

# Chapter 1

## Single-Phase Series A.C. Circuits

### Learning Outcomes

This chapter concerns the effect of resistors, inductors and capacitors when connected to an a.c. supply. It also deals with the methods used to analyse simple series a.c. circuits. At the end of the chapter, the concept of series resonance is introduced.

On completion of this chapter, you should be able to:

- 1 Draw the relevant phasor diagrams and waveform diagrams of voltage and current, for pure resistance, inductance and capacitance.
- 2 Understand and use the concepts of reactance and impedance to analyse simple a.c. series circuits.
- 3 Derive and use impedance and power triangles.
- 4 Calculate the power dissipation of an a.c. circuit, and understand the concept of power factor.
- 5 Explain the effect of series resonance, and its implications for practical circuits.

### 1.1 Pure Resistance

A pure resistor is one which exhibits only electrical resistance. This means that it has no inductance or capacitance. In practice, a carbon or metal film resistor is virtually perfect in these respects. Large wire-wound resistors can have a certain inductive and capacitive effect.

Consider a perfect (pure) resistor, connected to an a.c. supply, as shown in Fig. 1.1. The current flowing at any instant is directly proportional to the instantaneous applied voltage, and inversely proportional to the resistance value. The voltage is varying sinusoidally, and the resistance is a constant value. Thus the current flow will also be sinusoidal,

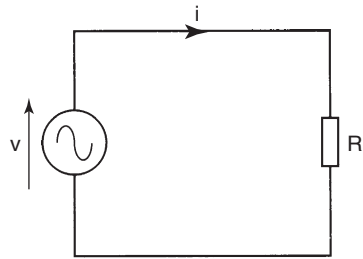


Fig. 1.1

and will be in phase with the applied voltage. This can be written as follows

$$i = \frac{v}{R} \text{ amp}$$

$$\text{but, } v = V_m \sin \omega t \text{ volt}$$

$$\text{therefore, } i = \frac{V_m}{R} \sin \omega t \text{ amp}$$

$$\text{but, } i = I_m \sin \omega t \text{ amp}$$

Thus, the current is a sinewave, of maximum value  $V_m/R$ , is of the same frequency as the voltage, and is in phase with it.

Hence,

$$I_m = \frac{V_m}{R} \text{ amp, or } I = \frac{V}{R} \text{ amp} \quad (1.1)$$

The relevant waveform and phasor diagrams are shown in Figs. 1.2 and 1.3 respectively.

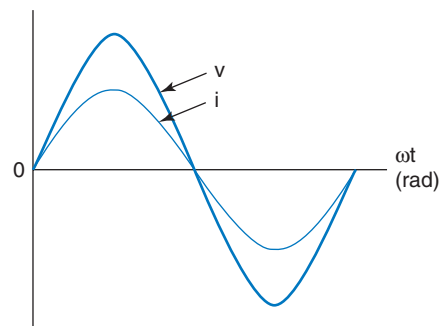


Fig. 1.2



Fig. 1.3

The instantaneous power ( $p$ ) is given by the product of the instantaneous values of voltage and current. Thus  $p = vi$ . The waveform diagram is shown in Fig. 1.4. From this diagram, it is

obvious that the power reaches its maximum and minimum values at the same time as both voltage and current. Therefore

$$P_m = V_m I_m$$

$$\text{hence, } P = VI = I^2R = \frac{V^2}{R} \text{ watt} \quad (1.2)$$

**Note:** When calculating the power, the *r.m.s.* values must be used.

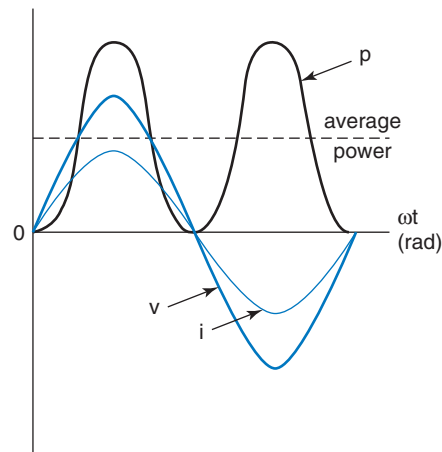


Fig. 1.4

From these results, we can conclude that a pure resistor, in an a.c. circuit, behaves in exactly the same way as in the equivalent d.c. circuit.

### Worked Example 1.1

**Q** Calculate the power dissipated by a  $560\ \Omega$  resistor, when connected to a  $v = 35 \sin 314 t$  volt supply.

**A**

$$R = 560\ \Omega; V_m = 35\text{V}$$

The r.m.s. value for the voltage,  $V = 0.707 V_m$  volt

$$\text{so, } V = 0.707 \times 35 = 24.75\text{ V}$$

$$P = \frac{V^2}{R} \text{ watt} = \frac{24.75^2}{560}$$

therefore,  $P = 1.09\text{ W}$  **Ans**

## 1.2 Pure Inductance

A pure inductor is one which possesses only inductance. It therefore has no electrical resistance or capacitance. Such a device is not

practically possible. Since the inductor consists of a coil of wire, then it must possess a finite value of resistance in addition to a very small amount of capacitance. However, let us assume for the moment that an inductor having zero resistance is possible. Consider such a perfect inductor, connected to an a.c. supply, as shown in Fig. 1.5.

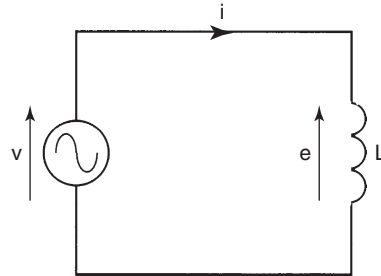


Fig. 1.5

An alternating current will now flow through the circuit. Since the current is continuously changing, then a back emf,  $e$  will be induced across the inductor. In this case,  $e$  will be exactly equal and opposite to the applied voltage,  $v$ . The equation for this back emf is

$$e = -L \frac{di}{dt} \text{ volt}$$

$e$  will have its maximum values when the rate of change of current,  $di/dt$ , is at its maximum values. These maximum rates of change occur as the current waveform passes through the zero position. Similarly,  $e$  will be zero when the rate of change is zero. This occurs when the current waveform is at its positive and negative peaks.

Thus  $e$  will reach its maximum *negative* value when the current waveform has its maximum *positive* slope. Similarly,  $e$  will be at its maximum positive value when  $i$  reaches its maximum negative slope. Also, since  $v = -e$ , then the applied voltage waveform will be the 'mirror image' of the waveform for the back emf. These waveforms are shown in Fig. 1.6. From this waveform diagram, it may be seen that the applied voltage,  $V$  leads the circuit current,  $I$ , by  $\pi/2$  rad, or  $90^\circ$ . The corresponding phasor diagram is shown in Fig. 1.7.

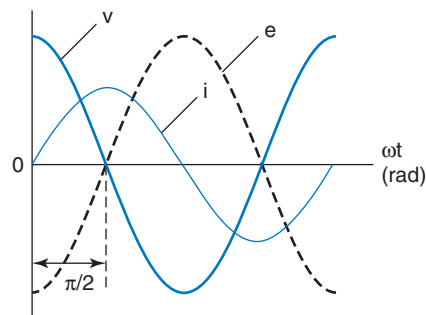


Fig. 1.6

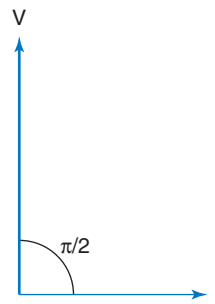


Fig. 1.7

Once more, the instantaneous power is given by the product of instantaneous values of voltage and current. In the first quarter cycle, both  $v$  and  $i$  are positive quantities. The power is therefore in the positive half of the diagram. In the next quarter cycle,  $i$  is still positive, but  $v$  is negative. The power waveform is therefore negative. This sequence is repeated every half cycle of the waveform. The average power is therefore zero. Thus, *a perfect inductor dissipates zero power*. These waveforms are shown in Fig. 1.8.

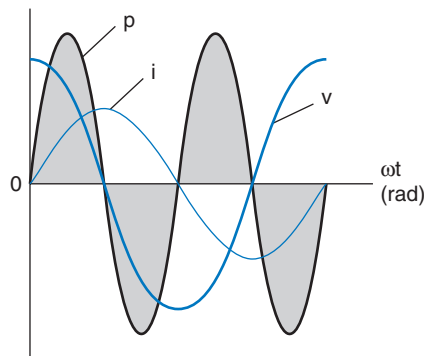


Fig. 1.8

In practical terms, the sequence is as follows. In the first quarter cycle, the magnetic field produced by the coil current, stores energy. In the next quarter cycle, the collapsing field returns all this energy back to the circuit. This sequence is repeated every half cycle. The net result is that the inductor returns as much energy as it receives. Thus no net energy is dissipated, so the power consumption is zero.

### 1.3 Inductive Reactance

A perfect inductor has no electrical resistance. However, there is some opposition to the flow of a.c. current through it. This opposition is of course due to the back emf induced in the coil. It would be most inconvenient to have to always express this opposition in terms of this

emf. It is much better to be able to express the opposition in a quantity that is measured in ohms. This quantity is called the inductive reactance.

Inductive reactance is defined as the opposition offered to the flow of a.c., by a perfect inductor. It is measured in ohms, and the quantity symbol is  $X_L$ . The reactance value for an inductor, at a particular frequency, can be determined from a simple equation. This equation may be derived either mathematically or graphically. Both methods will be shown here.

$$e = -L \frac{di}{dt} \text{ volt, and } e = -v$$

$$\text{therefore, } v = L \frac{di}{dt} \text{ volt}$$

$$\text{Now, } i = I_m \sin \omega t \text{ amp, so, } v = L \frac{d}{dt} (I_m \sin \omega t)$$

$$\text{therefore, } v = \omega L I_m \cos \omega t$$

$$\text{at time } t = 0, v = V_m; \quad \text{and } \cos \omega t = 1$$

$$\text{hence, } V_m = \omega L I_m; \quad \text{and dividing by } I_m$$

$$\frac{V_m}{I_m} = \frac{V}{I} = \omega L \text{ ohm}$$

so, inductive reactance is:

$$X_L = \omega L = 2\pi f L \text{ ohm} \quad (1.3)$$

Alternatively, consider the first quarter cycle of the waveform diagram of Fig. 1.9.

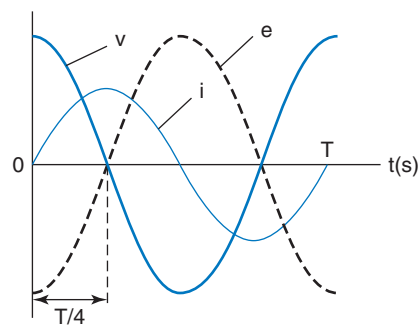


Fig. 1.9

The average emf induced in the coil,

$$e_{av} = \frac{-L di}{dt} \text{ volt}$$

$$\text{therefore, } e_{av} = \frac{-L(I_m - 0)}{T/4}; \quad \text{but } T = \frac{1}{f}$$

so,  $e_{av} = -4LI_m(f)$  volt; therefore  $v_{av} = 4LI_m(f)$  volt

also,  $v_{av} = 0.637V_m = \frac{2V_m}{\pi}$  so,  $V_m = v_{av} \times \frac{\pi}{2}$

therefore,  $V_m = 4LI_m(f) \times \frac{\pi}{2} = 2\pi fLI_m$

hence,  $\frac{V_m}{I_m} = \frac{V}{I} = 2\pi fL$  ohm

i.e. inductive reactance,  $X_L = 2\pi fL = \omega L$  ohm

From equation (1.3), it can be seen that the inductive reactance is directly proportional to both the inductance value and the frequency of the supply. This is logical, since the greater the frequency, the greater the rate of change of current, and the greater the back emf. Figure 1.10 shows the relationship between  $X_L$  and frequency  $f$ . Notice that the graph goes through the origin (0, 0), confirming that a *perfect* inductor has zero resistance. That is, a frequency of 0 Hz is *d.c.*, so no opposition would be offered to the flow of d.c.

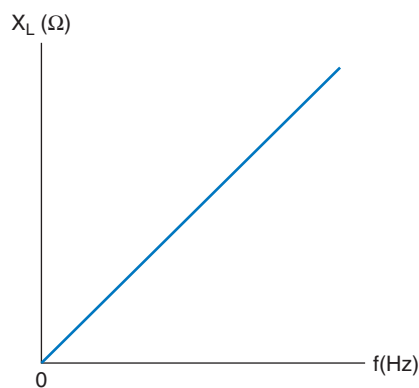


Fig. 1.10

### Worked Example 1.2

**Q** A pure 20 mH inductor is connected to a 30V, 50 Hz supply. Calculate (a) the reactance at this frequency, and (b) the resulting current flow.

**A**

$$L = 20 \times 10^{-3} \text{ H}; V = 30 \text{ V}; f = 50 \text{ Hz}$$

$$(a) X_L = 2\pi fL \text{ ohm} = 2 \times \pi \times 50 \times 20 \times 10^{-3}$$

$$\text{so } X_L = 6.283 \Omega \text{ Ans}$$

$$(b) I = \frac{V}{X_L} \text{ amp} = \frac{30}{6.283}$$

$$\text{so } I = 4.77 \text{ A Ans}$$

**Worked Example 1.3**

**Q** A current of 250 mA flows through a perfect inductor, when it is connected to a 5 V, 1 kHz supply. Determine the inductance value.

**A**

$$I = 0.25 \text{ A}; V = 5 \text{ V}; f = 1000 \text{ Hz}$$

Firstly, the inductive reactance must be calculated:

$$X_L = \frac{V}{I} \text{ ohm} = \frac{5}{0.25}$$

$$\text{therefore, } X_L = 20 \Omega$$

$$\text{Since } X_L = 2\pi fL \text{ ohm, then } L = \frac{X_L}{2\pi f} \text{ henry}$$

$$L = \frac{20}{2 \times \pi \times 1000}$$

$$\text{therefore, } L = 3.18 \text{ mH Ans}$$

**Worked Example 1.4**

**Q** A coil of inductance 400  $\mu\text{H}$ , and of negligible resistance, is connected to a 5 kHz supply. If the current flow is 15 mA, determine the supply voltage.

**A**

$$L = 400 \times 10^{-6} \text{ H}; f = 5 \times 10^3 \text{ Hz}; I = 15 \times 10^{-3} \text{ A}$$

$$X_L = 2\pi fL \text{ ohm} = 2 \times \pi \times 5 \times 10^3 \times 400 \times 10^{-6}$$

$$\text{so, } X_L = 12.57 \Omega$$

$$V = IX_L \text{ volt} = 15 \times 10^{-3} \times 12.57$$

$$\text{so, } V = 188.5 \text{ mV Ans}$$

**1.4 Pure Capacitance**

Consider a perfect capacitor, connected to an a.c. supply, as shown in Fig. 1.11. The charge on the capacitor is directly proportional to the p.d. across it. Thus, when the voltage is at its maximum, so too will be the charge, and so on. The waveform for the capacitor charge will therefore be in phase with the voltage. Current is the rate of change of charge. This means that when the rate of change of charge is a maximum, then the current will be at a maximum, and so on. Since the rate of change of charge is maximum as it passes through the zero axis, the current will be at its maximum values at these points. The resulting waveforms are shown in Fig. 1.12. It may therefore be seen that the current now leads the voltage by  $\pi/2$  rad, or



$90^\circ$ . To maintain consistency with the inductor previously considered, the circuit current will again be considered as the reference phasor. Thus, we can say that the voltage *lags* the current by  $\pi/2$  rad. This is illustrated in Fig. 1.13.

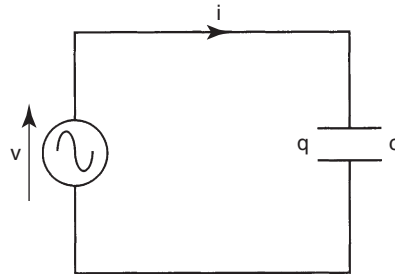


Fig. 1.11

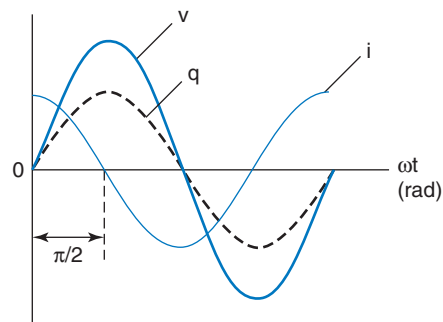


Fig. 1.12

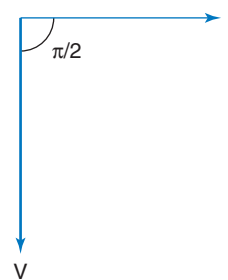


Fig. 1.13

The power at any instant is once more the product of the voltage and current at that instant. When the power waveform is plotted, it will be as shown in Fig. 1.14. Again, it can be seen that the average power is zero. We can therefore conclude that *a perfect capacitor dissipates zero power*. In a similar manner to the inductor, the energy stored in the electric field of the capacitor, in one quarter cycle, is returned to the supply in the next quarter cycle.

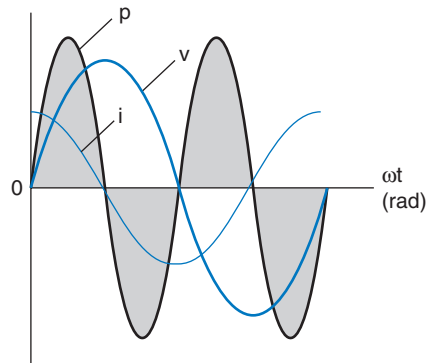


Fig. 1.14

## 1.5 Capacitive Reactance

The opposition offered to the a.c. current is due to the p.d. developed between the capacitor plates. Again, it is more convenient to refer to this opposition in terms of a quantity measured in ohms. This is known as the capacitive reactance,  $X_C$ , which is defined as the opposition offered to the flow of a.c. through a perfect capacitor. The equation for this reactance can be derived in one of two ways. Once more, both will be demonstrated here.

$$q = vC \text{ coulomb; and } i = \frac{dq}{dt} \text{ amp}$$

$$\text{therefore, } i = C \frac{dV}{dt}$$

and since  $v = V_m \sin \omega t$  volt, then

$$\begin{aligned} i &= C \frac{d}{dt} (V_m \sin \omega t) \\ &= \omega C V_m \cos \omega t \end{aligned}$$

when time  $t = 0$ ,  $I = I_m$ ; and  $\cos \omega t = 1$  (see Fig. 1.12)

$$\text{therefore, } I_m = \omega C V_m$$

$$\text{and } \frac{V_m}{I_m} = \frac{V}{I} = \frac{1}{\omega C} \text{ ohm}$$

capacity reactance,

$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C} \text{ ohm} \quad (1.4a)$$

Alternatively, consider the waveforms shown in Fig. 1.15.

In general:

$$\text{average charge} = \text{average current} \times \text{time}$$

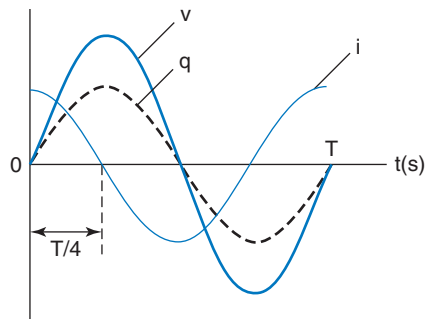


Fig. 1.15

therefore, in first quarter cycle:

$$q = i_{av} \times \frac{T}{4} \text{ coulomb}$$

$$\text{so } q = \frac{i_{av}}{4f}$$

$$\text{but } i_{av} = \frac{2I_m}{\pi} \text{ amp}$$

therefore,  $q = I_m 2\pi f$ ; and since  $q = CV$

$$\text{then, } CV_m = \frac{I_m}{2\pi f}$$

$$\text{and } \frac{V_m}{I_m} = \frac{V}{I} = \frac{1}{2\pi fC} = \frac{1}{\omega C} \text{ ohm}$$

i.e. capacitive reactance,

$$X_C = \frac{1}{2\pi fC} = \frac{1}{\omega C} \text{ ohm} \quad (1.4b)$$

Examination of equation (1.4) shows that the capacitive reactance is *inversely* proportional to both the capacitance and the frequency. Consider the two extremes of frequency. If  $f = 0$  Hz (d.c.), then  $X_c = \text{infinity}$ . This must be correct, since a capacitor does not allow d.c. current to flow through it. At the other extreme, if  $f = \text{infinity}$ , then  $X_c = 0$ . The graph of capacitive reactance versus frequency is therefore in the shape of a rectangular hyperbole, as shown in Fig. 1.16.

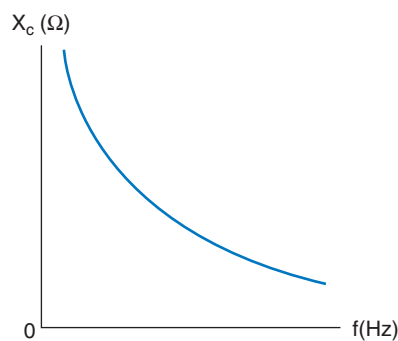


Fig. 1.16

**Worked Example 1.5**

**Q** A  $0.47\ \mu\text{F}$  capacitor is connected to a variable frequency signal generator, which provides an output voltage of  $25\text{V}$ . Calculate the current flowing when the frequency is set to (a)  $200\text{Hz}$ , and (b)  $4\text{kHz}$ .

**A**

$$C = 0.47 \times 10^{-6}\text{ F}; V = 25\text{ V}; f_1 = 200\text{ Hz}; f_2 = 4000\text{ Hz}$$

$$(a) \quad X_{C_1} = \frac{1}{2\pi f_1 C} \text{ ohm} = \frac{1}{2 \times \pi \times 200 \times 0.47 \times 10^{-6}}$$

$$X_{C_1} = 1.693\text{ k}\Omega$$

$$I_1 = \frac{V}{X_C} \text{ amp} = \frac{25}{1693}$$

therefore,  $I_1 = 14.77\text{ mA}$  **Ans**

$$(b) \quad X_{C_2} = \frac{1}{2\pi f_2 C} \text{ ohm} = \frac{1}{2 \times \pi \times 4000 \times 0.47 \times 10^{-6}}$$

$$X_{C_2} = 84.66\ \Omega$$

$$I_2 = \frac{V}{X_{C_2}} \text{ amp} = \frac{25}{84.66}$$

therefore,  $I_2 = 295.3\text{ mA}$  **Ans**

Alternatively, we can say that since

$$f_2 = 5 \times f_1, \text{ then } X_{C_2} = X_{C_1}/5 = 84.66\ \Omega$$

$$\text{Hence, } I_2 = 5 \times I_1 = 295.3\text{ mA}$$
 **Ans**

**Worked Example 1.6**

**Q** At what frequency will the reactance of a  $22\text{ pF}$  capacitor be  $500\ \Omega$ ?

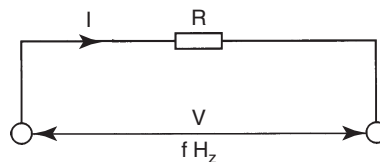
**A**

$$C = 22 \times 10^{-12}\text{ F}; X_C = 500\ \Omega$$

$$X_C = \frac{1}{2\pi f C} \text{ ohm; so } f = \frac{1}{2\pi C X_C} \text{ Hz}$$

$$\text{therefore, } f = \frac{1}{2 \times \pi \times 500 \times 22 \times 10^{-12}}$$

hence,  $f = 14.47\text{ MHz}$  **Ans**

**Summary**

**Fig. 1.17**

$$I = \frac{V}{R} \text{ amp; } V \text{ in phase with } I; \phi = 0; P = I^2R \text{ watt}$$

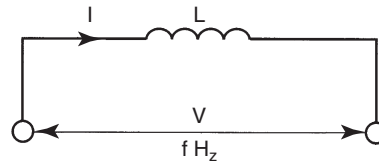


Fig. 1.18

$$I = \frac{V}{X_L} \text{ amp; } V \text{ leads } I \text{ by } 90^\circ; \phi = +\pi/2 \text{ rad } (+90^\circ)$$

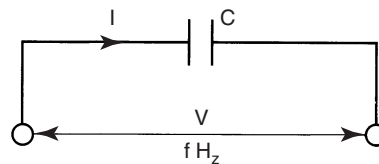


Fig. 1.19

$$X_L = 2\pi fL \text{ ohm; } P = \text{ZERO watt}$$

$$I = \frac{V}{X_C} \text{ amp; } V \text{ lags } I \text{ by } 90^\circ; \phi = -\pi/2 \text{ rad } (-90^\circ)$$

$$X_C = \frac{1}{2\pi fc} \text{ ohm; } P = \text{ZERO watt}$$

## CIVIL

In a capacitive  
(C) circuit

I before V  
(I leads V)

V before I in inductive  
(V leads I)/(L) circuit

The 'keyword' **CIVIL** is a convenient means of remembering whether voltage either leads or lags the circuit current, depending upon whether the circuit is capacitive or inductive. It is particularly useful when making the transition from series to parallel a.c. circuits. In a series circuit, the current is used as the reference phasor. In parallel circuits, the voltage is used as the reference phasor.

## 1.6 Impedance

This is the total opposition, offered to the flow of a.c. current, by a circuit that contains both resistance and reactance. It is measured in ohms, and has the quantity symbol  $Z$ .

$$\text{Thus, } Z = \frac{V}{I} \text{ ohm} \quad (1.5)$$

where  $V$  is the circuit applied voltage, and  $I$  is the resulting circuit current.

In order to describe the concept of impedance, let us consider a circuit comprising a pure resistor connected in series with a pure inductor. Please note that this arrangement could equally well represent a practical inductor. That is, one which possesses both resistance and inductance. In this latter case, it makes the circuit analysis simpler if we consider the component to consist of the combination of these two separate, pure elements.

## 1.7 Inductance and Resistance in Series

A pure resistor and a pure inductor are shown connected in series in Fig. 1.20. The circuit current,  $I$ , will produce the p.d.  $V_R$  across the resistor, due to its resistance,  $R$ . Similarly, the p.d.  $V_L$  results from the inductor's opposition, the inductive reactance,  $X_L$ . Thus, the only circuit quantity that is common, to both the resistor and the inductor, is the circuit current,  $I$ . For this reason, the current is chosen as the reference phasor.

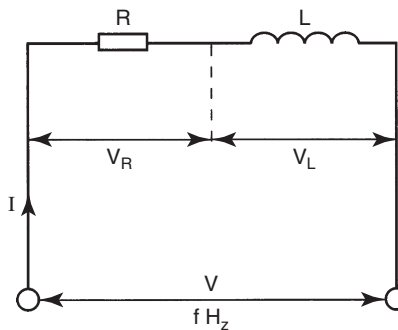


Fig. 1.20

The p.d. across the resistor will be in phase with the current through it ( $\phi = 0$ ). The p.d. across the inductor will lead the current by  $90^\circ$  ( $\phi = +90^\circ$ ). The total applied voltage,  $V$ , will be the *phasor sum* of  $V_R$  and  $V_L$ . This last statement may be considered as the 'a.c. version' of Kirchhoff's voltage law. In other words, the term 'phasor sum' has replaced the term 'algebraic sum', as used in d.c. circuits. The resulting phasor diagram is shown in Fig. 1.21. The angle  $\phi$ , shown

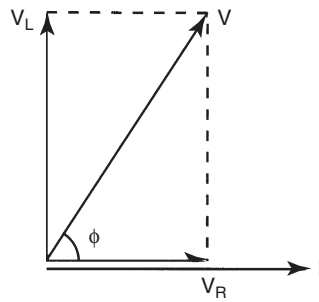


Fig. 1.21

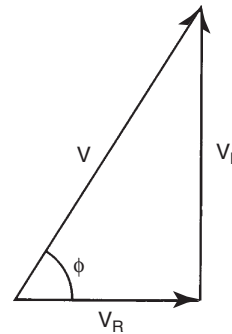


Fig. 1.22

on this diagram, is the angle between the circuit applied voltage,  $V$ , and the circuit current,  $I$ . It is therefore known as the circuit phase angle.

Phasors are vector quantities. They may therefore be drawn anywhere on the page, provided that they are of appropriate length, and are drawn in the appropriate direction. Thus, the voltage triangle, involving  $V$ ,  $V_R$  and  $V_L$ , as in Fig. 1.22, may be ‘extracted’ from the phasor diagram.

Considering Fig. 1.22, Pythagoras’ Theorem may be applied, as follows:

$$V^2 = V_R^2 + V_L^2 \dots\dots\dots [1]$$

Now,  $V_R = IR$ ;  $V_L = IX_L$ ; and  $V = IZ$  volt

and substituting these into equation [1] we have:

$$(IZ)^2 = (IR)^2 + (IX_L)^2$$

and dividing through by  $I^2$  we have:

$$Z^2 = R^2 + X_L^2$$

therefore,  $Z = \sqrt{R^2 + X_L^2}$  ohm **(1.6)**

From the last equation, it may be seen that  $Z$ ,  $R$  and  $X_L$  also form a right-angled triangle. This is known as the impedance triangle, and is shown in Fig. 1.23.

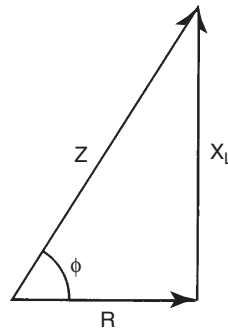


Fig. 1.23

From both the voltage and impedance triangles, the following expressions for the circuit phase angle,  $\phi$ , are obtained:

$$\begin{aligned} \cos \phi &= \frac{R}{Z} = \frac{V_R}{V} \\ \text{or, } \sin \phi &= \frac{X_L}{Z} = \frac{V_L}{V} \\ \text{or, } \tan \phi &= \frac{X_L}{R} = \frac{V_L}{V_R} \end{aligned} \quad (1.7)$$

Any one of the above three expressions may be used to obtain the value of  $\phi$ . However, as will be seen later, equation (1.7) is most frequently used.

### Worked Example 1.7

- Q** A resistor of  $250\ \Omega$ , is connected in series with a  $1.5\ \text{H}$  inductor, across a  $100\ \text{V}$ ,  $50\ \text{Hz}$  supply. Calculate (a) the current flowing, (b) the circuit phase angle, (c) the p.d. developed across each component and (d) the power dissipated.

**A**

$$R = 250\ \Omega; L = 1.5\ \text{H}; V = 100\ \text{V}; f = 50\ \text{Hz}$$

The relevant circuit and phasor diagrams are shown in Figs. 1.24 and 1.25.

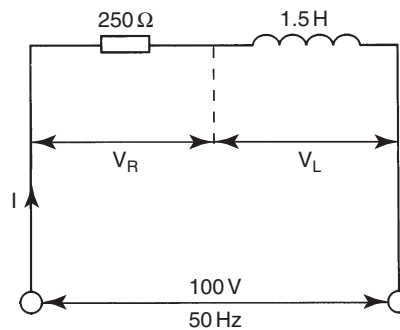


Fig. 1.24



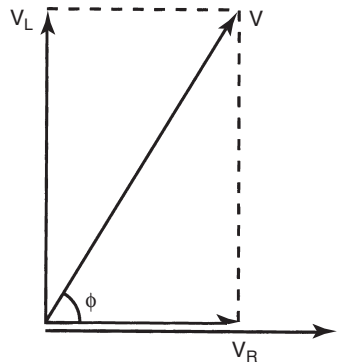


Fig. 1.25

**Note:** A sketch of these diagrams should normally accompany the written answer.

- (a) In order to calculate the current, the impedance must be found. In order to calculate the impedance, the inductive reactance must be known. This then is the starting point in the solution.

$$X_L = 2\pi fL \text{ ohm} = 2 \times \pi \times 50 \times 1.5$$

$$\text{so, } X_L = 471.2 \Omega$$

$$Z = \sqrt{R^2 + X_L^2} \text{ ohm} = \sqrt{250^2 + 471.2^2}$$

$$\text{so, } Z = 533.45 \Omega$$

$$I = \frac{V}{Z} \text{ amp} = \frac{100}{533.45}$$

therefore,  $I = 187.5 \text{ mA}$  **Ans**

$$(b) \quad \phi = \cos^{-1} \frac{R}{Z} = \frac{250}{533.45}$$

therefore,  $\phi = +62.05^\circ$  or  $+1.083 \text{ rad}$  **Ans**

$$(c) \quad V_R = IR \text{ volt} = 0.1875 \times 250$$

therefore,  $V_R = 46.88 \text{ V}$  **Ans**

$$V_L = IX_L \text{ volt} = 0.1875 \times 471.2$$

therefore,  $V_L = 88.35 \text{ V}$  **Ans**

$$(d) \quad P = I^2R \text{ watt} = 0.1875^2 \times 250$$

therefore  $P = 8.789 \text{ W}$  **Ans**

Alternatively, the power could be calculated using the  $V^2/R$  form, BUT in this case, the voltage must be the actual p.d. across the *resistor*, i.e.  $V_R$ . To avoid the possibility of, mistakenly, using  $V$ , or  $V_L$ , it is better to use the form of equation used above; so use  $P = I^2R$  watt.

**Worked Example 1.8**

**Q** A coil, of resistance  $35\ \Omega$  and inductance  $0.02\ \text{H}$ , carries a current of  $326.5\ \text{mA}$ , when connected to a  $400\ \text{Hz}$  a.c. supply. Determine the supply voltage.

**A**

$$R = 35\ \Omega; L = 0.02\ \text{H}; I = 0.3265\ \text{A}; f = 400\ \text{Hz}$$

$$X_L = 2\pi fL\ \text{ohm} = 2 \times \pi \times 400 \times 0.02$$

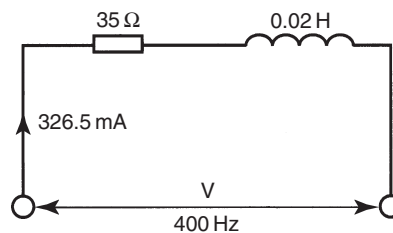
$$\text{therefore, } X_L = 50.27\ \Omega$$

$$Z = \sqrt{R^2 + X_L^2}\ \text{ohm} = \sqrt{35^2 + 50.27^2}$$

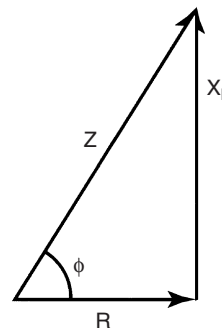
$$\text{therefore, } Z = 61.25\ \Omega$$

$$V = IZ\ \text{volt} = 0.3265 \times 61.25$$

$$\text{hence, } V = 20\ \text{V}\ \mathbf{Ans}$$



**Fig. 1.26**



**Fig. 1.27**

**Worked Example 1.9**

**Q** Two separate tests were carried out on a coil. In the first test, when connected to a  $24\ \text{V}$  d.c. supply, a current of  $2\ \text{A}$  flowed through it. In the second test, it was connected to a  $24\ \text{V}$ ,  $200\ \text{Hz}$  supply. In the latter test, the current flow was  $1.074\ \text{A}$ . Calculate the resistance and inductance of the coil.

**A**

When the coil is connected to a d.c. supply, the only opposition to current flow is the coil resistance. Thus the result of this test can be used to determine the resistance.

$$R = \frac{V}{I}\ \text{ohm} = \frac{24}{2}$$

$$\text{therefore, } R = 12\ \Omega\ \mathbf{Ans}$$

When connected to the a.c. supply, the opposition to current flow is the *impedance* of the coil. Thus:

$$Z = \frac{V}{I} \text{ ohm} = \frac{24}{1.074}$$

therefore,  $Z = 22.35 \Omega$

$$\text{Since } Z^2 = R^2 + X_L^2, \text{ then } X_L = \sqrt{Z^2 - R^2} \text{ ohm}$$

therefore,  $X_L = \sqrt{22.35^2 - 12^2} \text{ ohm} = 18.85 \Omega$

$$L = \frac{X_L}{2\pi f} \text{ henry} = \frac{18.85}{2 \times \pi \times 200}$$

hence,  $L = 15 \text{ mH}$  **Ans**

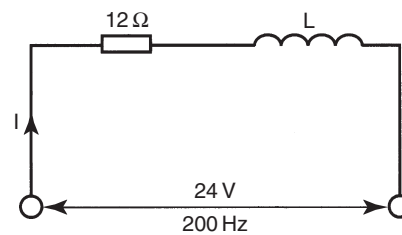


Fig. 1.28

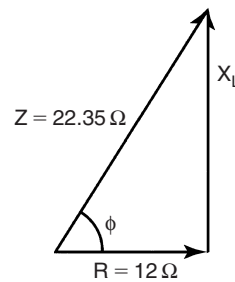


Fig. 1.29

## 1.8 Resistance and Capacitance in Series

Figure 1.30 shows a pure capacitor and resistor connected in series, across an a.c. supply. Again, being a series circuit, the circuit current is common to both components. Each will have a p.d. developed. In this case however, the p.d. across the capacitor will *lag* the current by  $90^\circ$ .

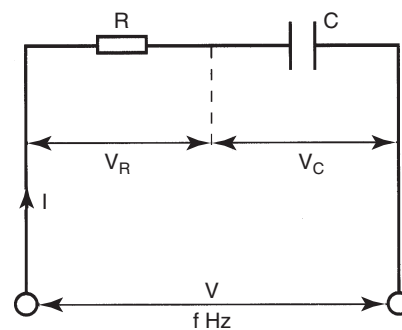


Fig. 1.30

The resulting phasor diagram is shown in Fig. 1.31.

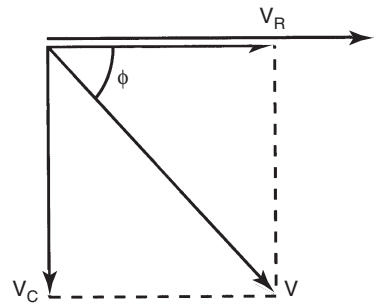


Fig. 1.31

Exactly the same techniques used previously, for the inductor–resistor circuit, may be used here. Thus, both voltage and impedance triangles can be derived. These are shown in Figs. 1.32 and 1.33.

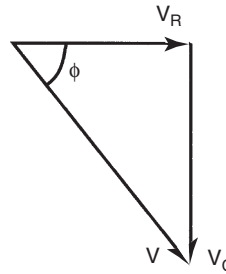


Fig. 1.32

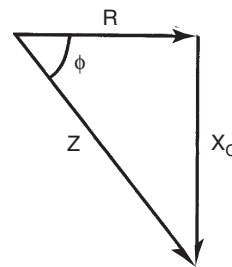


Fig. 1.33

The following equations result:

$$Z = \sqrt{R^2 + X_C^2} \text{ ohm} \quad (1.8)$$

$$\begin{aligned} \phi &= \cos^{-1} \frac{R}{Z} \\ \sin \phi &= \frac{X_C}{Z} = \frac{V_C}{V} \\ \text{and } \tan \phi &= \frac{X_C}{R} = \frac{V_C}{V_R} \end{aligned} \quad (1.9)$$

**Worked Example 1.10**

**Q** A 22 nF capacitor, and a 3.9 kΩ resistor, are connected in series across a 40V, 1 kHz supply. Determine, (a) the circuit current, (b) the circuit phase angle and (c) the power dissipated.

**A**

$$C = 22 \times 10^{-9} \text{ F}; R = 3900 \Omega; V = 40 \text{ V}; f = 10^3 \text{ Hz}$$

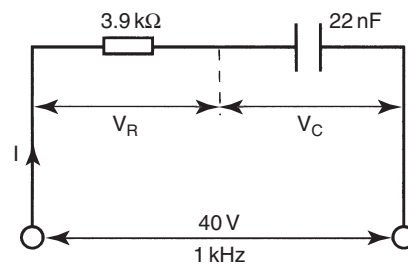


Fig. 1.34

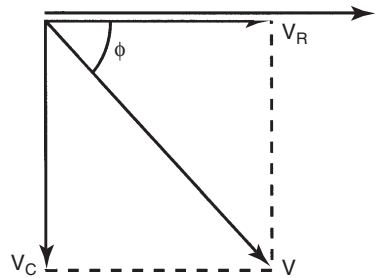


Fig. 1.35

$$(a) \quad X_C = \frac{1}{2\pi fC} \text{ ohm} = \frac{1}{2 \times \pi \times 10^3 \times 22 \times 10^{-9}}$$

$$\text{so, } X_C = 7.234 \text{ k}\Omega$$

$$Z \sqrt{R^2 + X_C^2} \text{ ohm} = \sqrt{(3.9 \times 10^3)^2 + (7.234 \times 10^3)^2}$$

$$\text{hence, } Z = 8.218 \text{ k}\Omega$$

$$I = \frac{V}{Z} \text{ amp} = \frac{40}{8.218 \times 10^3}$$

therefore,  $I = 4.87 \text{ mA}$  **Ans**

$$(b) \quad \phi = \cos^{-1} \frac{R}{Z} = \frac{3.9 \times 10^3}{8.218 \times 10^3}$$

therefore,  $\phi = -61.67^\circ$  or  $-1.076 \text{ rad}$  **Ans**

$$(c) \quad P = I^2 R \text{ watt} = (4.87 \times 10^{-3})^2 \times 3.9 \times 10^3$$

therefore,  $P = 92.5 \text{ mW}$  **Ans**

**Worked Example 1.11**

**Q** A  $470\ \Omega$  resistor is connected in series with a  $0.02\ \mu\text{F}$  capacitor, across a  $15\ \text{V}$  a.c. supply. If the current is  $28\ \text{mA}$ , calculate (a) the frequency of the supply, and (b) the p.d. across the capacitor.

**A**

$$C = 0.02 \times 10^{-6}\ \text{F}; R = 470\ \Omega; V = 15\ \text{V}; I = 0.028\ \text{A}$$

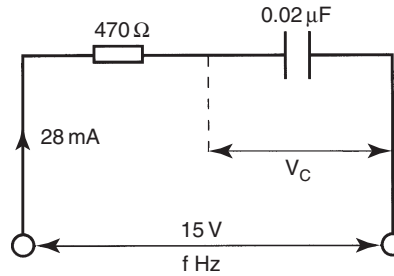


Fig. 1.36

$$(a) \quad Z = \frac{V}{I} \text{ ohm} = \frac{15}{0.028}$$

therefore,  $Z = 535.7\ \Omega$

$$\text{Now, } X_C = \sqrt{Z^2 - R^2} \text{ ohm} = \sqrt{535.7^2 - 470^2}$$

hence,  $X_C = 257.08\ \Omega$

$$\text{Since } X_C = \frac{1}{2\pi fC} \text{ ohm, then } f = \frac{1}{2\pi C X_C} \text{ Hz}$$

$$\text{therefore, } f = \frac{1}{2 \times \pi \times 257.08 \times 0.02 \times 10^{-6}}$$

hence,  $f = 30.95\ \text{kHz}$  **Ans**

$$(b) \quad V_C = I X_C \text{ volt} = 0.028 \times 257.08$$

therefore,  $V_C = 7.2\ \text{V}$  **Ans**

## 1.9 Resistance, Inductance and Capacitance in Series

These three elements, connected in series, are shown in Fig. 1.37. Of the three p.d.s,  $V_R$  will be in phase with the current  $I$ ,  $V_L$  will lead  $I$  by

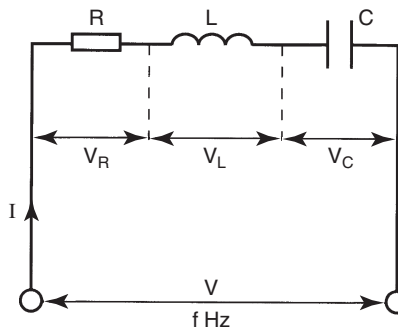


Fig. 1.37

$90^\circ$ , and  $V_C$  will lag  $I$  by  $90^\circ$ . The associated phasor diagram is shown in Fig. 1.38.

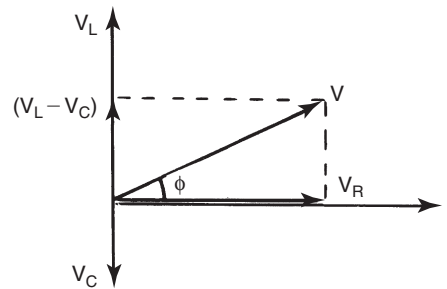


Fig. 1.38

The applied voltage  $V$  is the phasor sum of the circuit p.d.s. These p.d.s form horizontal and vertical components. It may be seen that the only horizontal voltage component is  $V_R$ . The vertical components,  $V_L$  and  $V_C$  are in direct opposition to each other. The resulting vertical component is therefore  $(V_L - V_C)$ . Applying Pythagoras' theorem, we can say that:

$$V^2 = V_R^2 + (V_L - V_C)^2$$

$$\text{but, } V = IZ, V_R = IR, V_L = IX_L \text{ and } V_C = IX_C \text{ volt}$$

$$\text{therefore, } (IZ)^2 = (IR)^2 + (IX_L - IX_C)^2$$

$$\text{hence, } Z^2 = R^2 + (X_L - X_C)^2$$

$$\text{and, } Z = \sqrt{R^2 + (X_L - X_C)^2} \text{ ohm} \quad (1.10)$$

The associated impedance triangle is shown in Fig. 1.39. Note that if  $X_C > X_L$ , then the circuit phase angle  $\phi$  will be lagging, instead of leading as shown.

$$\cos \phi = \frac{R}{Z}; \quad \sin \phi = \frac{X_L - X_C}{Z};$$

$$\text{and } \tan \phi = \frac{X_L - X_C}{R}$$

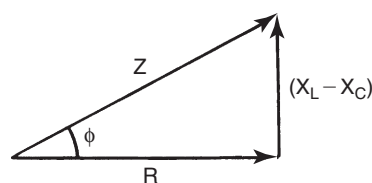


Fig. 1.39

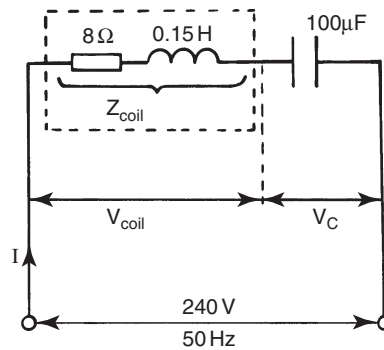
**Worked Example 1.12**

**Q** A coil of resistance  $8\ \Omega$  and inductance  $150\ \text{mH}$ , is connected in series with a  $100\ \mu\text{F}$  capacitor, across a  $240\ \text{V}$ ,  $50\ \text{Hz}$  a.c. supply. Calculate (a) the circuit current, (b) the circuit phase angle, (c) the p.d. across the coil, (d) the p.d. across the capacitor, and (e) the power dissipated.

**A**

$$R = 8\ \Omega; L = 0.15\ \text{H}; C = 10^{-4}\ \text{F}; V = 240\ \text{V}; f = 50\ \text{Hz}$$

The circuit diagram is shown in Fig. 1.40. Note that the coil has been considered as the combination of a pure resistor and a pure inductor.



**Fig. 1.40**

(a)  $X_L = 2\pi fL\ \text{ohm} = 2 \times \pi \times 50 \times 0.15$   
 so,  $X_L = 47.12\ \Omega$   
 $X_C = \frac{1}{2\pi fC}\ \text{ohm} = \frac{1}{2 \times \pi \times 50 \times 10^{-4}}$   
 so  $X_C = 31.83\ \Omega$   
 $Z = \sqrt{R^2 + (X_L - X_C)^2}\ \text{ohm}$   
 $= \sqrt{8^2 + (47.12 - 31.83)^2}$   
 therefore  $Z = 17.26\ \Omega$   
 $I = \frac{V}{Z}\ \text{amp} = \frac{240}{17.26}$   
 therefore  $I = 13.91\ \text{A}$  **Ans**

(b)  $\phi = \cos^{-1} \frac{R}{Z} = \cos^{-1} \frac{8}{17.26}$   
 therefore,  $\phi = +62.38^\circ$  or  $+1.089\ \text{rad}$  **Ans**

(c) The coil has both inductance and reactance. The coil itself therefore possesses impedance,  $Z_{\text{coil}}$  ohm. The p.d. across the coil is therefore due to its impedance, and NOT only  $R$  or  $X_L$ .

$$Z_{\text{coil}} = \sqrt{R^2 + X_L^2}\ \text{ohm} = \sqrt{8^2 + 47.12^2}$$

hence,  $Z_{\text{coil}} = 47.8\ \Omega$   
 $V_{\text{coil}} = IZ_{\text{coil}}\ \text{volt} = 13.91 \times 47.8$   
 therefore,  $V_{\text{coil}} = 664.9\ \text{V}$  **Ans**



(d)  $V_C = IX_C \text{ volt} = 13.91 \times 31.83$

hence,  $V_C = 442.8 \text{ V Ans}$

(e)  $P = I^2R \text{ watt} = 13.91^2 \times 8$

therefore,  $P = 1.547 \text{ kW Ans}$

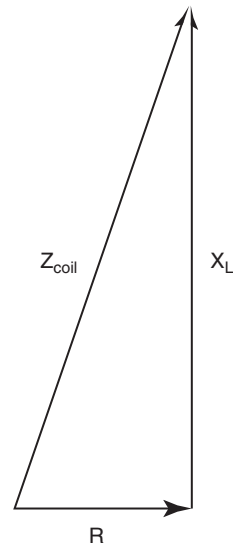


Fig. 1.41

**Note:** From the above calculations, we have the result that the p.d.s, across both the coil and the capacitor, are greater than the applied voltage,  $V$ . At first, this may seem to be incorrect, since it would not be possible in a d.c. circuit. In a d.c. circuit, the applied voltage equals the *algebraic* sum of the p.d.s. In the a.c. circuit, the applied voltage is the *phasor* sum of the p.d.s. Thus, it is perfectly possible for the p.d. across the capacitor, or the coil, or both, to be greater than the applied voltage. This is because the total vertical component of voltage is the *difference* between  $V_L$  and  $V_C$ . This condition is therefore possible only when *all three* components are in series.

## 1.10 Power in the A.C. Circuit

It has been shown that power is dissipated only by the *resistance* in the circuit. Pure inductance and capacitance do not dissipate any net power. Considering a resistor, we know that the p.d. across it is in phase with the current flowing through it ( $\phi = 0^\circ$ ). For both the inductor and the capacitor,  $\phi = 90^\circ$ . Thus, we can conclude that only the components of voltage and current, that are in phase with each other, dissipate power. Consider the phasor diagram for an  $R$ - $L$  series circuit, as shown in Fig. 1.42.

From this diagram, it can be seen that the only two components of current and voltage that are in phase with each other are  $I$  and  $V_R$ .

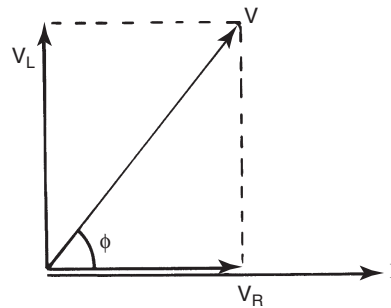


Fig. 1.42

Thus,  $P = V_R I$  watt

but,  $V_R = V \cos \phi$  volt

$$\text{therefore, } P = VI \cos \phi \text{ watt} \quad (1.11)$$

## 1.11 Power Factor

This is defined as the ratio of the true power, to the product of the r.m.s. values of applied voltage and current (apparent power).

$$\text{Thus, power factor (p.f.)} = \frac{VI \cos \phi}{VI}$$

$$\text{hence, p.f.} = \cos \phi \quad (1.12)$$

To illustrate the concepts of true power and apparent power, consider the  $R$ - $L$  circuit, shown in Fig. 1.43, where the components are inside a box, and therefore not visible. Let the applied voltage  $V$  and the current  $I$  be measured, using a voltmeter and ammeter respectively. Taking these two meter readings, it would *appear* that the circuit within the box consumes a power of  $VI$  watt.

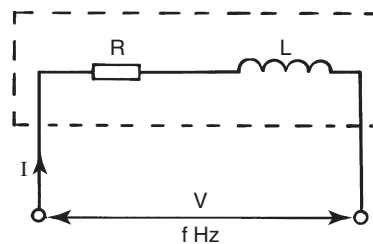


Fig. 1.43

However, if a wattmeter were connected, this would indicate the true power. This wattmeter reading would be found to be less than the product of the ammeter and voltmeter readings. Since the true and

apparent powers are related by the power factor,  $\cos \phi$ , then they must form two sides of a right-angled triangle.

## 1.12 Power Triangle

The true power is measured in watts (W). The apparent power is measured in volt-ampere (VA). The third side of the triangle is known as the reactive component of power, and is measured in reactive volt-ampere (VAr). The quantity symbols are  $P$ ,  $S$  and  $Q$  respectively. These are shown in Fig. 1.44.

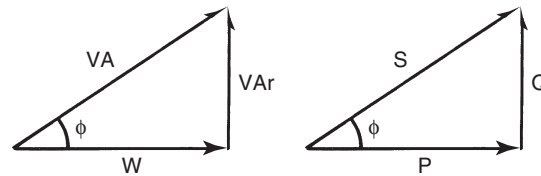


Fig. 1.44

### Worked Example 1.13

**Q** An R-C circuit consisting of a  $4.7 \mu\text{F}$  capacitor in series with a  $200 \Omega$  resistor, is connected to a  $250\text{V}$ ,  $50\text{Hz}$  supply. Determine, (a) the current, (b) the power factor, and (c) the values for true, apparent and reactive powers.

**A**

$$C = 4.7 \times 10^{-6} \text{ F}; R = 200 \Omega; V = 250 \text{ V}; f = 50 \text{ Hz}$$

$$(a) \quad X_C = \frac{1}{2\pi fC} \text{ ohm} = \frac{1}{2 \times \pi \times 50 \times 4.7 \times 10^{-6}}$$

$$\text{so, } X_C = 677.26 \Omega$$

$$Z = \sqrt{R^2 + X_C^2} \text{ ohm} = \sqrt{200^2 + 677.26^2}$$

$$\text{therefore, } Z = 706.2 \Omega$$

$$I = \frac{V}{Z} \text{ amp} = \frac{250}{706.2}$$

$$\text{hence } I = 0.354 \text{ A Ans}$$

$$(b) \quad \text{p.f.} = \cos \phi = \frac{R}{Z} = \frac{200}{706.2}$$

$$\text{therefore, p.f.} = 0.283 \text{ lagging Ans}$$

$$(c) \quad P = VI \cos \phi \text{ watt} = 250 \times 0.354 \times 0.283$$

$$\text{therefore, } P = 25.06 \text{ W Ans}$$

From the power triangle (Fig. 1.45)

$$S = VI \text{ volt-ampere} = 250 \times 0.354$$

$$\text{therefore, } S = 88.5 \text{ VA Ans}$$

$$Q = S \sin \phi \text{ reactive volt-ampere}$$

$$= 88.5 \times 0.9591$$

$$\text{hence } Q = 84.88 \text{ VAr Ans}$$

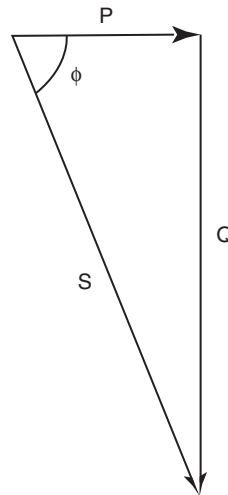


Fig. 1.45

### Worked Example 1.14

- Q** The stator winding of a single-phase a.c. motor consumes 250 W, at a leading power factor of 0.537, when connected to a 240 V, 50 Hz supply. Calculate (a) the current drawn, and (b) the resistance and inductance of the winding.

**A**

$$P = 250 \text{ W}; \cos \phi = +0.537; V = 240 \text{ V}; f = 50 \text{ Hz}$$

$$(a) \quad P = VI \cos \phi \text{ watt}$$

$$\text{so, } I = \frac{P}{V \cos \phi} \text{ amp} = \frac{250}{240 \times 0.537}$$

therefore,  $I = 1.94 \text{ A}$  **Ans**

$$(b) \quad Z = \frac{V}{I} \text{ ohm} = \frac{240}{1.94}$$

$$\text{so } Z = 123.72 \text{ } \Omega$$

$$\phi = \cos^{-1} 0.537 = 57.52^\circ; \text{ so } \sin \phi = 0.8436$$

The impedance triangle is shown in Fig. 1.46. From this:

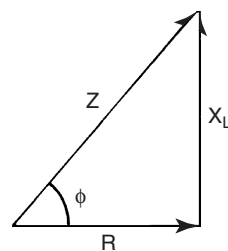


Fig. 1.46

$$R = Z \cos \phi = 123.72 \times 0.537$$

therefore,  $R = 66.44 \Omega$  **Ans**

$$X_L = Z \sin \phi = 123.72 \times 0.8436$$

so  $X_L = 104.37 \Omega$

$$L = \frac{X_L}{2\pi f} \text{ henry} = \frac{104.37}{2 \times \pi \times 50}$$

therefore,  $L = 0.332 \text{ H}$  **Ans**

### Worked Example 1.15

**Q** A circuit draws a power of 2.5 kW at a leading power factor of 0.866. Calculate the apparent power and the reactive component of power.

**A**

$$P = 2500 \text{ W}; \cos \phi = 0.866$$

Phase angle,  $\phi = \cos^{-1} 0.866 = 30^\circ$ , and the power triangle will be as shown in Fig. 1.47.

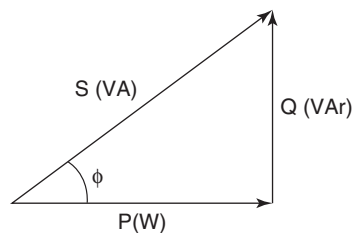


Fig. 1.47

$$\cos \phi = \frac{P}{S}$$

$$\begin{aligned} \text{so apparent power, } S &= \frac{P}{\cos \phi} \text{ volt-ampere} \\ &= \frac{2500}{0.866} \end{aligned}$$

hence,  $S = 2.89 \text{ kVA}$  **Ans**

$$\tan \phi = \frac{Q}{P}$$

so, reactive component,  $Q = P \tan \phi$  volt-ampere reactive

$$= 2.5 \times \tan 30^\circ \text{ kVAr}$$

therefore,  $Q = 1.44 \text{ kVAr}$  **Ans**

## 1.13 Series Resonance

Consider a series  $R$ - $L$ - $C$  circuit, connected to a constant amplitude, variable frequency supply, as in Fig. 1.48. Let the frequency be

initially set at a low value. In this case, the capacitive reactance will be relatively large. The inductive reactance will be relatively small. The resistance will remain constant throughout. The relevant phasor diagram is shown in Fig. 1.49(a).

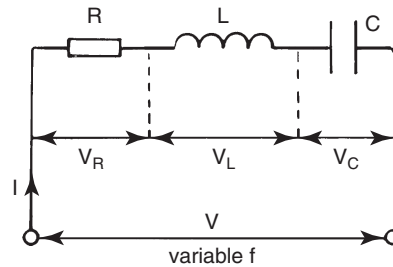


Fig. 1.48

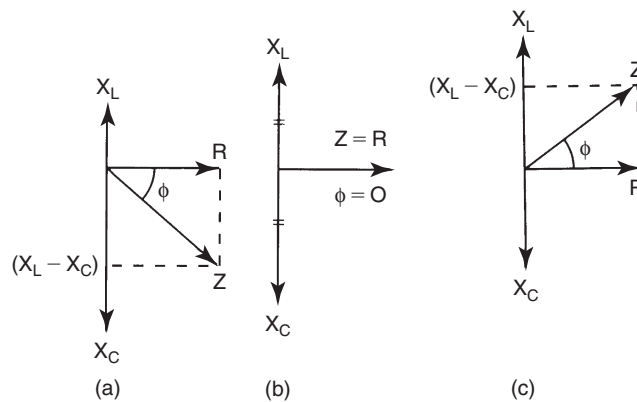


Fig. 1.49

Let the frequency of the supply now be increased. As this is done, so  $X_C$  will decrease, and  $X_L$  will increase. At one particular frequency, the values of  $X_C$  and  $X_L$  will be equal. This is known as the resonant frequency,  $f_0$ , and the general condition is known as resonance. This effect is illustrated in Fig. 1.49(b).

If the frequency continues to be increased, then  $X_L$  will continue to increase, whilst  $X_C$  continues to decrease. This is shown in Fig. 1.49(c).

Considering the circuit phase angle, it can be seen that it has, in turn, changed from lagging to zero (at resonance) to leading. This sequence, in terms of the voltage phasor diagram, is shown in Fig. 1.50(a), (b) and (c).

At the resonant frequency, the applied voltage is in phase with the circuit current. Additionally, under this condition,  $V = V_R$ . Thus, from

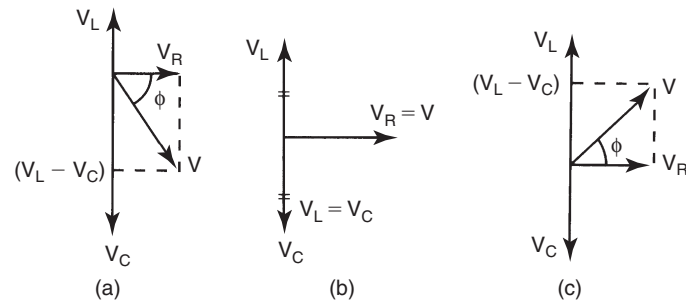


Fig. 1.50

the point of view of the supply, the circuit behaves as if it consists only of the resistor. This is because, at  $f_0$ ,  $X_L = X_C$ . These reactances therefore effectively 'cancel' each other. This is illustrated below:

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \text{ ohm}$$

and, at  $f_0$ ,  $X_L = X_C$   
therefore,  $Z = \sqrt{R^2}$

$$\text{hence, at resonance } Z = R \text{ ohm} \quad (1.13)$$

It should be apparent that this is the minimum possible value for the circuit impedance. So, at resonance, the circuit current must reach its maximum possible value. The variation of the reactances, and hence the impedance, is illustrated in Fig. 1.51. The variation of the circuit current, with frequency, is shown in Fig. 1.52.

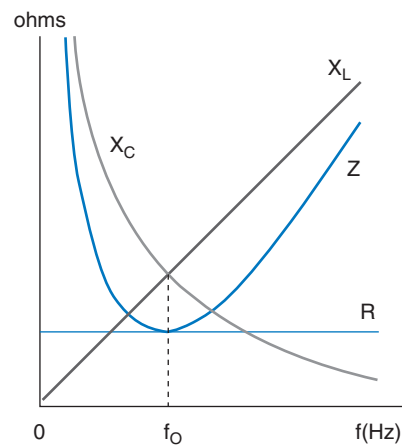


Fig. 1.51

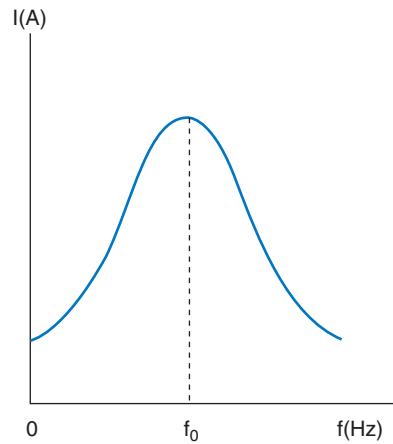


Fig. 1.52

The value for the resonant frequency, for a given circuit, can be determined as follows:

$$\text{At } f_0, \omega_0 L = \frac{1}{\omega_0 C} \text{ ohm}$$

$$\text{therefore, } \omega_0^2 LC = 1$$

$$\text{and, } \omega_0^2 = \frac{1}{LC}$$

$$\text{so, } \omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s}$$

$$\text{hence, } f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ hertz} \quad (1.14)$$

**Note:** Under resonant conditions, the p.d.s developed across the capacitor and inductor can be many times greater than the supply voltage. This could have serious consequences for the components used. The dielectric strength of the capacitor may well be exceeded. The insulation on the coil could also prove to be insufficient. For these reasons, the resonant condition is normally avoided, when dealing with ‘power’ circuits (i.e. 50 Hz supplies). At higher frequencies (e.g. radio frequencies), the applied voltages are normally fairly small. At these frequencies, resonance does not pose the same problem. Indeed, at these frequencies, the resonance effect can be put to good use. This aspect is dealt with in Chapter 2.

### Worked Example 1.16

**Q** A coil of resistance  $10\ \Omega$ , and inductance  $1.013\ \text{H}$ , is connected in series with a  $10\ \mu\text{F}$  capacitor. Calculate (a) the resonant frequency, (b) the circuit current, when connected to a  $240\ \text{V}$ ,  $50\ \text{Hz}$  supply, and (c) the p.d. developed across the capacitor.

**A**

$$R = 10\ \Omega; L = 1.013\ \text{H}; C = 10 \times 10^{-6}\ \text{F}; V = 240\ \text{V}$$



$$(a) \quad f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ hertz} = \frac{1}{2\pi\sqrt{1.013 \times 10^{-5}}}$$

thus,  $f_0 = 50 \text{ Hz}$  **Ans**

- (b) Since the supply frequency is 50 Hz, then the circuit will resonate. This means that the only opposition to current flow is the *resistance* of the coil,  $R$  ohm.

$$\text{Therefore, } I = \frac{V}{R} \text{ amp} = \frac{240}{10}$$

hence,  $I = 24 \text{ A}$  **Ans**

$$(c) \quad X_C = \frac{1}{2\pi fC} \text{ ohm} = \frac{1}{2 \times \pi \times 50 \times 10^{-5}}$$

$$\text{so, } X_C = 318.31 \Omega$$

$$V_C = IX_C \text{ volt} = 24 \times 318.31$$

therefore,  $V_C = 7639 \text{ V}$  **Ans**

The above example clearly illustrates the problem of resonance at power frequency. For example, suppose the capacitor dielectric has a thickness of 1 mm. The peak voltage between the plates will be  $\sqrt{2} \times 7639 \text{ V} = 10803 \text{ V}$ . The *minimum* dielectric strength required would therefore be  $10803/10^{-3} = 10.8 \text{ MV/m}$ .

If the frequency was increased to 200 Hz, the applied voltage remaining at 240 V, the current would be reduced to 0.518 A, and the p.d. across the capacitor to 46.25 V.

### Summary of Equations

**Resistance:**  $I = \frac{V}{R}$  amp;  $P = I^2R$  watt;  $\phi = 0^\circ$

**Inductance:** Reactance,  $X_L = 2\pi fL$  ohm;  $I = \frac{V}{X_L}$  amp;  $P = 0\text{W}$ ;  $\phi = +90^\circ$  or  $+\frac{\pi}{2}$  rad

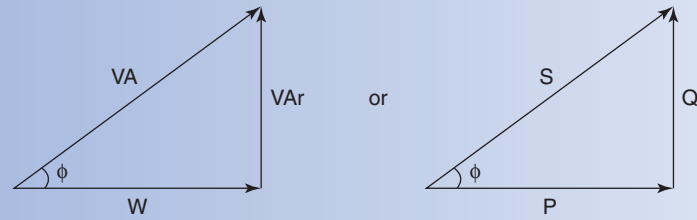
**Capacitance:** Reactance,  $X_C = \frac{1}{2f\pi C}$  ohm;  $I = \frac{V}{X_C}$  amp;  $P = 0\text{W}$ ;  $\phi = -90^\circ$  or  $-\frac{\pi}{2}$  rad

**Impedance:**  $Z = \sqrt{R^2 + (X_L - X_C)^2}$  ohm;  $\cos \phi = \frac{R}{Z}$ ;  $P = VI \cos \phi = I^2R$  watt

$$I = \frac{V}{Z} \text{ amp}$$

**Power triangle:** Power factor =  $\cos \phi$

True power in W; apparent power in VA; reactive 'power' in VAR.



**Series resonance:** Resonant frequency,  $f_0 = \frac{1}{2\pi\sqrt{LC}}$  hertz

$X_L = X_C$  ohm;  $Z = R$  ohm;  $I = \frac{V}{R}$  amp (max. possible value)

## Assignment Questions

- Calculate the reactance of a perfect capacitor, of  $0.47 \mu\text{F}$ , at the following frequencies. (a) 50 Hz, (b) 200 Hz, (c) 5 kHz, and (d) 300 kHz.
- Determine the reactance of a perfect inductor, of  $0.5 \text{ H}$ , at the following frequencies. (a) 40 Hz, (b) 600 Hz, (c) 4 kHz, and (d) 1 MHz.
- A pure inductor has a reactance of  $125 \Omega$ , when connected to a 5 kHz supply. Calculate the inductance.
- A perfect capacitor has a reactance of  $100 \Omega$ , when connected to a 60 Hz supply. Determine its capacitance.
- A 250 V, 50 Hz supply is connected to a pure inductor. The resulting current is 1.5 A. Calculate the value of the inductance.
- The current drawn by a perfect capacitor is 250 mA, when connected to a 25 V, 1.5 kHz supply. Determine the capacitance value.
- Determine the frequency at which a 40 mH inductor will have a reactance of  $60 \Omega$ .
- At what frequency will a 5000 pF capacitor have a reactance of  $2 \text{ k}\Omega$ ?
- A 24 V, 400 Hz a.c. supply is connected, in turn, to the following components. (a) a  $560 \Omega$  resistor, (b) a pure inductor, of 20 mH, and (c) a pure capacitor of 220 pF. For each case, calculate the current flow and power dissipation.
- Determine the impedance of a coil, which has a resistance of  $15 \Omega$  and a reactance of  $20 \Omega$ .
- A coil of inductance 85 mH, and resistance  $75 \Omega$ , is connected to a 120 V, 200 Hz supply. Calculate (a) the impedance, (b) the circuit current, (c) the circuit phase angle and power factor, and (d) the power dissipated.
- For the circuit shown in Fig. 1.53, determine (a) the supply voltage, and (b) the circuit phase angle.
- For the circuit of Fig. 1.53, assuming a supply frequency of 1.5 kHz, calculate (a) the current, (b) the inductance and (c) the power dissipated.
- A coil takes a current of 4.5 A, when connected to a 24 V d.c. supply. When connected to a 24 V, 50 Hz supply, it takes a current of 2.6 A. Explain why the current in the second case is less than in the first. Hence, calculate (a) the resistance of the coil and (b) its inductance.
- A voltage of 45 V, 80 Hz, is applied across a circuit consisting of a capacitor in series with a resistor. Determine the p.d. across the capacitor, if the p.d. across the resistor is 20 V.
- A  $75 \Omega$  resistor is connected in series with a  $15 \mu\text{F}$  capacitor. If this circuit is supplied at 180 V, 100 Hz, calculate (a) the impedance, (b) the current, (c) the power factor and (d) the power dissipated.
- For the circuit shown in Fig. 1.54, the reactance of the capacitor is  $1600 \Omega$ . Determine (a) the supply voltage, (b) the circuit current, (c) the supply frequency and (d) the value of the resistance.

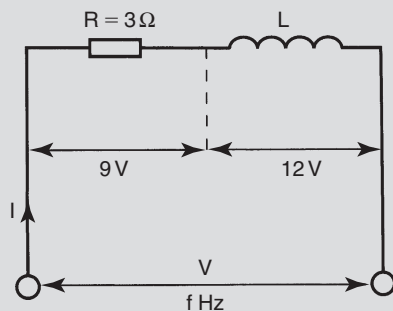


Fig. 1.53

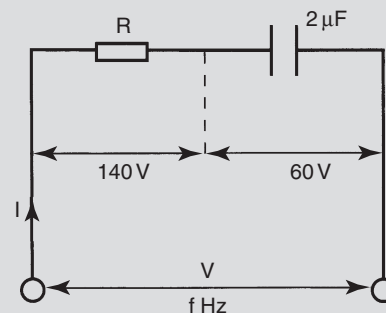


Fig. 1.54

- An alternating voltage of  $v = 180 \sin 628.4t$  volt is applied to a circuit containing a  $39 \Omega$  resistor, connected in series with a  $47 \mu\text{F}$  capacitor. Calculate (a) the circuit current, (b) the p.d.s across the resistor and capacitor, (c) the power dissipated, and (d) the power factor.

## Assignment Questions

- 19** For the circuit shown in Fig. 1.55, determine (a) the impedance, (b) the current, (c) the phase angle, (d) the p.d. across each component.

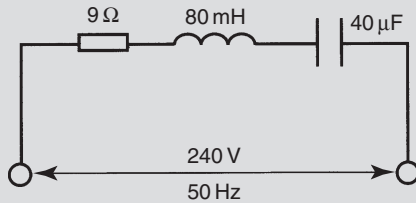


Fig. 1.55

- 20** The circuit shown in Fig. 1.56 dissipates  $162.5\ \text{W}$ . Calculate (a) the circuit current, (b) the p.d. across the coil, (c) the p.d. across the capacitor, and (d) the phase angle of the circuit.
- 21** Determine the resonant frequency for the circuit shown in Fig. 1.56. What would be the current flowing under this condition? Explain why this condition would be avoided at power frequencies.

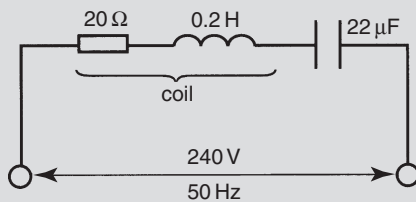


Fig. 1.56

- 22** An  $R$ - $L$ - $C$  circuit resonates at a frequency of  $10\ \text{kHz}$ . If the capacitor has a value of  $400\ \text{pF}$ , calculate the value of the inductor.
- 23** A transformer is rated at  $15\ \text{kVA}$ , at a lagging power factor of  $0.75$ . Determine the rated power output, in  $\text{kW}$ .
- 24** A load dissipates  $40\ \text{kW}$ , at a power factor of  $0.6$  lagging. Calculate the apparent power, and its reactive component.
- 25** The power consumed by an  $R$ - $C$  circuit, when connected to a  $100\ \text{V}$ ,  $2\ \text{kHz}$  supply, is  $800\ \text{W}$ . If the circuit current is  $12\ \text{A}$ , calculate (a) the impedance, (b) the resistance, (c) the reactance, (d) the capacitance, (e) the power factor, and (f) the circuit phase angle.
- 26** An a.c. motor draws a current of  $10\ \text{A}$ , when connected to a  $240\ \text{V}$ ,  $50\ \text{Hz}$  supply. If the power factor is  $0.6$  lagging, calculate (a) the true power, apparent power, and reactive component of power, (b) the resistance and inductance of the motor winding.

## Suggested Practical Assignments

### Assignment 1

To determine the variation of capacitive reactance, with variation of frequency.

#### Apparatus:

- 1 × 1  $\mu$ F capacitor
- 1 × signal generator
- 1 × ammeter
- 1 × voltmeter

#### Method:

- 1 Connect the capacitor and ammeter in series, across the output terminals of the signal generator.
- 2 Set the output voltage to 10V (as measured across the capacitor), and the frequency to 300 Hz. Measure the current flow. Maintain this voltage value throughout.
- 3 Alter the frequency, in 100 Hz steps, up to 1 kHz. Record the ammeter and voltmeter readings at each step.
- 4 Alter the frequency, in 500 Hz steps, up to 4 kHz. Record the meter readings at each step.
- 5 From your tabulated values of  $I$  and  $V$ , calculate and tabulate the values for  $X_C$ .
- 6 Plot two graphs. One graph to be  $X_C$  versus frequency; the other to be  $X_C$  versus reciprocal of frequency ( $1/f$ ).
- 7 Write an assignment report, and from your results, state the relationship between capacitive reactance and frequency.

### Assignment 2

To investigate the reactance and impedance of a coil.

#### Apparatus:

- 1 × coil
- 1 × signal generator
- 1 × d.c. supply
- 1 × ammeter
- 1 × voltmeter

#### Method:

- 1 Connect the coil to the d.c. supply.
- 2 Increase the voltage applied to the coil, in steps, and note the current at each step.
- 3 Tabulate these meter readings, and plot a graph of  $V$  versus  $I$ . From this graph, determine the resistance of the coil.
- 4 Connect the coil to the signal generator, set to a frequency of 1 kHz.
- 5 Vary the output voltage of the signal generator, in steps, and record the voltage and current readings at each step.
- 6 From your tabulated values, plot a graph of  $V$  versus  $I$ .
- 7 From this graph, determine the impedance of the coil. Hence, determine the reactance of the coil at 1 kHz, and its inductance.
- 8 Complete an assignment report.

### Assignment 3

To investigate the phase relationship between current and voltage in a capacitor circuit.

**Apparatus:**

- 1 × double-beam oscilloscope
- 1 × signal generator
- 1 × 0.22  $\mu\text{F}$  capacitor
- 1 × 1 k $\Omega$  resistor

**Method:**

- 1 Connect the resistor and capacitor, in series, across the output terminals of the signal generator.
- 2 Connect the oscilloscope, such that one channel monitors the p.d. across the capacitor, and the other the p.d. across the resistor.
- 3 Set the frequency to 1 kHz, and sketch the two waveforms observed.
- 4 Make a careful note of the time interval between the two waveforms, and hence calculate the phase difference between them.

**Note:** The current through a resistor is in phase with the p.d. across it. Thus, the waveform for the resistor p.d., may be taken to represent the circuit current.

**Assignment 4**

To investigate the phase relationship, between current and voltage, for an inductor.

**Apparatus:**

- 1 × low resistance coil
- 1 × 5 k $\Omega$  resistor
- 1 × signal generator
- 1 × double-beam oscilloscope

**Method:**

- 1 Connect the resistor and coil, in series, across the output terminals of the signal generator. Set the frequency to 10 kHz.
- 2 Using the two channels of the oscilloscope, monitor the p.d.s across the resistor and coil.
- 3 Carefully note the time interval between the two waveforms.
- 4 Calculate the phase angle between the two waveforms, and sketch the waveforms.

**Note:** The waveform of p.d. across the resistor, may again be taken to represent the circuit current. Provided that the coil resistance is very low compared to the reactance, then the coil may be considered to be almost ideal.

**Assignment 5**

To investigate the effect of series resonance.

**Apparatus:**

- 1 × signal generator
- 1 × inductor (of known value)
- 1 × capacitor (of known value)
- 1 × voltmeter
- 1 × ammeter

**Method:**

- 1 Connect the capacitor, inductor and ammeter in series, across the terminals of the signal generator.
- 2 Calculate the theoretical resonant frequency of the circuit.
- 3 Set the frequency to  $f_0/10$ . Record the circuit current, and the capacitor p.d. Ensure that the output voltage of the signal generator is maintained constant throughout.

- 4 Increase the frequency, in discrete steps, up to  $f_0 \times 10$ . Record the current and voltage readings at each step.

**Note:** Take 'extra' readings at, and on either side of the resonant frequency  $f_0$  Hz.

- 5 Plot a graph of circuit current versus frequency. Compare the actual resonant frequency to the calculated value.
- 6 Complete an assignment report.

