System Dynamics for Engineering Students
To all readers coming across this book,
with friendly consideration.
“Pro captu lectoris habent sua fata libelli.” (It is on the reader’s understanding that the fate of books depends).

Terentianus Maurus, Latin writer

*De litteris, De syllabis, De metris*
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Foreword

This text is a modern treatment of system dynamics and its relation to traditional mechanical engineering problems as well as modern microscale devices and machines. It provides an excellent course of study for students who want to grasp the fundamentals of dynamic systems and it covers a significant amount of material also taught in engineering modeling, systems dynamics, and vibrations, all combined in a dense form. The book is designed as a text for juniors and seniors in aerospace, mechanical, electrical, biomedical, and civil engineering. It is useful for understanding the design and development of micro- and macroscale structures, electric and fluidic systems with an introduction to transduction, and numerous simulations using MATLAB and SIMULINK.

The creation of machines is essentially what much of engineering is all about. Critical to almost all machines imaginable is a transient response, which is fundamental to their functionality and needs to be our primary concern in their design. This might be in the form of changing voltage levels in a sensor, the deflection of a spring supported mass, or the flow of fluid through a device. The phenomena which govern dynamics are not simply its mechanical components but often involve the dynamics of transducers as well, which are often electro-mechanical or fluidic based. This text discusses traditional electro-magnetic type actuators, but also ventures into electrostatics which are the dominant form of actuators in microelectromechanical systems (MEMs).

This book presents an opportunity for introducing dynamic systems to scientists and engineers who are concerned with the engineering of machines both at the micro- and macroscopic scale. Mechanism and movement are considered from the types of springs and joints that are critical to micro-machined, lithographic based devices to traditional models of macroscale electrical, fluidic, and electromechanical systems. The examples discussed and the problems at the end of each chapter have applicability at both scales. In essence this is a more modern treatment of dynamical systems, presenting views of modeling and substructures more consistent with the variety of problems that many engineers will face in the future. Any university with a substantive interest in microscale engineering would do well to consider a course that covers the material herein. Finally, this text lays the foundation and framework for the development of controllers applied to these dynamical systems.

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Preface

Engineering system dynamics is a discipline that focuses on deriving mathematical models based on simplified physical representations of actual systems, such as mechanical, electrical, fluid, or thermal, and on solving the mathematical models (most often consisting of differential equations). The resulting solution (which reflects the system response or behavior) is utilized in design or analysis before producing and testing the actual system. Because dynamic systems are characterized by similar mathematical models, a unitary approach can be used to characterize individual systems pertaining to different fields as well as to consider the interaction of systems from multiple fields as in coupled-field problems.

This book was designed to be utilized as a one-semester system dynamics text for upper-level undergraduate students with emphasis on mechanical, aerospace, or electrical engineering. Comprising important components from these areas, the material should also serve cross-listed courses (mechanical-electrical) at a similar study level. In addition to the printed chapters, the book contains an equal number of chapter extensions that have been assembled into a companion website section; this makes it useful as an introductory text for more advanced courses, such as vibrations, controls, instrumentation, or mechatronics. The book can also be useful in graduate coursework or in individual study as reference material. The material contained in this book most probably exceeds the time allotted for a one-semester course lecture, and therefore topical selection becomes necessary, based on particular instruction emphasis and teaching preferences.

While the book maintains its focus on the classical approach to system dynamics, a new feature of this text is the introduction of examples from compliant mechanisms and micro- and nano-electromechanical systems (MEMS/NEMS), which are recognized as increasingly important application areas. As demonstrated in the book, and for the relatively simple examples that have been selected here, this inclusion can really be treated within the regular system dynamics lumped-parameter (pointlike) modeling; therefore, the students not so familiar with these topics should face no major comprehension difficulties. Another central point of this book is proposing a chapter on coupled-field (or multiple-field) systems, whereby interactions between the mechanical, electrical, fluid, and thermal fields occur and generate means for actuation or sensing applications, such as in piezoelectric, electromagnetomechanical, or electrothermomechanical applications.

Another key objective was to assemble a text that is structured, balanced, cohesive, and providing a fluent and logical sequence of topics along the following lines:

1. It starts from simple objects (the components), proceeds to the objects’ assembly (the individual system), and arrives at the system interaction level (coupled-field systems).
2. It uses modeling and solution techniques that are familiar from other disciplines, such as physics or ordinary differential equations, and subsequently introduces new modeling and solution procedures.

3. It provides a rather even coverage (space) to each book chapter.

4. While various chapter structures are possible in a system dynamics text, this book proposes a sequence that was intended to be systematic and consistent with the logical structure and progression of the presented material.

As such, the book begins with an introductory Chapter 1, which offers an overview of the main aspects of a system dynamics course for engineering students. The next four chapters—Chapters 2, 3, 4, and 5—are dedicated, in order, to mechanical (Chapters 2 and 3), electrical (Chapter 4), and fluid and thermal (Chapter 5) system modeling. They contain basic information on components, systems, and the principal physical and mathematical tools that make it possible to model a dynamic system and determine its solution.

Once the main engineering dynamic systems have been studied, Chapter 6 presents the Laplace transform technique, a mathematical tool that allows simplifying the differential equation solution process for any of the individual systems. This chapter is directly connected to the next segment of the book, containing Chapters 7, 8, and 9. Chapter 7 introduces the transfer function approach, which facilitates finding the time-domain response (solution) of a dynamic system after the corresponding unknowns have been determined in the Laplace domain. The complex impedance, which is actually a transfer function connecting the Laplace-transformed input and output of a specific system element, is also introduced and thoroughly treated in this chapter. Chapter 8 studies the state space modeling and solution approach, which is also related to the Laplace transform of Chapter 6 and the transfer function of Chapter 7. Chapter 9 discusses modeling system dynamics in the frequency domain by means of the sinusoidal (harmonic) transfer function.

Chapter 10 analyzes coupled-field (or multiple-field) dynamic systems, which are combinations of mechanical, electrical, magnetic, piezoelectric, fluid, or thermal systems. In this chapter, dynamic models are formulated and solved by means of the procedures studied in previous chapters. Because of the partial and natural overlap between system dynamics and controls, the majority of textbooks on either of these two areas contain coverage of material from the adjoining domain. Consistent with this approach, the companion website contains one chapter, Chapter 11, on introductory controls, where basic time-domain and frequency-domain topics are addressed.

The book also includes four appendices: Appendix A presents the solutions to linear differential equations with constant coefficients, Appendix B is a review of matrix algebra, Appendix C contains basic MATLAB® commands that have been used throughout this text, and Appendix D gives a summary of equations for calculating deformations, strains, and stresses of deformable mechanical components.
The book introduces several topics that are new to engineering system dynamics, as highlighted here:

Chapter 3, Mechanical Systems II
- Lumped-parameter inertia properties of basic compliant (flexible) members.
- Lumped-parameter dynamic modeling of simple compliant mechanical microsystems.
- Mass detection in MEMS by the resonance shift method.

Chapter 4, Electrical Systems
- Capacitive sensing and actuation in MEMS.

Chapter 5, Fluid and Thermal Systems
- Comprehensive coverage of liquid, pneumatic, and thermal systems.
- Natural response of fluid systems.

Chapters 3, 4, and 5
- Notion of degrees of freedom (DOFs) for defining the system configuration of dynamic systems.
- Application of the energy method to calculate the natural frequencies of single- and multiple-DOF conservative systems.
- Utilization of the vector-matrix method to calculate the eigenvalues either analytically or using MATLAB®.

Chapter 6, Laplace Transform
- Linear ordinary differential equations with time-varying coefficients.
- Use of the convolution theorem to solve integral and integral-differential equations.
- Time-domain system identification.

Chapter 7, Transfer Function Approach
- Extension of the single-input, single-output (SISO) transfer function approach to multiple-input, multiple-output (MIMO) systems by means of the transfer function matrix.
- Application of the transfer function approach to solve the forced and the free responses with nonzero initial conditions.
- Systematic introduction and comprehensive application of the complex impedance approach to electrical, mechanical, and fluid and thermal systems.
- MATLAB® conversion between zero-pole-gain (zpk) and transfer function (tf) models.
Chapter 8, State Space Approach

- Treatment of the descriptor state equation.
- Application of the state space approach to solve the forced and free responses with nonzero initial conditions.
- MATLAB® conversion between state space (ss) models and zpk or tf models.

Chapter 9, Frequency-Domain Approach

- State space approach and the frequency domain.
- MATLAB® conversion from zpk, tf, or ss models to frequency response data (frd) models.
- Steady-state response of cascading unloading systems.
- Mechanical and electrical filters.

Chapter 10, Coupled-Field Systems

- Formulation of the coupled-field (multiple-field) problem.
- Principles and applications of sensing and actuation.
- Strain gauge and Wheatstone bridge circuits for measuring mechanical deformation.
- Applications of electromagnetomechanical system dynamics.
- Principles and applications of piezoelectric coupling with mechanical deformable systems.
- Nonlinear electrothermomechanical coupling.

Within this printed book’s space limitations, attention has been directed at generating a balanced coverage of minimally necessary theory presentation, solved examples, and end-of-chapter proposed problems. Whenever possible, examples are solved analytically, using hand calculation, so that any mathematical software can be used in conjunction with any model developed here. The book is not constructed on MATLAB®, but it uses this software to determine numerical solutions and to solve symbolically mathematical models too involved to be obtained by hand. It would be difficult to overlook the built-in capabilities of MATLAB®’s tool boxes (really programs within the main program, such as the ones designed for symbolic calculation or controls), which many times use one-line codes to solve complex system dynamics problems and which have been used in this text. Equally appealing solutions to system dynamics problems are the ones provided by Simulink®, a graphical user interface program built atop MATLAB®, and applications are included in almost all the chapters of solved and proposed exercises that can be approached by Simulink®.

Through a companion website, the book comprises more ancillary support material, including companion book chapters with extensions to the printed book (with more advanced topics, details of the printed book material, and additional solved examples, this section could be of interest and assistance to both the instructor and the student). The sign ☀️ is used in the printed book to signal associated
material on the companion website. The companion website chapters address the following topics:

Chapter 3, Mechanical Systems II
- Details on lumped-parameter stiffness and inertia properties of basic compliant (flexible) members.
- Additional springs for macro and micro system applications.
- Pulley systems.

Chapter 4, Electrical Systems
- Equivalent resistance method.
- Transformer elements and electrical circuits.
- Operational amplifier circuits as analog computers.

Chapter 5, Fluid and Thermal Systems
- Capacitance of compressible pipes.

Chapter 6, Laplace Transform
- Thorough presentation of the partial-fraction expansion.
- Application of the Laplace transform method to calculating natural frequencies.
- Method of integrating factor and the Laplace transform.

Chapter 7, Transfer Function Approach
- System identification from time response.
- Cascading loading systems.
- Mutual inductance impedance.
- Impedance node analysis.

Chapter 8, State Space Approach
- State space modeling of MIMO systems with input time derivative.
- Calculation of natural frequencies and determination of modes.
- Matrix exponential method.

Chapter 10, Coupled-Field Systems
- Three-dimensional piezoelectricity.
- Energy coupling in piezoelectric elements.
- Time stepping algorithms for the solution of coupled-field nonlinear differential equations.

Whenever possible, alternative solution methods have been provided in the text to enable using the algorithm that best suits various individual approaches to the same problem. Examples include Newton’s second law and the energy method for the free response of systems, which have been used in Chapters 3, 4, and 5, or the mesh analysis and the node analysis methods for electrical systems in Chapter 4.
The ancillary material also comprises an instructor’s manual, an image bank of figures from the book, MATLAB® code for the book’s solved examples, PowerPoint lecture slides, and a longer project whereby the material introduced in the chapter sequence is applied progressively. After publication and as a result of specific requirements or suggestions expressed by instructors who adopted the text and feedback from students, additional problems resulting from this interaction will be provided on the website, as well as corrections of the unwanted but possible errors.

To make distinction between variables, small-cap symbols are generally used for the time domain (such as \( f \) for force, \( m \) for moment, or \( v \) for voltage), whereas capital symbols denote Laplace transforms (such as \( F \) for force, \( M \) for moment, or \( V \) for voltage). With regard to matrix notation, the probably old-fashioned symbols \( \{ \} \) for vectors and \( [ \] \) for matrices are used here, which can be replicated easily on the board.

Several solved examples and end-of-chapter problems in this book resulted from exercises that I have used and tested in class over the last years while teaching courses such as system dynamics, controls, or instrumentation, and I am grateful to the students who contributed to enhancing the scope and quality of the original variants. I am indebted to the anonymous academic reviewers who critically checked this project at its initial (proposal) phase, as well as at two intermediate stages. They have made valid suggestions for improvement of this text, which were well taken and applied to this current version. I appreciate the valuable suggestions by Mr. Tzuliang Loh from the MathWorks Inc. on improving the presentation of the MATLAB® material in this book. I am very thankful to Steven Merken, Associate Acquisition Editor at Elsevier Science & Technology Books, whose quality and timely assistance have been instrumental in converting this project from its embryonic to its current stage.

In closing, I would like again to acknowledge and thank the unwavering support of my wife, Simona, and my daughters, Diana and Ioana—they definitely made this project possible. As always, my thoughts and profound gratitude for everything they gave me go to my parents.
Resources That Accompany This Book

System dynamics instructors and students will find additional resources at the book’s Web site: www.booksite.academicpress.com/lobontiu

Available to All

**Bonus Online Chapter**  For courses that include lectures on controls, Chapter 11, Introduction to Modeling and Design of Feedback Control Systems, is an online chapter available free to instructors and students.

**Additional Online Content**  Linked to specific sections of the book by an identifiable Web icon, extra content includes advanced topics, additional worked examples, and more.

**Downloadable MATLAB® Code**  For the book’s solved examples.

For Instructors Only

**Instructor's Manual**  The book itself contains a comprehensive set of exercises. Worked-out solutions to the exercises are available online to instructors who adopt this book.

**Image Bank**  The Image Bank provides adopting instructors with various electronic versions of the figures from the book that may be used in lecture slides and class presentations.

**PowerPoint Lecture Slides**  Use the available set of lecture slides in your own course as provided, or edit and reorganize them to meet your individual course needs.

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