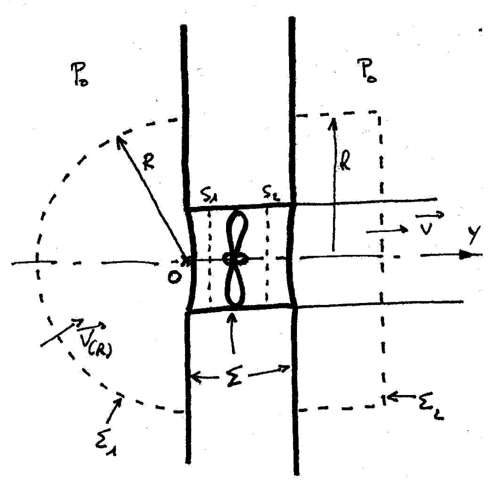
Bow thruster

The sketch shows a schematic sketch of the bow thruster flow. The hull is limited by two parallel planes which are penetrated by a cylindrical tunnel with cross section S. The tunnel has axis in y-direction and is orthogonal to the hull walls. We consider the case where the flow induced by the bow thruster is from left to right, i.e. in direction of the y-axis. The problem is considered as stationary. The thruster propeller turns around axis y.



The basic model simplifications are:

- The fluid is ideal and gravity can be neglected.
- The flow is uniform inside the thruster tunnel (except in the vicinity of the propeller and the openings)
- Seen from far away, the inlet to the tunnel can be approximated by a sink located at the origin O, with volume flow Q. Thus the flow is radial and the velocity depends only on the distance R from the origin O.
- At the outlet of the tunnel, a cylindrical jet of cross section S exits with uniform velocity V
- The pressure at infinity (both in $\pm y$ direction) is p_0.
- The fluid is at rest far upstream ($y \to -\infty$).
- The fluid is at rest outside the exiting jet.

Derivation of force of water on propeller

We apply Bernoulli's equation to calculate the pressure between far upstream and the section S_1 directly before the propeller:

$$\frac{p_0}{\rho} = \frac{V^2}{2} + \frac{p_1}{\rho}$$

We apply Bernoulli's equation to calculate the pressure between section S_2 directly behind the propeller and far downstream:

$$\frac{V^2}{2} + \frac{p_2}{\rho} = \frac{V^2}{2} + \frac{p_0}{\rho}$$

Thus the pressure difference between the upstream section S_1 and the downstream section S_2 is:

$$p_2 - p_1 = \frac{1}{2}\rho V^2$$

The volume flow through the cross section S is $Q = V \cdot S$ [m^3/s].

The force, the water exerts on the propeller, follows from the pressure jump upstream and downstream of the propeller:

$$T_p = -(p_2 - p_1)S = -\frac{\rho}{2}V^2 S = -\frac{\rho}{2}QV = -\frac{\rho}{2}\frac{Q^2}{S}$$

Derivation of force on propeller and ship

We apply the Euler equation to the volume Ω encased by the boundaries Σ_1, Σ and Σ_2, letting $R \to \infty$, to get the force T the propeller exerts on the water. Remember that the theorem of Gauss-Ostrogradsky allows to convert a volume integral to a surface integral: $\int(\nabla\vec{V})d\Omega = \int(\vec{V}\vec{n})d\Gamma$.

$$\rho \int\limits_{\Gamma} \vec{V}(\vec{V}\vec{n})\, dS = -\int\limits_{\Gamma} p\vec{n}\, dS$$

$\Gamma = \Sigma_1 + P_1 + P_2 + \Sigma + \Sigma_2$. P_1 and P_2 are suction and pressure surfaces of the propeller.

Now we consider each of the boundaries separately:

- Σ_1: $\vec{V} \to 0$ and $p \to p_0$ as $R \to \infty$
- P_1, P_2: $\vec{V} \cdot \vec{n} = 0$
- Σ: $\vec{V} \cdot \vec{n} = 0$
- Σ_2: $\vec{V} \cdot \vec{n} = 0$ except on the cross section S, where we have $p = p_0$

This yields:

$$\rho \int\limits_{\Sigma_1+\Sigma_2} \vec{V}(\vec{V}\cdot\vec{n})\, dS = -\int\limits_{\Sigma_1+\Sigma_2} p\vec{n}\, dS - \int\limits_{P_1+P_2+\Sigma} p\vec{n}\, dS$$

The component in y-direction gives for the three terms:

$$\rho \int\limits_{\Sigma_2} \vec{V}(\vec{V}\cdot\vec{n})\, dS \quad \to \quad \rho S V^2 = \frac{\rho Q^2}{S}$$

$$\left.\begin{array}{lll} \int\limits_{\Sigma_2} p\vec{n}\, dS & \to & p_0\pi R^2 \\ \lim\limits_{R\to\infty} \int\limits_{\Sigma_1} p\vec{n}\, dS & \to & -p_0\pi R^2 \end{array}\right\} \int\limits_{\Sigma_1+\Sigma_2} p\vec{n}\, dS \quad \to \quad 0$$

At a large distance from the tunnel, we can approximate the effect of the tunnel as a sink. For a sink, the flux to the sink is constant over each circumference Σ_1 (each R). The part in y-direction is decreasing with increasing circumference, thus:

$$\lim_{R\to\infty} \rho \int\limits_{\Sigma_1} \vec{V}(\vec{V}\cdot\vec{n})\, dS \quad \to \quad 0$$

T is the total force the water exerts on the propeller and the tunnel:

$$T = \int\limits_{P_1+P_2+\Sigma} p\vec{n}\, dS$$

Combining the above, we get then:

$$T = -\frac{\rho Q^2}{S}$$

T is thus twice as big as T_p; the difference between T_p and T is the force on ship (tunnel). The thrust of a propeller in a tunnel is twice the thrust of a propeller in free stream.