# Mathematical Statistics with Applications

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*Dedicated to our families:*
*Usha, Vikas, Vilas, and Varsha Ramachandran*
*and*
*Debbie, Matthew, Jonathan, and Maria Tsokos*

# Contents

# Preface

This textbook is of an interdisciplinary nature and is designed for a two- or one-semester course in probability and statistics, with basic calculus as a prerequisite. The book is primarily written to give a sound theoretical introduction to statistics while emphasizing applications. If teaching statistics is the main purpose of a two-semester course in probability and statistics, this textbook covers all the probability concepts necessary for the theoretical development of statistics in two chapters, and goes on to cover all major aspects of statistical theory in two semesters, instead of only a portion of statistical concepts. What is more, using the optional section on computer examples at the end of each chapter, the student can also simultaneously learn to utilize statistical software packages for data analysis. It is our aim, without sacrificing any rigor, to encourage students to apply the theoretical concepts they have learned. There are many examples and exercises concerning diverse application areas that will show the pertinence of statistical methodology to solving real-world problems. The examples with statistical software and projects at the end of the chapters will provide good perspective on the usefulness of statistical methods. To introduce the students to modern and increasingly popular statistical methods, we have introduced separate chapters on Bayesian analysis and empirical methods.

One of the main aims of this book is to prepare advanced undergraduates and beginning graduate students in the theory of statistics with emphasis on interdisciplinary applications. The audience for this course is regular full-time students from mathematics, statistics, engineering, physical sciences, business, social sciences, materials science, and so forth. Also, this textbook is suitable for people who work in industry and in education as a reference book on introductory statistics for a good theoretical foundation with clear indication of how to use statistical methods. Traditionally, one of the main prerequisites for this course is a semester of the introduction to probability theory. A working knowledge of elementary (descriptive) statistics is also a must. In schools where there is no statistics major, imposing such a background, in addition to calculus sequence, is very difficult. Most of the present books available on this subject contain full one-semester material for probability and then, based on those results, continue on to the topics in statistics. Also, some of these books include in their subject matter only the theory of statistics, whereas others take the cookbook approach of covering the mechanics. Thus, even with two full semesters of work, many basic and important concepts in statistics are never covered. This book has been written to remedy this problem. We fuse together both concepts in order for students to gain knowledge of the theory and at the same time develop the expertise to use their knowledge in real-world situations.

Although statistics is a very applied subject, there is no denying that it is also a very abstract subject. The purpose of this book is to present the subject matter in such a way that anyone with exposure to basic calculus can study statistics without spending two semesters of background preparation. To prepare students, we present an optional review of the elementary (descriptive) statistics in Chapter 1. All the probability material required to learn statistics is covered in two chapters. Students with a probability background can either review or skip the first three chapters. It is also our belief that any statistics course is not complete without exposure to computational techniques. At

the end of each chapter, we give some examples of how to use Minitab, SPSS, and SAS to statistically analyze data. Also, at the end of each chapter, there are projects that will enhance the knowledge and understanding of the materials covered in that chapter. In the chapter on the empirical methods, we present some of the modern computational and simulation techniques, such as bootstrap, jackknife, and Markov chain Monte Carlo methods. The last chapter summarizes some of the steps necessary to apply the material covered in the book to real-world problems. The first eight chapters have been class tested as a one-semester course for more than 3 years with five different professors teaching. The audience was junior- and senior-level undergraduate students from many disciplines who had had two semesters of calculus, most of them with no probability or statistics background. The feedback from the students and instructors was very positive. Recommendations from the instructors and students were very useful in improving the style and content of the book.

## AIM AND OBJECTIVE OF THE TEXTBOOK

This textbook provides a calculus-based coverage of statistics and introduces students to methods of theoretical statistics and their applications. It assumes no prior knowledge of statistics or probability theory, but does require calculus. Most books at this level are written with elaborate coverage of probability. This requires teaching one semester of probability and then continuing with one or two semesters of statistics. This creates a particular problem for non-statistics majors from various disciplines who want to obtain a sound background in mathematical statistics and applications. It is our aim to introduce basic concepts of statistics with sound theoretical explanations. Because statistics is basically an interdisciplinary applied subject, we offer many applied examples and relevant exercises from different areas. Knowledge of using computers for data analysis is desirable. We present examples of solving statistical problems using Minitab, SPSS, and SAS.

## FEATURES

- During years of teaching, we observed that many students who do well in mathematics courses find it difficult to understand the concept of statistics. To remedy this, we present most of the material covered in the textbook with well-defined step-by-step procedures to solve real problems. This clearly helps the students to approach problem solving in statistics more logically.
- The usefulness of each statistical method introduced is illustrated by several relevant examples.
- At the end of each section, we provide ample exercises that are a good mix of theory and applications.
- In each chapter, we give various projects for students to work on. These projects are designed in such a way that students will start thinking about how to apply the results they learned in the chapter as well as other issues they will need to know for practical situations.
- At the end of the chapters, we include an optional section on computer methods with Minitab, SPSS, and SAS examples with clear and simple commands that the student can use to analyze

data. This will help students to learn how to utilize the standard methods they have learned in the chapter to study real data.

- We introduce many of the modern statistical computational and simulation concepts, such as the jackknife and bootstrap methods, the EM algorithms, and the Markov chain Monte Carlo methods such as the Metropolis algorithm, the Metropolis–Hastings algorithm, and the Gibbs sampler. The Metropolis algorithm was mentioned in *Computing in Science and Engineering* as being among the top 10 algorithms having the "greatest influence on the development and practice of science and engineering in the 20th century."
- We have introduced the increasingly popular concept of Bayesian statistics and decision theory with applications.
- A separate chapter on design of experiments, including a discussion on the Taguchi approach, is included.
- The coverage of the book spans most of the important concepts in statistics. Learning the material along with computational examples will prepare students to understand and utilize software procedures to perform statistical analysis.
- Every chapter contains discussion on how to apply the concepts and what the issues are related to applying the theory.
- A student's solution manual, instructor's manual, and data disk are provided.
- In the last chapter, we discuss some issues in applications to clearly demonstrate in a unified way how to check for many assumptions in data analysis and what steps one needs to follow to avoid possible pitfalls in applying the methods explained in the rest of this textbook.

# Acknowledgments

We express our sincere appreciation to our late colleague, co-worker, and dear friend, Professor A. N. V. Rao, for his helpful suggestions and ideas for the initial version of the subject textbook. In addition, we thank Bong-jin Choi and Yong Xu for their kind assistance in the preparation of the manuscript. Finally, we acknowledge our students at the University of South Florida for their useful comments and suggestions during the class testing of our book. To all of them, we are very thankful.

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# About the Authors

**Kandethody M. Ramachandran** is Professor of Mathematics and Statistics at the University of South Florida. He received his B.S. and M.S. degrees in Mathematics from the Calicut University, India. Later, he worked as a researcher at the Tata Institute of Fundamental Research, Bangalore center, at its Applied Mathematics Division. Dr. Ramachandran got his Ph.D. in Applied Mathematics from Brown University.

His research interests are concentrated in the areas of applied probability and statistics. His research publications span a variety of areas such as control of heavy traffic queues, stochastic delay equations and control problems, stochastic differential games and applications, reinforcement learning methods applied to game theory and other areas, software reliability problems, applications of statistical methods to microarray data analysis, and mathematical finance.

Professor Ramachandran is extensively involved in activities to improve statistics and mathematics education. He is a recipient of the Teaching Incentive Program award at the University of South Florida. He is a member of the MEME Collaborative, which is a partnership among mathematics education, mathematics, and engineering faculty to address issues related to mathematics and mathematics education. He was also involved in the calculus reform efforts at the University of South Florida.

**Chris P. Tsokos** is Distinguished University Professor of Mathematics and Statistics at the University of South Florida. Dr. Tsokos received his B.S. in Engineering Sciences/Mathematics, his M.A. in Mathematics from the University of Rhode Island, and his Ph.D. in Statistics and Probability from the University of Connecticut. Professor Tsokos has also served on the faculties at Virginia Polytechnic Institute and State University and the University of Rhode Island.

Dr. Tsokos's research has extended into a variety of areas, including stochastic systems, statistical models, reliability analysis, ecological systems, operations research, time series, Bayesian analysis, and mathematical and statistical modeling of global warming, among others. He is the author of more than 250 research publications in these areas.

Professor Tsokos is the author of several research monographs and books, including *Random Integral Equations with Applications to Life Sciences and Engineering, Probability Distribution: An Introduction to Probability Theory with Applications, Mainstreams of Finite Mathematics with Applications, Probability with the Essential Analysis, and Applied Probability Bayesian Statistical Methods with Applications to Reliability,* among others.

Dr. Tsokos is the recipient of many distinguished awards and honors, including Fellow of the American Statistical Association, USF Distinguished Scholar Award, Sigma Xi Outstanding Research Award, USF Outstanding Undergraduate Teaching Award, USF Professional Excellence Award, URI Alumni Excellence Award in Science and Technology, Pi Mu Epsilon, and election to the International Statistical Institute, among others.

# Flow Chart

This flow chart gives some options on how to use the book in a one-semester or two-semester course. For a two-semester course, we recommend coverage of the complete textbook. However, Chapters 1, 9, and 14 are optional for both one- and two-semester courses and can be given as reading exercises. For a one-semester course, we suggest the following options: A, B, C, D.