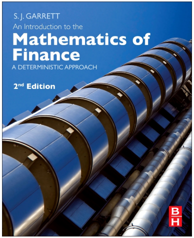
Essential Topic: Forwards and futures

Chapter 10



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Summary

Forwards and futures

- *Forwards* and *futures* are simple examples of *derivatives*, i.e. contracts that derive their value from an underlying asset.
- A forward/future is an agreement between two parties to trade a specified asset on a set date in the future for an agreed price.
- Futures and forwards can be considered as identical other than that futures are standardized and exchange tradable.
- We do not distinguish between the two in what follows and ignore the subtle effects of the clearing-house margins that apply to futures.
- The term “forward” will be used throughout, although this can be substituted with “future” as appropriate.

FORWARD PRICE, NON-INCOME PAYING ASSET

- The trade of the asset underlying a forward necessarily has a buyer and seller, these positions define the *long* and *short* positions of the forward, respectively.
- A key aspect of a forward is the *forward price*, K. This is the price that both parties agree at time $t = 0$ to trade the underlying at the *expiry date* $t = T$.
- To determine K, we require the *risk-free force of interest*, r, and the *current market price of the underlying*, S_0.
- It is then possible to use a *no-arbitrage* argument to determine that

$$K = S_0 e^{rT}.$$

- This is the only price acceptable to both parties. Neither party is favoured and so the contract has zero value at $t = 0$.

EXAMPLE

Calculate the forward price under a 6-month forward contract on a non-income paying asset currently trading at £50. The risk-free force of interest is 5% per annum. State the value of contract to the long and short positions at the time the contract is entered.

Answer

We have $S_0 = 50$, $T = 0.5$ and $r = 0.05$. The forward price is then

$$K = 50 \times e^{0.05 \times 0.5} = £51.27.$$

The values of the contract to the long and short positions at $t = 0$ are

$$V_L(0) = 0 \qquad V_S(0) = 0$$

respectively.

Value of the contract

- As time moves towards the expiry date, the price of the underlying asset S_t will also move.
- At maturity, the value of the forward is given by the *payoff*

$$V_L(T) = S_T - K \quad \text{and} \quad V_S(T) = K - S_T = -V_L(T).$$

- At general time $t < T$, the value of the forward is determined by the favourability of the movement of the asset price:
  - an increase in the asset price beyond the risk-free rate favours the long position and $V_L(t) > 0$,
  - a decrease in the asset price below the risk-free rate favours the short position and $V_S(t) > 0$.
- In particular, the value of the contract at $t < T$ is

$$V_L(t) = S_t - S_0 e^{rt} = S_t - Ke^{-r(T-t)} \quad \text{and} \quad V_S(t) = -V_L(t).$$

- These expressions are consistent with the known values at $t = 0$ and T and apply for all $t \in [0, T]$.

Example

A 2-year forward contract was struck on a non-income paying asset currently trading at £100. Calculate the value of the contract to the long position after 6 months if the underlying asset is trading at the following prices.

a.) $S_{0.5} = £110$

b.) $S_{0.5} = £50$

c.) $S_{0.5} = £102.02$

The risk-free force of interest is 4% per annum throughout the 2 years.

Answer

Note that we do not require K.

a.) $V_L(0.5) = S_{0.5} - S_0 e^{rt} = 110 - 100e^{0.04\times 0.5} = £7.98$, the market has moved in favour of the long position.

b.) $V_L(0.5) = 50 - 100e^{0.04\times 0.5} = -£50.02$, the market has moved significantly against the long position.

c.) $V_L(0.5) = 102.02 - 100e^{0.04\times 0.5} = £0$, the market has moved in line with the initial risk-free rate assumption.

Forward price, fixed-income asset

- We now assume that the underlying asset pays a fixed income between $t = 0$ and T.
- Since the current holder of the asset will receive the income, the long position will disagree on the current market price of the underlying.
- It is therefore appropriate to modify the current market price to account for this "missed" income.
- Under the *discounted cash flow pricing model*, the modified price of the underlying is

$$S_0' = S_0 - I_0$$

  where I_0 is the present value at $t = 0$ of the missed income between $t = 0$ and T.
- We extend the previous no-arbitrage argument and deduce that

$$K = S_0' e^{rT} = (S_0 - I_0)\, e^{rT}.$$

VALUE OF THE CONTRACT

- We can modify the previous expression for the contract value using the idea of missed income at the relevant times.
- The value of the contract at $t < T$ is given by

$$\begin{aligned} V_L(t) =& (S_t - I_t) - (S_0 - I_0)e^{rt} = (S_t - I_t) - Ke^{-r(T-t)} \\ V_S(t) =& - V_L(t) \end{aligned}$$

- These expressions are again consistent with the known values at $t = 0$ and T and so apply for all $t \in [0, T]$.

EXAMPLE

A 1-year forward contract was written on a fixed-income paying asset then trading at £50. If the asset is known to generate cash flows of £3 at times $t = 0.25$ and 0.75 and the risk-free *rate* of interest is 7% per annum, calculate

a.) the forward price,

b.) the value of the contract to the short position at $t = 0.6$ if $S_{0.6} = £48$.

Answer

We have $S_0 = 50$, $T = 1$ and $i = 0.07$.

a.) $K = (S_0 - I_0)(1+i)^1 = \left(50 - 3\nu^{0.25} - 3\nu^{0.75}\right) \times 1.07 = £47.29.$

b.) $V_L(0.6) = (S_{0.6} - I_{0.6}) - Ke^{-r(T-t)} = 48 - 3\nu^{0.15} - 47.29\nu^{0.4} = -£1.00$ and so $V_S(0.6) = £1.00$.

FORWARD PRICE, CONSTANT DIVIDEND YIELD

- Implicit in the above is that the fixed income generated by an asset is invested at the risk-free rate.
- However, when an asset pays a constant *dividend yield*, D, the value of the proceeds is unknown in monetary terms. We therefore assume that proceeds are invested in holdings of the asset.
- The "modified" asset price for the long position at $t = 0$ is then $S_0 e^{-DT}$ and the forward price is

$$K = S_0 e^{-DT} \times e^{rT} = S_0 e^{(r-D)T}$$

- A similar argument is used to determine the value of the contract at some interim time, $t < T$.

EXAMPLE

A 3-year forward contract was written on an asset paying a dividend yield of 2% per annum then trading at £20. If the risk-free force of interest is 4% per annum, calculate

a.) the forward price,

b.) the value of the contract to the long position at $t = 0.3$ if $S_{0.3} = £19.45$.

Answer

We have $S_0 = 20$, $T = 3$ and $r = 0.04$.

a.) $K = S_0 e^{(r-D)T} = 20e^{(0.04-0.02)\times 3} = £21.24$

b.) $V_L(0.3) = S_{0.3}e^{-D(T-0.3)} - Ke^{-r(T-0.3)} = 19.45e^{-0.02\times 2.7} - 21.24e^{-0.04\times 2.7} = -£0.64.$

Summary

- *Forwards* and *futures* are simple examples of *derivatives*.
- Using a no-arbitrage argument, the forward price is calculated from the prevailing risk-free interest rate, the current price of the underlying asset and any receipts due from the asset during the term of the contract
$$K = S_0' e^{rT}.$$
S_0' is the *modified market price of the underlying* that takes into account the long position's "missed" income between $t = 0$ and T.
- The forward price is agreeable to both parties at the outset and the contract has zero initial value.
- Subsequent market movements in the price of the underlying relative to the risk-free rate favour either the long or the short position and lead to a non-zero value at $t \leq T$.
- This value can be calculated using time value of money arguments at the risk-free rate and modified asset prices.