

Nonlinear Fiber Optics

Fourth Edition

Nonlinear Fiber Optics

Fourth Edition

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*In the memory of my mother and
for Anne, Sipra, Caroline, and Claire*

Contents

| | |
|---|-----------|
| Preface | xv |
| 1 Introduction | 1 |
| 1.1 Historical Perspective | 1 |
| 1.2 Fiber Characteristics | 3 |
| 1.2.1 Material and Fabrication | 4 |
| 1.2.2 Fiber Losses | 5 |
| 1.2.3 Chromatic Dispersion | 6 |
| 1.2.4 Polarization-Mode Dispersion | 11 |
| 1.3 Fiber Nonlinearities | 13 |
| 1.3.1 Nonlinear Refraction | 14 |
| 1.3.2 Stimulated Inelastic Scattering | 15 |
| 1.3.3 Importance of Nonlinear Effects | 17 |
| 1.4 Overview | 18 |
| Problems | 20 |
| References | 21 |
| 2 Pulse Propagation in Fibers | 25 |
| 2.1 Maxwell's Equations | 25 |
| 2.2 Fiber Modes | 27 |
| 2.2.1 Eigenvalue Equation | 28 |
| 2.2.2 Single-Mode Condition | 29 |
| 2.2.3 Characteristics of the Fundamental Mode | 30 |
| 2.3 Pulse-Propagation Equation | 31 |
| 2.3.1 Nonlinear Pulse Propagation | 32 |
| 2.3.2 Higher-Order Nonlinear Effects | 36 |
| 2.4 Numerical Methods | 41 |
| 2.4.1 Split-Step Fourier Method | 41 |
| 2.4.2 Finite-Difference Methods | 45 |
| Problems | 46 |
| References | 47 |

| | | |
|----------|---|-----------|
| 3 | Group-Velocity Dispersion | 51 |
| 3.1 | Different Propagation Regimes | 51 |
| 3.2 | Dispersion-Induced Pulse Broadening | 53 |
| 3.2.1 | Gaussian Pulses | 54 |
| 3.2.2 | Chirped Gaussian Pulses | 56 |
| 3.2.3 | Hyperbolic Secant Pulses | 58 |
| 3.2.4 | Super-Gaussian Pulses | 58 |
| 3.2.5 | Experimental Results | 61 |
| 3.3 | Third-Order Dispersion | 62 |
| 3.3.1 | Evolution of Chirped Gaussian Pulses | 63 |
| 3.3.2 | Broadening Factor | 65 |
| 3.3.3 | Arbitrary-Shape Pulses | 67 |
| 3.3.4 | Ultrashort-Pulse Measurements | 69 |
| 3.4 | Dispersion Management | 71 |
| 3.4.1 | GVD-Induced Limitations | 71 |
| 3.4.2 | Dispersion Compensation | 73 |
| 3.4.3 | Compensation of Third-Order Dispersion | 74 |
| | Problems | 76 |
| | References | 77 |
| 4 | Self-Phase Modulation | 79 |
| 4.1 | SPM-Induced Spectral Changes | 79 |
| 4.1.1 | Nonlinear Phase Shift | 80 |
| 4.1.2 | Changes in Pulse Spectra | 82 |
| 4.1.3 | Effect of Pulse Shape and Initial Chirp | 85 |
| 4.1.4 | Effect of Partial Coherence | 87 |
| 4.2 | Effect of Group-Velocity Dispersion | 89 |
| 4.2.1 | Pulse Evolution | 90 |
| 4.2.2 | Broadening Factor | 91 |
| 4.2.3 | Optical Wave Breaking | 94 |
| 4.2.4 | Experimental Results | 97 |
| 4.2.5 | Effect of Third-Order Dispersion | 98 |
| 4.2.6 | SPM Effects in Fiber Amplifiers | 100 |
| 4.3 | Semianalytic Techniques | 102 |
| 4.3.1 | Moment Method | 102 |
| 4.3.2 | Variational Method | 103 |
| 4.3.3 | Specific Analytic Solutions | 104 |
| 4.4 | Higher-Order Nonlinear Effects | 106 |
| 4.4.1 | Self-Steepening | 107 |
| 4.4.2 | Effect of GVD on Optical Shocks | 109 |
| 4.4.3 | Intrapulse Raman Scattering | 111 |
| | Problems | 114 |
| | References | 116 |

| | | |
|----------|---|------------|
| 5 | Optical Solitons | 120 |
| 5.1 | Modulation Instability | 120 |
| 5.1.1 | Linear Stability Analysis | 121 |
| 5.1.2 | Gain Spectrum | 122 |
| 5.1.3 | Experimental Results | 124 |
| 5.1.4 | Ultrashort Pulse Generation | 125 |
| 5.1.5 | Impact on Lightwave Systems | 127 |
| 5.2 | Fiber Solitons | 129 |
| 5.2.1 | Inverse Scattering Method | 130 |
| 5.2.2 | Fundamental Soliton | 132 |
| 5.2.3 | Higher-Order Solitons | 134 |
| 5.2.4 | Experimental Confirmation | 136 |
| 5.2.5 | Soliton Stability | 137 |
| 5.3 | Other Types of Solitons | 140 |
| 5.3.1 | Dark Solitons | 140 |
| 5.3.2 | Dispersion-Managed Solitons | 144 |
| 5.3.3 | Bistable Solitons | 144 |
| 5.4 | Perturbation of Solitons | 146 |
| 5.4.1 | Perturbation Methods | 146 |
| 5.4.2 | Fiber Losses | 147 |
| 5.4.3 | Soliton Amplification | 149 |
| 5.4.4 | Soliton Interaction | 152 |
| 5.5 | Higher-Order Effects | 156 |
| 5.5.1 | Moment Equations for Pulse Parameters | 156 |
| 5.5.2 | Third-Order Dispersion | 158 |
| 5.5.3 | Self-Steepening | 160 |
| 5.5.4 | Intrapulse Raman Scattering | 162 |
| 5.5.5 | Propagation of Femtosecond Pulses | 167 |
| | Problems | 169 |
| | References | 170 |
| 6 | Polarization Effects | 177 |
| 6.1 | Nonlinear Birefringence | 177 |
| 6.1.1 | Origin of Nonlinear Birefringence | 178 |
| 6.1.2 | Coupled-Mode Equations | 180 |
| 6.1.3 | Elliptically Birefringent Fibers | 181 |
| 6.2 | Nonlinear Phase Shift | 182 |
| 6.2.1 | Nondispersive XPM | 182 |
| 6.2.2 | Optical Kerr Effect | 183 |
| 6.2.3 | Pulse Shaping | 187 |
| 6.3 | Evolution of Polarization State | 189 |
| 6.3.1 | Analytic Solution | 189 |
| 6.3.2 | Poincaré-Sphere Representation | 191 |
| 6.3.3 | Polarization Instability | 194 |
| 6.3.4 | Polarization Chaos | 196 |
| 6.4 | Vector Modulation Instability | 197 |

| | | |
|----------|--|------------|
| 6.4.1 | Low-Birefringence Fibers | 197 |
| 6.4.2 | High-Birefringence Fibers | 200 |
| 6.4.3 | Isotropic Fibers | 202 |
| 6.4.4 | Experimental Results | 203 |
| 6.5 | Birefringence and Solitons | 206 |
| 6.5.1 | Low-Birefringence Fibers | 206 |
| 6.5.2 | High-Birefringence Fibers | 207 |
| 6.5.3 | Soliton-Dragging Logic Gates | 211 |
| 6.5.4 | Vector Solitons | 212 |
| 6.6 | Random Birefringence | 213 |
| 6.6.1 | Polarization-Mode Dispersion | 214 |
| 6.6.2 | Vector Form of the NLS Equation | 215 |
| 6.6.3 | Effects of PMD on Solitons | 216 |
| | Problems | 220 |
| | References | 221 |
| 7 | Cross-Phase Modulation | 226 |
| 7.1 | XPM-Induced Nonlinear Coupling | 227 |
| 7.1.1 | Nonlinear Refractive Index | 227 |
| 7.1.2 | Coupled NLS Equations | 228 |
| 7.2 | XPM-Induced Modulation Instability | 229 |
| 7.2.1 | Linear Stability Analysis | 229 |
| 7.2.2 | Experimental Results | 232 |
| 7.3 | XPM-Paired Solitons | 233 |
| 7.3.1 | Bright–Dark Soliton Pair | 233 |
| 7.3.2 | Bright–Gray Soliton Pair | 234 |
| 7.3.3 | Periodic Solutions | 235 |
| 7.3.4 | Multiple Coupled NLS Equations | 237 |
| 7.4 | Spectral and Temporal Effects | 238 |
| 7.4.1 | Asymmetric Spectral Broadening | 239 |
| 7.4.2 | Asymmetric Temporal Changes | 244 |
| 7.4.3 | Higher-Order Nonlinear Effects | 247 |
| 7.5 | Applications of XPM | 248 |
| 7.5.1 | XPM-Induced Pulse Compression | 248 |
| 7.5.2 | XPM-Induced Optical Switching | 251 |
| 7.5.3 | XPM-Induced Nonreciprocity | 252 |
| 7.6 | Polarization Effects | 254 |
| 7.6.1 | Vector Theory of XPM | 254 |
| 7.6.2 | Polarization Evolution | 255 |
| 7.6.3 | Polarization-Dependent Spectral Broadening | 257 |
| 7.6.4 | Pulse Trapping and Compression | 260 |
| 7.6.5 | XPM-Induced Wave Breaking | 262 |
| 7.7 | XPM Effects in Birefringent Fibers | 264 |
| 7.7.1 | Fibers with Low Birefringence | 264 |
| 7.7.2 | Fibers with High Birefringence | 267 |
| | Problems | 268 |

| | |
|--|------------|
| References | 270 |
| 8 Stimulated Raman Scattering | 274 |
| 8.1 Basic Concepts | 274 |
| 8.1.1 Raman-Gain Spectrum | 275 |
| 8.1.2 Raman Threshold | 276 |
| 8.1.3 Coupled Amplitude Equations | 279 |
| 8.1.4 Effect of Four-Wave Mixing | 281 |
| 8.2 Quasi-Continuous SRS | 283 |
| 8.2.1 Single-Pass Raman Generation | 283 |
| 8.2.2 Raman Fiber Lasers | 285 |
| 8.2.3 Raman Fiber Amplifiers | 288 |
| 8.2.4 Raman-Induced Crosstalk | 292 |
| 8.3 SRS with Short Pump Pulses | 294 |
| 8.3.1 Pulse-Propagation Equations | 294 |
| 8.3.2 Nondispersive Case | 295 |
| 8.3.3 Effects of GVD | 297 |
| 8.3.4 Experimental Results | 300 |
| 8.3.5 Synchronously Pumped Raman Lasers | 304 |
| 8.3.6 Short-Pulse Raman Amplification | 305 |
| 8.4 Soliton Effects | 306 |
| 8.4.1 Raman Solitons | 306 |
| 8.4.2 Raman Soliton Lasers | 311 |
| 8.4.3 Soliton-Effect Pulse Compression | 313 |
| 8.5 Polarization Effects | 315 |
| 8.5.1 Vector Theory of Raman Amplification | 315 |
| 8.5.2 PMD Effects on Raman Amplification | 319 |
| Problems | 321 |
| References | 322 |
| 9 Stimulated Brillouin Scattering | 329 |
| 9.1 Basic Concepts | 329 |
| 9.1.1 Physical Process | 330 |
| 9.1.2 Brillouin-Gain Spectrum | 330 |
| 9.2 Quasi-CW SBS | 333 |
| 9.2.1 Brillouin Threshold | 333 |
| 9.2.2 Polarization Effects | 334 |
| 9.2.3 Techniques for Controlling the SBS Threshold | 335 |
| 9.2.4 Experimental Results | 338 |
| 9.3 Brillouin Fiber Amplifiers | 340 |
| 9.3.1 Gain Saturation | 341 |
| 9.3.2 Amplifier Design and Applications | 342 |
| 9.4 SBS Dynamics | 344 |
| 9.4.1 Coupled Amplitude Equations | 345 |
| 9.4.2 SBS with Q-Switched Pulses | 346 |
| 9.4.3 SBS-Induced Index Changes | 350 |

| | | |
|-----------|--|------------|
| 9.4.4 | Relaxation Oscillations | 352 |
| 9.4.5 | Modulation Instability and Chaos | 354 |
| 9.5 | Brillouin Fiber Lasers | 356 |
| 9.5.1 | CW Operation | 356 |
| 9.5.2 | Pulsed Operation | 360 |
| | Problems | 362 |
| | References | 363 |
| 10 | Four-Wave Mixing | 368 |
| 10.1 | Origin of Four-Wave Mixing | 368 |
| 10.2 | Theory of Four-Wave Mixing | 370 |
| 10.2.1 | Coupled Amplitude Equations | 371 |
| 10.2.2 | Approximate Solution | 371 |
| 10.2.3 | Effect of Phase Matching | 373 |
| 10.2.4 | Ultrafast Four-Wave Mixing | 374 |
| 10.3 | Phase-Matching Techniques | 376 |
| 10.3.1 | Physical Mechanisms | 376 |
| 10.3.2 | Phase Matching in Multimode Fibers | 377 |
| 10.3.3 | Phase Matching in Single-Mode Fibers | 380 |
| 10.3.4 | Phase Matching in Birefringent Fibers | 383 |
| 10.4 | Parametric Amplification | 387 |
| 10.4.1 | Review of Early Work | 387 |
| 10.4.2 | Gain Spectrum and Its Bandwidth | 389 |
| 10.4.3 | Single-Pump Configuration | 391 |
| 10.4.4 | Dual-Pump Configuration | 394 |
| 10.4.5 | Effects of Pump Depletion | 399 |
| 10.5 | Polarization Effects | 401 |
| 10.5.1 | Vector Theory of Four-Wave Mixing | 401 |
| 10.5.2 | Polarization Dependence of Parametric Gain | 403 |
| 10.5.3 | Linearly and Circularly Polarized Pumps | 405 |
| 10.5.4 | Effect of Residual Fiber Birefringence | 408 |
| 10.6 | Applications of Four-Wave Mixing | 411 |
| 10.6.1 | Parametric Oscillators | 412 |
| 10.6.2 | Ultrafast Signal Processing | 413 |
| 10.6.3 | Quantum Noise and Correlation | 415 |
| | Problems | 417 |
| | References | 418 |
| 11 | Highly Nonlinear Fibers | 424 |
| 11.1 | Nonlinear Parameter | 424 |
| 11.1.1 | Units and Values of n_2 | 425 |
| 11.1.2 | SPM-Based Techniques | 426 |
| 11.1.3 | XPM-Based Technique | 429 |
| 11.1.4 | FWM-Based Technique | 430 |
| 11.1.5 | Variations in n_2 Values | 431 |
| 11.2 | Fibers with Silica Cladding | 434 |

| | |
|--|------------|
| Contents | xiii |
| 11.3 Tapered Fibers with Air Cladding | 436 |
| 11.4 Microstructured Fibers | 440 |
| 11.5 Non-Silica Fibers | 444 |
| Problems | 448 |
| References | 449 |
| 12 Novel Nonlinear Phenomena | 453 |
| 12.1 Intrapulse Raman Scattering | 453 |
| 12.1.1 Enhanced RIFS and Wavelength Tuning | 454 |
| 12.1.2 Nonsolitonic Radiation | 457 |
| 12.1.3 Effects of Birefringence | 459 |
| 12.1.4 Suppression of Raman-Induced Frequency Shifts | 461 |
| 12.2 Four-Wave Mixing | 464 |
| 12.2.1 FWM in Highly Nonlinear Fibers | 464 |
| 12.2.2 Effects of Fiber Birefringence | 467 |
| 12.3 Supercontinuum Generation | 469 |
| 12.3.1 Pumping with Picosecond Pulses | 470 |
| 12.3.2 Continuous-Wave Pumping | 474 |
| 12.3.3 Pumping with Femtosecond Pulses | 475 |
| 12.4 Temporal and Spectral Evolution | 477 |
| 12.4.1 Numerical Modeling of Supercontinuum | 477 |
| 12.4.2 Soliton Fission and Nonsolitonic Radiation | 480 |
| 12.4.3 Effects of Cross-Phase Modulation | 484 |
| 12.4.4 Polarization Effects | 488 |
| 12.4.5 Coherence Properties of a Supercontinuum | 492 |
| 12.5 Harmonic Generation | 495 |
| 12.5.1 Second-Harmonic Generation | 495 |
| 12.5.2 Third-Harmonic Generation | 502 |
| Problems | 506 |
| References | 507 |
| A System of Units | 514 |
| B Numerical Code for the NLS Equation | 516 |
| C List of Acronyms | 519 |
| Index | 521 |

Preface

Since the publication of the first edition of this book in 1989, the field of *nonlinear fiber optics* has remained an active area of research and has thus continued to grow at a rapid pace. During the 1990s, a major factor behind such a sustained growth was the advent of fiber amplifiers and lasers, made by doping silica fibers with rare-earth materials such as erbium and ytterbium. Erbium-doped fiber amplifiers revolutionized the design of fiber-optic communication systems, including those making use of optical solitons, whose very existence stems from the presence of nonlinear effects in optical fibers. Optical amplifiers permit propagation of lightwave signals over thousands of kilometers as they can compensate for all losses encountered by the signal in the optical domain. At the same time, fiber amplifiers enable the use of massive wavelength-division multiplexing, a technique that led by 1999 to the development of lightwave systems with capacities exceeding 1 Tb/s. Nonlinear fiber optics plays an important role in the design of such high-capacity lightwave systems. In fact, an understanding of various nonlinear effects occurring inside optical fibers is almost a prerequisite for a lightwave-system designer.

Starting around 2000, a new development occurred in the field of *nonlinear fiber optics* that changed the focus of research and has led to a number of advances and novel applications in recent years. Several kinds of new fibers, classified as highly nonlinear fibers, have been developed. They are referred to with names such as microstructured fibers, holey fibers, or photonic crystal fibers, and share the common property that a relatively narrow core is surrounded by a cladding containing a large number of air holes. The nonlinear effects are enhanced dramatically in such fibers to the extent that they can be observed even when the fiber is only a few centimeters long. Their dispersive properties are also quite different compared with those of conventional fibers developed for telecommunication applications. Because of these changes, microstructured fibers exhibit a variety of novel nonlinear effects that are finding applications in fields as diverse as optical coherence tomography and high-precision frequency metrology.

The fourth edition is intended to bring the book up-to-date so that it remains a unique source of comprehensive coverage on the subject of nonlinear fiber optics. It retains most of the material that appeared in the third edition. However, an attempt was made to include recent research results on all topics relevant to the field of nonlinear fiber optics. Such an ambitious objective has increased the size of the book considerably. Two new chapters, Chapters 11 and 12, have been added to cover the recent research advances. Chapter 11 describes the properties of highly nonlinear fibers, and the novel nonlinear effects that have been observed since 2000 in such fibers are cov-

ered in Chapter 12. Although all other chapters have been updated, Chapters 8 to 10 required major additions because of the recent advances in the research areas covered by them. For example, polarization issues have become increasingly more important for stimulated Raman scattering and four-wave mixing, and thus they are discussed in detail in Chapters 8 and 10. It is important that students learn about such polarization effects in a course devoted to nonlinear fiber optics.

The potential readership is likely to consist of senior undergraduate students, graduate students enrolled in the M.S. and Ph.D. degree programs, engineers and technicians involved with the fiber-optics industry, and scientists working in the fields of fiber optics and optical communications. This revised edition should continue to be a useful text for graduate and senior-level courses dealing with nonlinear optics, fiber optics, or optical communications that are designed to provide mastery of the fundamental aspects. Some universities may even opt to offer a high-level graduate course devoted to solely nonlinear fiber optics. The problems provided at the end of each chapter should be useful to instructors of such a course.

Many individuals have contributed, either directly or indirectly, to the completion of the third edition. I am thankful to all of them, especially to my graduate students whose curiosity and involvement led to several improvements. Several of my colleagues have helped me in preparing the fourth edition. I especially thank F. Omenetto, Q. Lin, and F. Yaman for reading drafts of selected chapters and for making helpful suggestions. I am grateful to many readers for their occasional feedback. Last, but not least, I thank my wife, Anne, and my daughters, Sipra, Caroline, and Claire, for understanding why I needed to spend many weekends on the book instead of spending time with them.

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