

## 15.9 TWO-PORTS\*

It should be obvious by now that circuits with dependent sources can perform much more interesting and useful signal processing than those constructed solely from two-terminal resistive elements. But inclusion of dependent sources has brought about a modest increase in circuit complexity, so it is useful at this point to generalize some of the concepts introduced in previous chapters. In particular, let us examine how to generalize the Thévenin calculations to three-terminal or four-terminal systems.

We start with a linear network containing resistors, voltage sources, and current sources as we did in Figure 3.55, but now we assume *two* pairs of external terminals, as shown in Figure 15.36. This network is called a *two-port*, or a *two-terminal-pair* network. For the purposes of the present discussion, it doesn't matter whether the two negative leads are tied together or go to some common ground, or both terminal pairs are floating with respect to ground. We wish to find a *two-port* Thévenin equivalent of this network. The derivation is a simple extension of the method in Section 3.6.1. We apply current sources at each of the ports, as in Figure 15.37a, then solve the problem by superposition. We first set all the independent sources, both internal and external, to zero except  $i_2$ , and measure the resulting  $v_{2a}$  as in the subcircuit of Figure 15.37b. Because there is nothing left of the network except resistors (and possibly dependent sources),  $v_{2a}$  must be linearly dependent on  $i_2$  without offsets. In other words, the ratio  $v_{2a}/i_2$  is a pure resistance, a Thévenin-equivalent output resistance:

$$R_{\text{Thout}} = \frac{v_{2a}}{i_2}. \quad (15.114)$$

Then we set  $i_1$  and  $i_2$  to zero, leave the internal sources active, as in Figure 15.37c, and measure  $v_{2b} = v_{2oc}$ , (this is what we previously called the open-circuit voltage). Finally we set  $i_2$  and the internal sources to zero, leaving  $i_1$  active, and measure  $v_{2c}$ , which must be linearly dependent on  $i_1$ , and hence can be written as

$$v_{2c} = i_1 R_{21}. \quad (15.115)$$

This is clearly a *dependent source* relationship: an output voltage dependent on an input current. Now by superposition, the total output voltage is the sum of these three terms:

$$v_2 = i_1 R_{21} + i_2 R_{\text{Thout}} + v_{2oc}. \quad (15.116)$$

A completely analogous argument yields for the input terminals:

$$v_1 = i_1 R_{\text{Thin}} + i_2 R_{12} + v_{1oc} \quad (15.117)$$

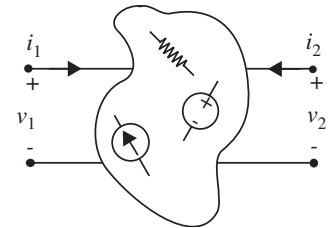


FIGURE 15.36 Linear two-port network.

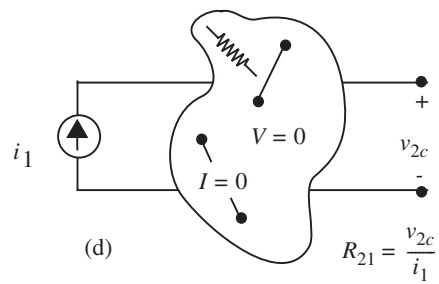
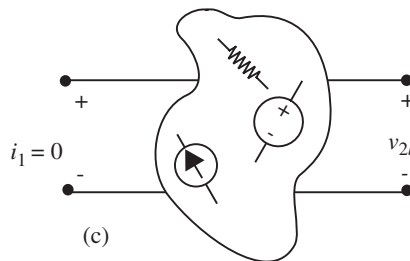
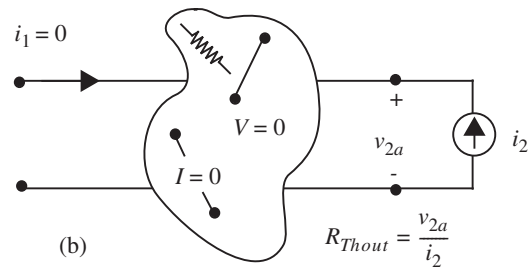
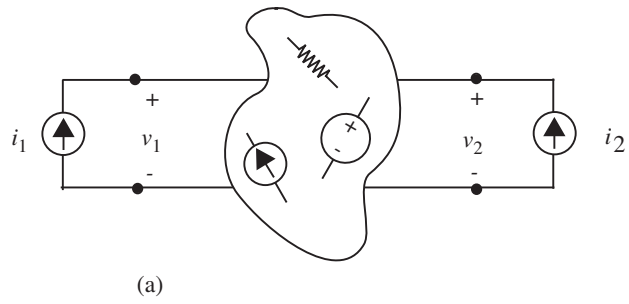
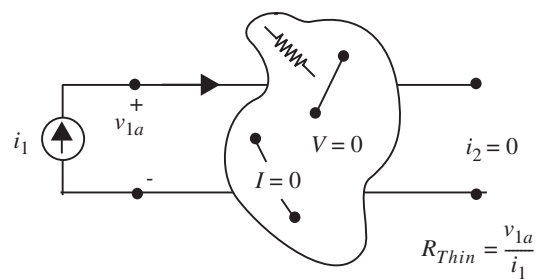
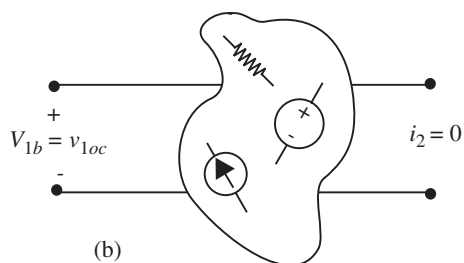


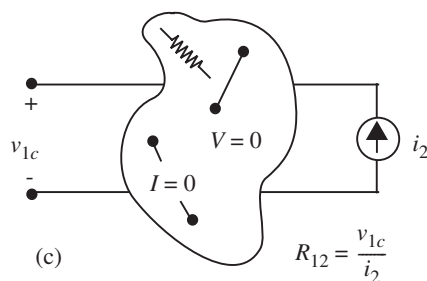
FIGURE 15.37 Two-port calculations.



(a)



(b)



(c)

FIGURE 15.38 Two-port input calculations.

where  $R_{Thin}$ ,  $v_{1oc}$ , and  $R_{12}$  are measured or calculated using the subcircuits in Figures 15.38a, 15.38b, and 15.38c, respectively.

Equations 15.116 and 15.117 taken together, are a complete representation of the network as viewed from the two terminal pairs or two ports. It is common practice in linear network theory to assume that there are no independent sources inside the network. In this case a rather simple generalization of the Thévenin equivalent circuit emerges. Equations 15.116 and 15.117 simplify to

$$v_1 = i_1 R_{Thin} + i_2 R_{12} \quad (15.118)$$

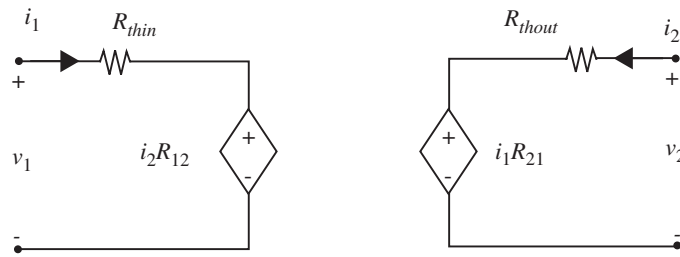
$$v_2 = i_1 R_{21} + i_2 R_{Thout} \quad (15.119)$$

and a simple circuit interpretation is now apparent. The term  $i_2 R_{12}$  in the equation for the input port, Equation 15.118, is a voltage, dependent on the current at the output. That is, it is a dependent voltage source, under the control of  $i_2$ . The first term in Equation 15.118 is the Thévenin input resistance. Hence the equation can be represented in circuit form by the left half of Figure 15.39a. The expression for the output port, Equation 15.119 has similar structure, except the role of input and output variables have been reversed. Hence the right half of Figure 15.39a. The circuits and equations for calculating the four parameters (called  $z$  parameters in linear network theory) are given in Figures 15.37b and 15.37d, and Figures 15.38a and 15.38c.

If we had chosen to drive the two-port with two voltage sources, rather than two current sources as in Figure 15.37, then from Section 3.6.2, the two-port version of the Norton equivalent would have emerged. The equations analogous to Equations 15.118 and 15.119 are

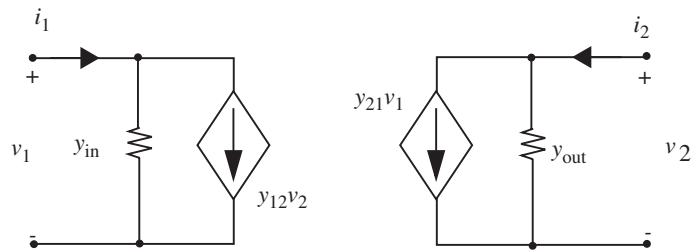
$$i_1 = y_{in} v_1 + y_{12} v_2 \quad (15.120)$$

$$i_2 = y_{21} v_1 + y_{out} v_2 \quad (15.121)$$



(a)  $z$  parameter model

FIGURE 15.39  $z$  and  $y$  parameter models.



(b)  $y$  parameter model

where the  $Y$  terms are conductances for resistive circuits. The circuit equivalent, called the  $y$  parameter model, is shown in Figure 15.39b. The expressions for each of the  $y$  parameters are readily derivable from Equations 15.120 and 15.121, or from first principles, as in Figures 15.37 and 15.38, or by a linear transformation on the  $z$  parameters.

Two other representations, the  $g$  parameters and the  $h$  parameters, arise if one excites the two-port with a voltage source at one port and a current source at the other. All four representations are related by linear transformations.

It is helpful to re-examine the calculation of Op Amp input and output resistance in Section 15.4 from the more general two-port point of view of this section. Because in Figures 15.11 and 15.12 we used a test voltage source at the input and a test current source at the output, we in fact were calculating the  $g$ -parameters, defined in Figure 15.40a. To complete the calculation, we assume that in the Op Amp circuit, Figure 15.12, the *reverse* signal flow through the circuit is negligibly small. Hence  $g_{12}$  in Figure 15.40a is zero. Also, from Figure 15.12 the forward dependent source  $g_{21}$  is approximately  $A$ , if we neglect the drop in  $r_t$  caused by current through  $R_f$ . On this basis the  $g$ -parameter representation for the inverting Op Amp connection is as shown in Figure 15.40b, assuming  $R_s$  is external to this model.

It is now (finally) possible to justify the omission of the  $\pm 12$  V power supplies in all calculations in this chapter. In terms of a two-port model, the

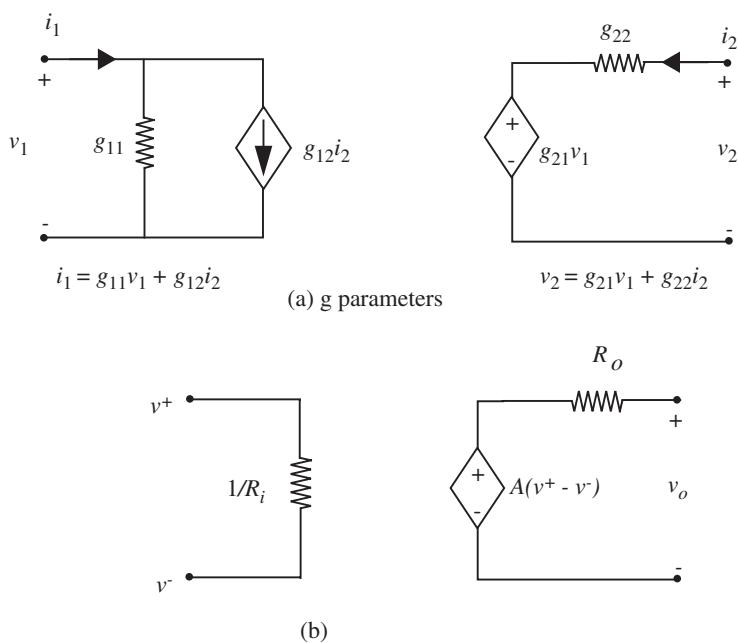


FIGURE 15.40  $g$  parameter model for inverting Op Amp.

power supplies would produce no measurable voltage or current at either the input or the output of the circuit, because of the balanced nature of the circuit in the active region. Hence inclusion of the power supplies would not change the model parameters we have just derived, so it is correct to neglect these supplies in all Op Amp active-region calculations.