} \end{bmatrix} = \frac{\begin{bmatrix} g(G_3 + G_4) & -gG_2 G_4(1 - rG_3) \\ r(G_1 G_4 + gG_3) & rG_4(G_1 G_2 + G_1 G_3 + G_2 G_3) \end{bmatrix} \begin{bmatrix} I \\ V \end{bmatrix}}{\Delta + rG_1 G_2 G_4 - gG_2(1 - rG_3)}. \tag{3.74}$$

The actual values of the dependent sources are now known. Finally, to complete the node analysis, at least to the point of determining e_1 and e_2, Equation 3.74 is substituted into Equation 3.67 to yield

$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \frac{\begin{bmatrix} G_3(1+rg)+G_4(1+rG_1) & \Delta - G_2G_4 - rG_1G_3G_4 - gG_2(1-rG_3) \\ g - G_1 & G_1G_2 + G_1G_3 + G_2G_3 - gG_2(1-rG_3) \end{bmatrix} \begin{bmatrix} I \\ V \end{bmatrix}}{\Delta + rG_1G_2G_4 - gG_2(1-rG_3)}. \tag{3.75}$$

Now, with Equations 3.74 and 3.75, all node voltages are known and so all branch variables may be computed explicitly.

As was the case for the circuit in Figure 3.26, it is also possible to apply the simple node analysis described in Subsection 3.3 to the circuit in Figure 3.28. However, for the latter circuit, the savings in time is not as great because some effort and thought is needed to express i and v explicitly in terms of e_1 and e_2. Furthermore, since these expressions can be obtained in several different ways, the simple analysis becomes somewhat ad hoc when applied to the circuit in Figure 3.28.

To begin the simple node analysis of the circuit in Figure 3.28, we express i and v explicitly in terms of e_1 and e_2. The ability to do so will be needed to carry out the spirit of Step 3. From the definition of v in Figure 3.28, it is apparent that

$$v = e_1 - V - ri. \tag{3.76}$$

Thus, v can easily be expressed explicitly in terms of e_1 and e_2 once i is so expressed. One relatively convenient way to express i explicitly in terms of e_1 and e_2 is to combine KCL applied at Nodes 1, 3, and 4. This results in

$$i = I + G_2(e_2 - e_1) + G_3(e_2 - V). \tag{3.77}$$

The first term on the right-hand side of Equation 3.77 is the current through the independent current source, and the second term on the right-hand side is the current through the resistor labeled G_2. These two currents combine at Node 1, and their sum exits Node 1 through the resistor labeled G_1. Finally, the combined current passes through Node 4 and the CCVS, before entering Node 3. At Node 3, the combined current also combines with the current through the resistor labeled G_3, and together they exit Node 3 as i. The last term on the right-hand side of Equation 3.77 is the current through the resistor labeled G_3. Thus, Equation 3.77 does express KCL applied to Nodes 1, 3, and 4. Finally, the substitution of Equation 3.77 into Equation 3.76 yields

$$v = e_1 - V - r(I + G_2(e_2 - e_1) + G_3(e_2 - V)), \tag{3.78}$$

which expresses v explicitly in terms of e_1 and e_2.

Next, we apply the simple node method, beginning with Step 3, yielding

$$0 = G_1 v + G_2(e_1 - e_2) - I, \tag{3.79}$$

for Node 1 and

$$0 = I + G_2(e_2 - e_1) + G_3(e_2 - V) + G_4 e_2 - g v \tag{3.80}$$

for Node 2. At this point Equations 3.79 and 3.80 still contain v. However, upon substitution of Equation 3.78, they can be rewritten as

$$\begin{bmatrix} G_1 + G_2 + rG_1G_2 & -G_2 - rG_1(G_2 + G_3) \\ -G_2 - g - rgG_2 & G_4 + (1 + rg)(G_2 + G_3) \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} 1 + rG_1 & G_1(1 - rG_3) \\ -1 - rg & G_3(1 + rg) - g \end{bmatrix} \begin{bmatrix} I \\ V \end{bmatrix}. \tag{3.81}$$

Finally, following Step 4, Equation 3.81 can be solved to yield

$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \frac{\begin{bmatrix} G_3(1 + rg) + G_4(1 + rG_1) & \Delta - G_2G_4 - rG_1G_3G_4 - gG_2(1 - rG_3) \\ g - G_1 & G_1G_2 + G_1G_3 + G_2G_3 - gG_2(1 - rG_3) \end{bmatrix} \begin{bmatrix} I \\ V \end{bmatrix}}{\Delta + rG_1G_2G_4 - gG_2(1 - rG_3)}, \tag{3.82}$$

which is identical to Equation 3.75, as it should be. The main point here is that while the application of the simple node analysis described in Section 3.3 to circuits containing dependent sources can result in less work, it also generally becomes less structured. This is because, as part of the analysis, it is necessary to determine the variables that control the dependent sources explicitly in terms of the unknown node voltages before the node analysis is actually completed. It may not always be obvious how to do this in a simple way. For this reason, when it is necessary to carry out a well-structured node analysis, such as when the analysis is to be computerized, then the node analysis presented in this subsection is preferred.