EXAMPLE 3.9 EVEN MORE ON THE NODE METHOD As we discussed earlier, it is often inefficient to use only Kirchhoff's laws to analyze complicated circuits. For example, if we simply modify the example we saw on page 192 to the circuit shown in Figure 3.18, Kirchhoff's laws alone will not be able to solve it easily. We will use the node method to solve the problem and the node assignment in Figure 3.19.

With respect to the node assignment in Figure 3.19, we have the following equations:

$$\frac{V - e_1}{R_1} + \frac{e_2 - e_1}{R_2} + \frac{e_3 - e_1}{R_3} = 0 \tag{3.29}$$

$$\frac{0 - e_2}{R_4} + \frac{e_1 - e_2}{R_2} + \frac{e_3 - e_2}{R_6} = 0 \tag{3.30}$$

$$\frac{e_1 - e_3}{R_3} + \frac{e_2 - e_3}{R_6} + \frac{0 - e_3}{R_5} = 0. \tag{3.31}$$

We can rearrange the terms and express the equations in matrix form:

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} & -\frac{1}{R_2} & -\frac{1}{R_3} \\ -\frac{1}{R_2} & \left(\frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{R_6}\right) & -\frac{1}{R_6} \\ -\frac{1}{R_3} & -\frac{1}{R_6} & \left(\frac{1}{R_3} + \frac{1}{R_5} + \frac{1}{R_6}\right) \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \frac{V}{R_1} \\ 0 \\ 0 \end{bmatrix}$$

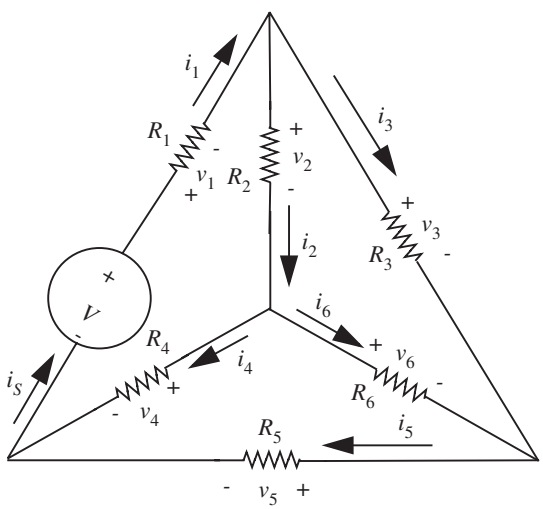

FIGURE 3.18 Circuit with appropriate branch variables.

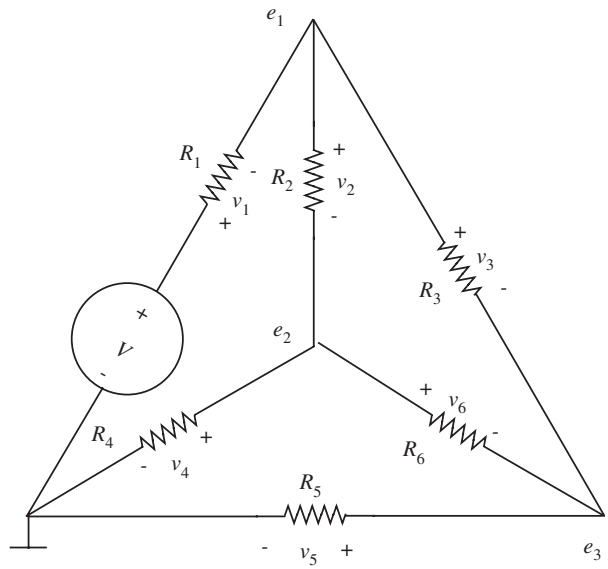

FIGURE 3.19 Node assignments of the circuit.

Standard matrix techniques can be used to solve for the unknowns. Let us assign the following values to the resistors and voltage source:

$$V = 5 \text{ V}$$

$$R_1 = 50\ \Omega$$

$$R_2 = 100\ \Omega$$

$$R_3 = 100\ \Omega$$

$$R_4 = 75\ \Omega$$

$$R_5 = 75\ \Omega$$

$$R_6 = 150\ \Omega.$$

Then we have:

$$\begin{bmatrix} \frac{1}{25} & \frac{-1}{100} & \frac{-1}{100} \\ \frac{-1}{100} & \frac{3}{100} & \frac{-1}{150} \\ \frac{-1}{100} & \frac{-1}{150} & \frac{3}{100} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{10} \\ 0 \\ 0 \end{bmatrix}$$

Solving for e_1, e_2, and e_3, we have $e_1 = 35/11$ V, $e_2 = 15/11$ V, and $e_3 = 15/11$ V. Notice that $e_2 = e_3$, that is, there is no current going through resistor R_6. Since $R_2 = R_3$ and $R_4 = R_5$, the symmetry of the network between node e_1 and ground splits the current going into node e_1 evenly, thus causing the same voltage drop.