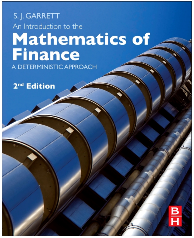
Essential Topic: Fixed-interest securities

Chapters 7 and 8



CONTENTS PAGE

FIXED-INTEREST SECURITIES

- We consider securities that pay fixed monetary amounts at known times.
- The cash flows arising from investment in a £100 nominal of a security are determined by the security parameters
  - purchase price, P (outflow)
  - annual coupon, D, paid $p$thly (inflow)
  - redemption payment, R (inflow)
- We assume that the investor is subject to *income tax* at rate t_1.
- We begin by neglecting capital gains tax.

EQUATION OF VALUE

- In the simplest case, the equation of value for an n-year fixed-interest security is

$$P = D(1 - t_1)a_{\overline{n}|}^{(p)} + R\nu^n$$

- If the price is known, the EoV can be solved to give the yield, i.
- If the desired yield is known, the EoV can be evaluated to determine the maximum price, P.

Example

Consider a 10-year fixed-interest security that pays coupons of 5% per annum at 6-monthly intervals and is redeemed at par.

a.) Calculate the maximum price than an investor should pay to achieve a net yield of 6% per annum.

b.) Calculate the net yield if the investor actually purchases the security for £75%.

You should assume that the investor pays income tax at a rate of 20% per annum and no capital gains tax.

EXAMPLE

Answer

The EoV for £100 nominal is written as

$$P = 5 \times (1 - 0.20) \times a^{(2)}_{\overline{10|}} + 100\nu^{10}$$

a.) We evaluate this at $i = 6\%$ to determine that $P = £85.71\%$.

b.) Setting $P = 75$, the EoV is solved (by trial and error or Excel's *Goalseek*) to give $i = 7.8\%$.

MAKEHAM'S FORMULA

- Consider a general nominal amount, N, of the security and define $C = RN$ to be the redemption cash flow.
- The annual coupon payment is of amount DN and we define $g = D/R$ to be the annual coupon expressed as a percentage of the redemption payment.
- If $A = PN$, the EoV can be written as

$$\begin{aligned} A =& NR\nu^n + (1 - t_1)DNa^{(p)}_{\overline{n}|} \\ =& C\nu^n + (1 - t_1)gC\frac{1 - \nu^n}{i^{(p)}} \\ A =& K + \frac{g(1 - t_1)}{i^{(p)}}(C - K) \end{aligned}$$

  where $K = C\nu^n$ is the PV of the redemption payment.
- This is *Makeham's formula* for pricing fixed-interest securities.

EXAMPLE

Consider a 10-year fixed-interest security that pays coupons of 5% per annum at 6-monthly intervals and is redeemed at par. If an investor pays income tax at a rate of 20% per annum, use Makeham's formula to determine the maximum price that he should pay for the security to achieve a net yield of at least 6% per annum.

Answer

We have $p = 2$, $n = 10$, $t_1 = 0.20$, $g = 0.05/1$ and, for £100 nominal, $C = 100$.

$$A = 100\nu^{10} + \frac{0.05 \times (1 - 0.20)}{i^{(2)}} \times \left(100 - 100\nu^{10}\right) = £85.71$$

This price is identical to that found earlier.

CAPITAL GROWTH/LOSS

- Makeham's formula is simply an alternative form of the EoV, it contains no new information.
- However, its use can simplify problems. In particular, K is considered as the PV of redemption payments irrespective of how complicated they may be distributed.
- Furthermore, we immediately see a simple test for a capital gain/loss at redemption:
  - if $g(1-t_1) = i^{(p)}$, $A = C$ and there is no capital gain or loss,
  - if $g(1-t_1) > i^{(p)}$, $A > C$ and there is a capital loss,
  - if $g(1-t_1) < i^{(p)}$, $A < C$ and there is a capital gain.
- In some sense, $i^{(p)}$ is the *internalized* return and $g(1-t_1)$ the *externalized* return prior to redemption. The size of the externalized return relative to the target yield determines whether there is a further return required from a capital gain at redemption.

CAPITAL GROWTH/LOSS

- In the previous example we had $i^{(2)} = 5.9126\%$ and $g(1 - t_1) = 4\%$ and so $g(1 - t_1) < i^{(p)}$. This implies that a capital gain at redemption is needed to achieve the required 6% per annum from the investment.
- Indeed we had $A = £85.71$ and $C = £100$, i.e. a capital gain.

- If the net yield demanded were instead $i = 3\%$, then $i^{(2)} = 2.9778\% < g(1 - t_1) = 4\%$. We would then expect a capital loss at redemption to offset the large externalized return from the coupon payments.
- In fact, Makeham's formula gives $A = £108.78$ and $C = £100$. Which would indeed give a capital loss at redemption.

CAPITAL GAINS TAX

- We now consider investors that are liable to *capital gains tax* at rate t_2.
- Capital gains tax will be due when $A < C$, i.e. the redemption cash flow is greater than the price paid. In this case, the investor is liable to pay an amount $t_2 \times (C - A)$ at time $t = n$.
- Makeham's formula for the EoV is then modified to

$$A = K + \frac{g(1 - t_1)}{i^{(p)}}(C - K) - t_2 \frac{C - A}{C} K$$

- Which is rearranged to give an expression for A

$$A = \frac{(1 - t_2)K + (1 - t_1)(g/i^{(p)})(C - K)}{1 - t_2 K/C}$$

CAPITAL GAINS TAX

- We now have a situation where the expression for determining the price to pay for the security is dependent on whether a capital gain will occur. However, whether a capital gain occurs depends on the price paid.
- Fortunately, this circular argument is broken by the comparison of $i^{(p)}$ to $g(1-t_1)$, as discussed above.
- We therefore have the following generalized form of Makeham's formula for an investor liable to income tax at rate t_1 and capital gains tax at rate t_2

$$A = \begin{cases} K + \frac{g(1-t_1)}{i^{(p)}}(C-K) & \text{if } i^{(p)} \leq g(1-t_1) \\ \frac{(1-t_2)K+(1-t_1)(g/i^{(p)})(C-K)}{1-t_2K/C} & \text{if } i^{(p)} > g(1-t_1) \end{cases}$$

EXAMPLE

Consider a 5-year fixed-interest security that pays coupons of 3% per annum at 3-monthly intervals and is redeemed at 105%. If an investor pays income tax at a rate of 40% per annum and capital gains tax at 25%, determine the maximum price to pay so that the net yield is at least 4% per annum.

Answer

We have $p = 4$, $n = 5$, $t_1 = 0.40$, $t_2 = 0.25$, $g = 0.03/1.05$ and, for £100 nominal, $C = 105$. If $i = 4\%$, $i^{(4)} = 3.9414\% > g(1 - t_1)$ and there is a capital gain. The appropriate pricing formula is then

$$A = \frac{(1 - 0.25)105\nu^5 + (1 - 0.4)(g/i^{(p)})\left(105 - 105\nu^5\right)}{1 - 0.25\nu^5} = £91.70$$

Optional redemption dates

- The redemption date of a security can be at the option of the borrower (i.e. issuer).
- In this case, the value of n is not known in advance and the investor's decision to invest is made more complicated.
- The investor should take a *prudent* approach and assume that redemption will occur at the time that gives the *lowest* yield.
  - If there is a capital gain, $i^{(p)} > g(1-t_1)$, the lowest yield will be achieved with redemption at the latest possible date.
  - If there is a capital loss, $i^{(p)} < g(1-t_1)$, the lowest yield will be achieved with redemption at the earliest possible date.
  - If there is no capital change, $i^{(p)} = g(1-t_1)$, the yield will be independent of the redemption date.
- These considerations are irrespective of if the investor pays capital gains tax.

EXAMPLE

If an investor requires a net yield of 8% per annum, calculate the maximum price he should pay for a fixed-interest security that pays coupons of 10% per annum at 6-monthly intervals and is redeemed at 103%. You are given that the issuer can redeem the security at any coupon date between $n = 5$ and $n = 10$ and the investor pays no tax.

Answer

We have $p = 2$, $t_1 = 0$, $g = 0.10/1.03 = 9.7087\%$ and, for £100 nominal, $C = 103$. If $i = 0.08\%$, $i^{(2)} = 7.8461\% < g(1 - t_1)$ and there is a capital loss. It is then prudent to assume that the security is redeemed at $t = 5$. The price is then

$$A = 103\nu^5 + (g/i^{(p)})\left(103 - 103\nu^5\right) = £110.81$$

Note that if the issuer actually redeems the security at $t > 5$, the investor will earn a yield greater than 8% per annum.

SUMMARY

- Fixed-interest securities can be valued from first principles using annuity notation and compound interest tables.
- Makeham's formula is a standardized expression for the EoV of a fixed-interest security.
- Makeham's formula can simplify the study of complicated security issues, but essentially contains no new information.
- The form of Makeham's formula enables one to show that
  - if $g(1-t_1) = i^{(p)}$, $A = C$ and there is no capital gain or loss,
  - if $g(1-t_1) > i^{(p)}$, $A > C$ and there is a capital loss,
  - if $g(1-t_1) < i^{(p)}$, $A < C$ and there is a capital gain.
- These expressions are also useful in pricing securities with redemption dates at the option of the issuer.
- An investor's income tax and capital gains tax rates can be incorporated into Makeham's formulation and net yields obtained.