3.3.4 THE CONDUCTANCE AND SOURCE MATRICES *

As we saw earlier in Equation 3.27, when a resistive circuit is linear (that is, when its resistors and dependent sources are all linear), the equations resulting from Step 3 of a node analysis can be formulated as a matrix equation, which takes the form

$$\bar{G}\bar{e} = \bar{S}\bar{s}. \tag{3.83}$$

Here, $\bar{e}$ is a vector of the unknown node voltages, $\bar{s}$ is a vector of the known independent source amplitudes, and $\bar{G}$ and $\bar{S}$ are known matrices, referred to here as the conductance and source matrices, respectively. Examples of such equations can be seen in Equations 3.27 and 3.66.

As previewed in the discussion following Equation 3.27, the matrices $\bar{G}$ and $\bar{S}$ have a very special structure. This structure allows us to skip the details of Step 3 of a node analysis, and derive the two matrices directly from the topology of the circuit. This also facilitates the computerization of a node analysis. Alternatively, the special structure of the two matrices can be used to check our work during Step 3. For simplicity, in this subsection we will examine the structure of $\bar{G}$ and $\bar{S}$ that arises from circuits that contain neither floating voltage sources nor dependent sources. However, it is possible to extend our observations to accommodate these sources as well.

The special structure of $\bar{G}$ and $\bar{S}$ can be exposed by studying the partial circuit shown in Figure 3.30. By the end of Step 3 of a node analysis, one expression of KCL has been derived in terms of the unknown node voltages for each node having an unknown node voltage. In the case of the partial circuit in

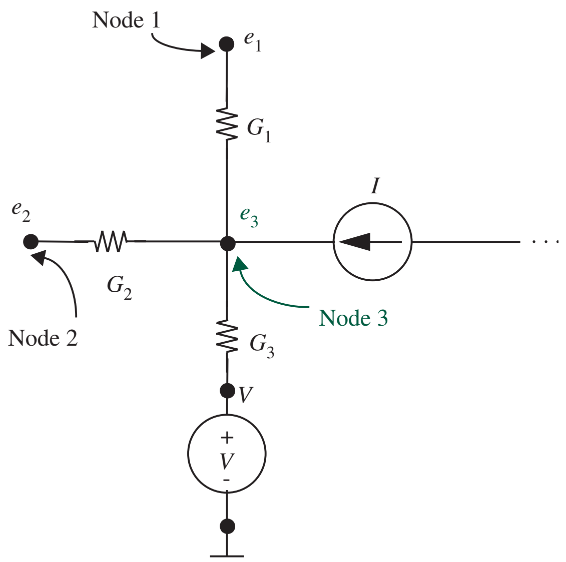

FIGURE 3.30 A partial circuit.

Figure 3.30, the corresponding expression of KCL for Node 3 is

$$G_1(e_3 - e_1) + G_2(e_3 - e_2) + G_3(e_3 - V) - I = 0. \tag{3.84}$$

In writing Equation 3.84, KCL has been taken to state that the sum of the currents exiting a node must vanish. Next, we rearrange Equation 3.84 as

$$-G_1e_1 - G_2e_2 + (G_1 + G_2 + G_3)e_3 = G_3V + I. \tag{3.85}$$

By writing KCL as in Equation 3.85, the special structure of the expression becomes apparent. For example, the conductance of each resistor connected to Node 3 contributes positively to the coefficient of e_3, and negatively to the coefficient of the node voltage at the other end of the resistor. This is because e_3 acts to drive currents out from Node 3, while the other node voltages act to drive currents in to Node 3. The same observation holds for the coefficient of the grounded independent voltage source, except for a change in sign due to the fact that the corresponding term is moved to the opposite side of the equal sign. We also see that the current source enters positively into Equation 3.85, once its term is moved to the opposite side of the equal sign, since it sources current into Node 3.

Now consider assembling Equation 3.85, and its counterparts from the other nodes in the circuit, in the form of Equation 3.83. Each expression of KCL becomes a row within Equation 3.83. For the sake of discussion, let us assume that these rows are ordered according to the number of the node for which they are written, and further that the node voltages in $\bar{e}$ are listed in order of their corresponding node numbers. In this case, Equation 3.85 enters into Equation 3.83 as

$$\begin{bmatrix} \vdots & & & & \\ -G_1 & -G_2 & G_1 + G_2 + G_3 & 0 & \cdots \\ \vdots & & & & \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ \vdots \end{bmatrix} = \begin{bmatrix} \vdots & & \\ 1 & G_3 & \cdots \\ \vdots & & \end{bmatrix} \begin{bmatrix} I \\ V \\ \vdots \end{bmatrix}. \tag{3.86}$$

Thus, we see that $\bar{G}$ is a matrix of conductances. A diagonal element at the position $[m, m]$ in $\bar{G}$ is the sum of the conductances connected Node m. An off-diagonal element at the position $[m, n]$ in $\bar{G}$, $m \neq n$, is the negative of the conductance connecting Nodes m and n. This is true even for the zero elements within $\bar{G}$ since a zero conductance indicates the absence of a resistor, or no connection. As a consequence of this structure, $\bar{G}$ is symmetric about its main diagonal, at least in the absence of dependent sources.

Similarly, the matrix $\bar{S}$ contains the coefficients of the sources. For each independent current source, there will be a $+1$ in its column in $\bar{S}$ at Row m if the source enters Node m, a -1 if the source exits Node m, and a 0 otherwise. For each grounded independent voltage source, the conductance connecting it to Node m will appear in Row m of its column in $\bar{S}$, including zeros to indicate the absence of a connecting resistor.

Again, the structure of $\bar{G}$ and $\bar{S}$ can be seen in Equations 3.27 and 3.66. Consider, for example, the matrices in Equation 3.27. The [1,1] element of $\bar{G}$ is $G_1 + G_2 + G_3$ because the resistors labeled G_1, G_2, and G_3 are all connected to Node 1. Similarly, the [2,2] element in $\bar{G}$ is $G_3 + G_4$ because the resistors labeled G_3 and G_4 are both connected to Node 2. The [1,2] and [2,1] elements in $\bar{G}$ are both $-G_3$ since G_3 connects Nodes 1 and 2. Since the voltage source connects to Node 1 through the resistor labeled G_1, but does not connect to Node 2, the [1,1] element of $\bar{S}$ is G_1 and the [2,1] element is zero. Similarly, since the current source enters Node 2, but does not connect to Node 1, the [2,2] element of $\bar{S}$ is $+1$ and the [1,2] element is zero. Thus, the matrices in Equation 3.27 could have been derived by inspection of the circuit topology only.