EXAMPLE 13.8 NON-IDEAL TRANSFORMERS Real transformers are never ideal, and at times their nonidealities are important. Such nonidealities include the resistance of the coils, the leakage inductance of the coils and the magnetizing inductance of the core. These non-idealities can be added to the model shown in Figure 9.29 to arrive at the model shown in Figure 13.60; note that the transformer symbol in Figure 13.60 represents the original ideal transformer from Figure 9.29.

In Figure 13.60, R_1 and R_2 represent the resistances of the two coils, and L_{L1} and L_{L2} represent the leakage inductances of the two coils. The inductance L_0 represents the magnetizing inductance of the core given a single-turn coil, and so must be multiplied by N_1^2 if placed on the primary side of the ideal transformer, or by N_2^2 if placed on the secondary side of the ideal transformer. In either case, it represents the effect of the non-infinite permeability of the core.

Let us now examine the effect of the magnetizing inductance on the results of Example 13.7. Consider the case of $N_1 = 100$, $N_2 = 600$, and $L_0 = 8\ \mu$H, as might be the case for a small-signal transformer. This case is shown in Figure 13.61.

Following the results of Example 13.7, we can replace the combination of the ideal transformer and the 1800-Ω load resistor with a 50-Ω resistor, and compute the magnitude of v_1 to be

$$|v_1| = \left| \frac{(j\omega\ 80\ \text{mH}) \| (50\ \Omega)}{50\ \Omega\ + (j\omega\ 80\ \text{mH}) \| (50\ \Omega)} \right| 1\text{V} = \left| \frac{\omega}{\sqrt{4\omega^2 + (625\ \text{rad/s})^2}} \right| 1\text{V}.$$

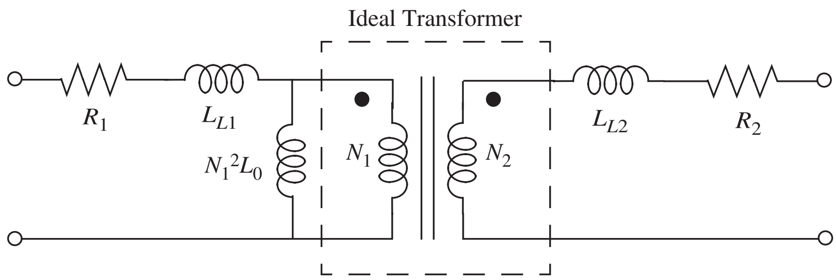

FIGURE 13.60 A non-ideal transformer.

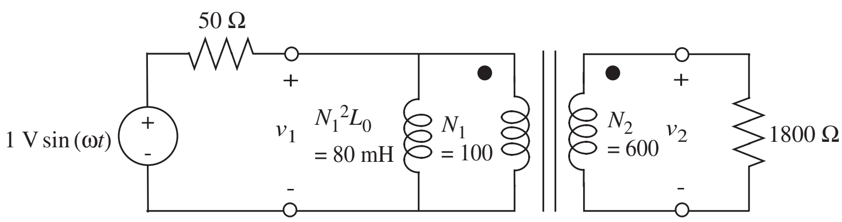

FIGURE 13.61 Power transfer with a non-ideal transformer.

Thus, for $\omega \gg 312.5$ rad/s, that is, for source frequencies well above 50 Hz, the voltage magnitude across the transformer primary is approximately 0.5 V peak. In this case, the maximum power is transfered to the resistor load. However, as the frequency goes below 50 Hz, the inductor behaves like a relative short circuit in comparison to the 50-Ω resistance of the transformed load resistor, and so the magnitude of v_1 drops. The magnitude of v_2 and the power delivered to the load drops accordingly. In general, the time-average power delivered to the load resistor is that which flows into the primary of the ideal transformer, namely:

$$\frac{\omega^2}{4\omega^2 + (625 \text{ rad/s})^2}\ 10 \text{ mW}.$$