1.1 Introduction

The term microcomputer is used to describe a system that includes at minimum a microprocessor, program memory, data memory, and an input-output (I/O) device. Some microcomputer systems include additional components such as timers, counters, and analog-to-digital converters. Thus, a microcomputer system can be anything from a large computer having hard disks, floppy disks, and printers to a single-chip embedded controller.

In this book we are going to consider only the type of microcomputers that consist of a single silicon chip. Such microcomputer systems are also called microcontrollers, and they are used in many household goods such as microwave ovens, TV remote control units, cookers, hi-fi equipment, CD players, personal computers, and refrigerators. Many different microcontrollers are available on the market. In this book we shall be looking at programming and system design for the PIC (programmable interface controller) series of microcontrollers manufactured by Microchip Technology Inc.

1.2 Microcontroller Systems

A microcontroller is a single-chip computer. Micro suggests that the device is small, and controller suggests that it is used in control applications. Another term for microcontroller is embedded controller, since most of the microcontrollers are built into (or embedded in) the devices they control.

A microprocessor differs from a microcontroller in a number of ways. The main distinction is that a microprocessor requires several other components for its operation,
such as program memory and data memory, input-output devices, and an external clock circuit. A microcontroller, on the other hand, has all the support chips incorporated inside its single chip. All microcontrollers operate on a set of instructions (or the user program) stored in their memory. A microcontroller fetches the instructions from its program memory one by one, decodes these instructions, and then carries out the required operations.

Microcontrollers have traditionally been programmed using the assembly language of the target device. Although the assembly language is fast, it has several disadvantages. An assembly program consists of mnemonics, which makes learning and maintaining a program written using the assembly language difficult. Also, microcontrollers manufactured by different firms have different assembly languages, so the user must learn a new language with every new microcontroller he or she uses.

Microcontrollers can also be programmed using a high-level language, such as BASIC, PASCAL, or C. High-level languages are much easier to learn than assembly languages. They also facilitate the development of large and complex programs. In this book we shall be learning the programming of PIC microcontrollers using the popular C language known as mikroC, developed by mikroElektronika.

In theory, a single chip is sufficient to have a running microcontroller system. In practical applications, however, additional components may be required so the microcomputer can interface with its environment. With the advent of the PIC family of microcontrollers the development time of an electronic project has been reduced to several hours.

Basically, a microcomputer executes a user program which is loaded in its program memory. Under the control of this program, data is received from external devices (inputs), manipulated, and then sent to external devices (outputs). For example, in a microcontroller-based oven temperature control system the microcomputer reads the temperature using a temperature sensor and then operates a heater or a fan to keep the temperature at the required value. Figure 1.1 shows a block diagram of a simple oven temperature control system.

The system shown in Figure 1.1 is very simple. A more sophisticated system may include a keypad to set the temperature and an LCD to display it. Figure 1.2 shows a block diagram of this more sophisticated temperature control system.
Figure 1.1: Microcontroller-based oven temperature control system

Figure 1.2: Temperature control system with a keypad and LCD
We can make the design even more sophisticated (see Figure 1.3) by adding an alarm that activates if the temperature goes outside the desired range. Also, the temperature readings can be sent to a PC every second for archiving and further processing. For example, a graph of the daily temperature can be plotted on the PC. As you can see, because microcontrollers are programmable the final system can be as simple or as complicated as we like.

A microcontroller is a very powerful tool that allows a designer to create sophisticated input-output data manipulation under program control. Microcontrollers are classified by the number of bits they process. Microcontrollers with 8 bits are the most popular and are used in most microcontroller-based applications. Microcontrollers with 16 and 32 bits are much more powerful, but are usually more expensive and not required in most small- or medium-size general purpose applications that call for microcontrollers.

The simplest microcontroller architecture consists of a microprocessor, memory, and input-output. The microprocessor consists of a central processing unit (CPU) and a
control unit (CU). The CPU is the brain of the microcontroller; this is where all the arithmetic and logic operations are performed. The CU controls the internal operations of the microprocessor and sends signals to other parts of the microcontroller to carry out the required instructions.

Memory, an important part of a microcontroller system, can be classified into two types: program memory and data memory. Program memory stores the program written by the programmer and is usually nonvolatile (i.e., data is not lost after the power is turned off). Data memory stores the temporary data used in a program and is usually volatile (i.e., data is lost after the power is turned off).

There are basically six types of memories, summarized as follows:

1.2.1 RAM

RAM, random access memory, is a general purpose memory that usually stores the user data in a program. RAM memory is volatile in the sense that it cannot retain data in the absence of power (i.e., data is lost after the power is turned off). Most microcontrollers have some amount of internal RAM, 256 bytes being a common amount, although some microcontrollers have more, some less. The PIC18F452 microcontroller, for example, has 1536 bytes of RAM. Memory can usually be extended by adding external memory chips.

1.2.2 ROM

ROM, read only memory, usually holds program or fixed user data. ROM is nonvolatile. If power is removed from ROM and then reapplied, the original data will still be there. ROM memory is programmed during the manufacturing process, and the user cannot change its contents. ROM memory is only useful if you have developed a program and wish to create several thousand copies of it.

1.2.3 PROM

PROM, programmable read only memory, is a type of ROM that can be programmed in the field, often by the end user, using a device called a PROM programmer. Once a PROM has been programmed, its contents cannot be changed. PROMs are usually used in low production applications where only a few such memories are required.
1.2.4 EPROM

EPROM, erasable programmable read only memory, is similar to ROM, but EPROM can be programmed using a suitable programming device. An EPROM memory has a small clear-glass window on top of the chip where the data can be erased under strong ultraviolet light. Once the memory is programmed, the window can be covered with dark tape to prevent accidental erasure of the data. An EPROM memory must be erased before it can be reprogrammed. Many developmental versions of microcontrollers are manufactured with EPROM memories where the user program can be stored. These memories are erased and reprogrammed until the user is satisfied with the program. Some versions of EPROMs, known as OTP (one time programmable), can be programmed using a suitable programmer device but cannot be erased. OTP memories cost much less than EPROMs. OTP is useful after a project has been developed completely and many copies of the program memory must be made.

1.2.5 EEPROM

EEPROM, electrically erasable programmable read only memory, is a nonvolatile memory that can be erased and reprogrammed using a suitable programming device. EEPROMs are used to save configuration information, maximum and minimum values, identification data, etc. Some microcontrollers have built-in EEPROM memories. For instance, the PIC18F452 contains a 256-byte EEPROM memory where each byte can be programmed and erased directly by applications software. EEPROM memories are usually very slow. An EEPROM chip is much costlier than an EPROM chip.

1.2.6 Flash EEPROM

Flash EEPROM, a version of EEPROM memory, has become popular in microcontroller applications and is used to store the user program. Flash EEPROM is nonvolatile and usually very fast. The data can be erased and then reprogrammed using a suitable programming device. Some microcontrollers have only 1K flash EEPROM while others have 32K or more. The PIC18F452 microcontroller has 32K bytes of flash memory.

1.3 Microcontroller Features

Microcontrollers from different manufacturers have different architectures and different capabilities. Some may suit a particular application while others may be totally
unsuitable for the same application. The hardware features common to most microcontrollers are described in this section.

1.3.1 Supply Voltage

Most microcontrollers operate with the standard logic voltage of +5V. Some microcontrollers can operate at as low as +2.7V, and some will tolerate +6V without any problem. The manufacturer’s data sheet will have information about the allowed limits of the power supply voltage. PIC18F452 microcontrollers can operate with a power supply of +2V to +5.5V.

Usually, a voltage regulator circuit is used to obtain the required power supply voltage when the device is operated from a mains adapter or batteries. For example, a 5V regulator is required if the microcontroller is operated from a 5V supply using a 9V battery.

1.3.2 The Clock

All microcontrollers require a clock (or an oscillator) to operate, usually provided by external timing devices connected to the microcontroller. In most cases, these external timing devices are a crystal plus two small capacitors. In some cases they are resonators or an external resistor-capacitor pair. Some microcontrollers have built-in timing circuits and do not require external timing components. If an application is not time-sensitive, external or internal (if available) resistor-capacitor timing components are the best option for their simplicity and low cost.

An instruction is executed by fetching it from the memory and then decoding it. This usually takes several clock cycles and is known as the instruction cycle. In PIC microcontrollers, an instruction cycle takes four clock periods. Thus the microcontroller operates at a clock rate that is one-quarter of the actual oscillator frequency. The PIC18F series of microcontrollers can operate with clock frequencies up to 40MHz.

1.3.3 Timers

Timers are important parts of any microcontroller. A timer is basically a counter which is driven from either an external clock pulse or the microcontroller’s internal oscillator. A timer can be 8 bits or 16 bits wide. Data can be loaded into a timer under program control, and the timer can be stopped or started by program control. Most timers can be
configured to generate an interrupt when they reach a certain count (usually when they
overflow). The user program can use an interrupt to carry out accurate timing-related
operations inside the microcontroller. Microcontrollers in the PIC18F series have at
least three timers. For example, the PIC18F452 microcontroller has three built-in
timers.

Some microcontrollers offer capture and compare facilities, where a timer value can be
read when an external event occurs, or the timer value can be compared to a preset
value, and an interrupt is generated when this value is reached. Most PIC18F
microcontrollers have at least two capture and compare modules.

1.3.4 Watchdog

Most microcontrollers have at least one watchdog facility. The watchdog is basically a
timer that is refreshed by the user program. Whenever the program fails to refresh
the watchdog, a reset occurs. The watchdog timer is used to detect a system problem,
such as the program being in an endless loop. This safety feature prevents runaway
software and stops the microcontroller from executing meaningless and unwanted
code. Watchdog facilities are commonly used in real-time systems where the
successful termination of one or more activities must be checked regularly.

1.3.5 Reset Input

A reset input is used to reset a microcontroller externally. Resetting puts the
microcontroller into a known state such that the program execution starts from address
0 of the program memory. An external reset action is usually achieved by connecting
a push-button switch to the reset input. When the switch is pressed, the microcontroller
is reset.

1.3.6 Interrupts

Interrupts are an important concept in microcontrollers. An interrupt causes the
microcontroller to respond to external and internal (e.g., a timer) events very quickly.
When an interrupt occurs, the microcontroller leaves its normal flow of program
execution and jumps to a special part of the program known as the interrupt service
routine (ISR). The program code inside the ISR is executed, and upon return from the
ISR the program resumes its normal flow of execution.
The ISR starts from a fixed address of the program memory sometimes known as the interrupt vector address. Some microcontrollers with multi-interrupt features have just one interrupt vector address, while others have unique interrupt vector addresses, one for each interrupt source. Interrupts can be nested such that a new interrupt can suspend the execution of another interrupt. Another important feature of multi-interrupt capability is that different interrupt sources can be assigned different levels of priority. For example, the PIC18F series of microcontrollers has both low-priority and high-priority interrupt levels.

### 1.3.7 Brown-out Detector

Brown-out detectors, which are common in many microcontrollers, reset the microcontroller if the supply voltage falls below a nominal value. These safety features can be employed to prevent unpredictable operation at low voltages, especially to protect the contents of EEPROM-type memories.

### 1.3.8 Analog-to-Digital Converter

An analog-to-digital converter (A/D) is used to convert an analog signal, such as voltage, to digital form so a microcontroller can read and process it. Some microcontrollers have built-in A/D converters. External A/D converter can also be connected to any type of microcontroller. A/D converters are usually 8 to 10 bits, having 256 to 1024 quantization levels. Most PIC microcontrollers with A/D features have multiplexed A/D converters which provide more than one analog input channel. For example, the PIC18F452 microcontroller has 10-bit 8-channel A/D converters.

The A/D conversion process must be started by the user program and may take several hundred microseconds to complete. A/D converters usually generate interrupts when a conversion is complete so the user program can read the converted data quickly.

A/D converters are especially useful in control and monitoring applications, since most sensors (e.g., temperature sensors, pressure sensors, force sensors, etc.) produce analog output voltages.

### 1.3.9 Serial Input-Output

Serial communication (also called RS232 communication) enables a microcontroller to be connected to another microcontroller or to a PC using a serial cable. Some
microcontrollers have built-in hardware called USART (universal synchronous-asynchronous receiver-transmitter) to implement a serial communication interface. The user program can usually select the baud rate and data format. If no serial input-output hardware is provided, it is easy to develop software to implement serial data communication using any I/O pin of a microcontroller. The PIC18F series of microcontrollers has built-in USART modules. We shall see in Chapter 6 how to write mikroC programs to implement serial communication with and without a USART module.

Some microcontrollers (e.g., the PIC18F series) incorporate SPI (serial peripheral interface) or I²C (integrated interconnect) hardware bus interfaces. These enable a microcontroller to interface with other compatible devices easily.

1.3.10 EEPROM Data Memory

EEPROM-type data memory is also very common in many microcontrollers. The advantage of an EEPROM memory is that the programmer can store nonvolatile data there and change this data whenever required. For example, in a temperature monitoring application, the maximum and minimum temperature readings can be stored in an EEPROM memory. If the power supply is removed for any reason, the values of the latest readings are available in the EEPROM memory. The PIC18F452 microcontroller has 256 bytes of EEPROM memory. Other members of the PIC18F family have more EEPROM memory (e.g., the PIC18F6680 has 1024 bytes). The mikroC language provides special instructions for reading and writing to the EEPROM memory of a PIC microcontroller.

1.3.11 LCD Drivers

LCD drivers enable a microcontroller to be connected to an external LCD display directly. These drivers are not common since most of the functions they provide can be implemented in software. For example, the PIC18F6490 microcontroller has a built-in LCD driver module.

1.3.12 Analog Comparator

Analog comparators are used where two analog voltages need to be compared. Although these circuits are implemented in most high-end PIC microcontrollers, they are not common in other microcontrollers. The PIC18F series of microcontrollers has built-in analog comparator modules.
1.3.13 Real-time Clock

A real-time clock enables a microcontroller to receive absolute date and time information continuously. Built-in real-time clocks are not common in most microcontrollers, since the same function can easily be implemented by either a dedicated real-time clock chip or a program written for this purpose.

1.3.14 Sleep Mode

Some microcontrollers (e.g., PICs) offer built-in sleep modes, where executing this instruction stops the internal oscillator and reduces power consumption to an extremely low level. The sleep mode’s main purpose is to conserve battery power when the microcontroller is not doing anything useful. The microcontroller is usually woken up from sleep mode by an external reset or a watchdog time-out.

1.3.15 Power-on Reset

Some microcontrollers (e.g., PICs) have built-in power-on reset circuits which keep the microcontroller in the reset state until all the internal circuitry has been initialized. This feature is very useful, as it starts the microcontroller from a known state on power-up. An external reset can also be provided, where the microcontroller is reset when an external button is pressed.

1.3.16 Low-Power Operation

Low-power operation is especially important in portable applications where microcontroller-based equipment is operated from batteries. Some microcontrollers (e.g., PICs) can operate with less than 2mA with a 5V supply, and around 15μA at a 3V supply. Other microcontrollers, especially microprocessor-based systems with several chips, may consume several hundred milliamperes or even more.

1.3.17 Current Sink/Source Capability

Current sink/source capability is important if the microcontroller is to be connected to an external device that might draw a large amount of current to operate. PIC microcontrollers can source and sink 25mA of current from each output port pin. This current is usually sufficient to drive LEDs, small lamps, buzzers, small relays, etc. The
current capability can be increased by connecting external transistor switching circuits or relays to the output port pins.

1.3.18 USB Interface

USB is currently a very popular computer interface specification used to connect various peripheral devices to computers and microcontrollers. Some PIC microcontrollers provide built-in USB modules. The PIC18F2x50, for example, has built-in USB interface capabilities.

1.3.19 Motor Control Interface

Some PIC microcontrollers, for example the PIC18F2x31, provide motor control interface capability.

1.3.20 CAN Interface

CAN bus is a very popular bus system used mainly in automation applications. Some PIC18F-series microcontrollers (e.g., the PIC18F4680) provide CAN interface capability.

1.3.21 Ethernet Interface

Some PIC microcontrollers (e.g., the PIC18F97J60) provide Ethernet interface capabilities and thus are easily used in network-based applications.

1.3.22 ZigBee Interface

ZigBee, an interface similar to Bluetooth, is used in low-cost wireless home automation applications. Some PIC18F-series microcontrollers provide ZigBee interface capabilities, making the design of such wireless systems very easy.

1.4 Microcontroller Architectures

Two types of architectures are conventional in microcontrollers (see Figure 1.4). Von Neumann architecture, used by a large percentage of microcontrollers, places all memory space on the same bus; instruction and data also use the same bus.
In *Harvard* architecture (used by PIC microcontrollers), code and data are on separate buses, which allows them to be fetched simultaneously, resulting in an improved performance.

### 1.4.1 RISC and CISC

RISC (reduced instruction set computer) and CISC (complex instruction computer) refer to the instruction set of a microcontroller. In an 8-bit RISC microcontroller, data is 8 bits wide but the instruction words are more than 8 bits wide (usually 12, 14, or 16 bits) and the instructions occupy one word in the program memory. Thus the instructions are fetched and executed in one cycle, which improves performance.

In a CISC microcontroller, both data and instructions are 8 bits wide. CISC microcontrollers usually have over two hundred instructions. Data and code are on the same bus and cannot be fetched simultaneously.

### 1.5 Number Systems

To use a microprocessor or microcontroller efficiently requires a working knowledge of binary, decimal, and hexadecimal numbering systems. This section provides background information about these numbering systems for readers who are unfamiliar with them or do not know how to convert from one number system to another.

Number systems are classified according to their bases. The numbering system used in everyday life is base 10, or the decimal number system. The numbering system most
commonly used in microprocessor and microcontroller applications is base 16, or hexadecimal. Base 2, or binary, and base 8, or octal, number systems are also used.

1.5.1 Decimal Number System

The numbers in the decimal number system, of course, are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. The subscript 10 indicates that a number is in decimal format. For example, the decimal number 235 is shown as \(235_{10}\).

In general, a decimal number is represented as follows:

\[ a_n \times 10^n + a_{n-1} \times 10^{n-1} + a_{n-2} \times 10^{n-2} + \ldots + a_0 \times 10^0 \]

For example, decimal number \(825_{10}\) can be shown as:

\[825_{10} = 8 \times 10^2 + 2 \times 10^1 + 5 \times 10^0\]

Similarly, decimal number \(26_{10}\) can be shown as:

\[26_{10} = 2 \times 10^1 + 6 \times 10^0\]

or

\[3359_{10} = 3 \times 10^3 + 3 \times 10^2 + 5 \times 10^1 + 9 \times 10^0\]

1.5.2 Binary Number System

The binary number system consists of two numbers: 0 and 1. A subscript 2 indicates that a number is in binary format. For example, the binary number 1011 would be \(1011_2\).

In general, a binary number is represented as follows:

\[ a_n \times 2^n + a_{n-1} \times 2^{n-1} + a_{n-2} \times 2^{n-2} + \ldots + a_0 \times 2^0 \]

For example, binary number \(1110_2\) can be shown as:

\[1110_2 = 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0\]
Similarly, binary number $1001110_2$ can be shown as:

$$1001110_2 = 1 \times 2^7 + 0 \times 2^6 + 0 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$$

### 1.5.3 Octal Number System

In the octal number system, the valid numbers are 0, 1, 2, 3, 4, 5, 6, 7. A subscript 8 indicates that a number is in octal format. For example, the octal number 23 appears as $23_8$.

In general, an octal number is represented as:

$$a_n \times 8^n + a_{n-1} \times 8^{n-1} + a_{n-2} \times 8^{n-2} + \ldots \ldots + a_0 \times 8^0$$

For example, octal number $237_8$ can be shown as:

$$237_8 = 2 \times 8^2 + 3 \times 8^1 + 7 \times 8^0$$

Similarly, octal number $1777_8$ can be shown as:

$$1777_8 = 1 \times 8^3 + 7 \times 8^2 + 7 \times 8^1 + 7 \times 8^0$$

### 1.5.4 Hexadecimal Number System

In the hexadecimal number system, the valid numbers are: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F. A subscript 16 or subscript H indicates that a number is in hexadecimal format. For example, hexadecimal number 1F can be written as $1F_{16}$ or as $1F_{H}$.

In general, a hexadecimal number is represented as:

$$a_n \times 16^n + a_{n-1} \times 16^{n-1} + a_{n-2} \times 16^{n-2} + \ldots \ldots + a_0 \times 16^0$$

For example, hexadecimal number $2AC_{16}$ can be shown as:

$$2AC_{16} = 2 \times 16^2 + 10 \times 16^1 + 12 \times 16^0$$

Similarly, hexadecimal number $3FFE_{16}$ can be shown as:

$$3FFE_{16} = 3 \times 16^3 + 15 \times 16^2 + 15 \times 16^1 + 14 \times 16^0$$
1.6 Converting Binary Numbers into Decimal

To convert a binary number into decimal, write the number as the sum of the powers of 2.

Example 1.1

Convert binary number 1011₂ into decimal.

Solution 1.1

Write the number as the sum of the powers of 2:

\[
1011₂ = 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0
\]
\[
= 8 + 0 + 2 + 1
\]
\[
= 11
\]

or, \(1011₂ = 11_{10}\)

Example 1.2

Convert binary number 11001110₂ into decimal.

Solution 1.2

Write the number as the sum of the powers of 2:

\[
11001110₂ = 1 \times 2^7 + 1 \times 2^6 + 0 \times 2^5 + 0 \times 2^4
\]
\[
+ 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0
\]
\[
= 128 + 64 + 0 + 0 + 8 + 4 + 2 + 0
\]
\[
= 206
\]

or, \(11001110₂ = 206_{10}\)

Table 1.1 shows the decimal equivalent of numbers from 0 to 31.

1.7 Converting Decimal Numbers into Binary

To convert a decimal number into binary, divide the number repeatedly by 2 and take the remainders. The first remainder is the least significant digit (LSD), and the last remainder is the most significant digit (MSD).

Example 1.3

Convert decimal number 28₁₀ into binary.
Table 1.1: Decimal equivalent of binary numbers

<table>
<thead>
<tr>
<th>Binary</th>
<th>Decimal</th>
<th>Binary</th>
<th>Decimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>00000000</td>
<td>0</td>
<td>00010000</td>
<td>16</td>
</tr>
<tr>
<td>00000001</td>
<td>1</td>
<td>00010001</td>
<td>17</td>
</tr>
<tr>
<td>00000010</td>
<td>2</td>
<td>00010010</td>
<td>18</td>
</tr>
<tr>
<td>00000011</td>
<td>3</td>
<td>00010011</td>
<td>19</td>
</tr>
<tr>
<td>00000100</td>
<td>4</td>
<td>00010100</td>
<td>20</td>
</tr>
<tr>
<td>00000101</td>
<td>5</td>
<td>00010101</td>
<td>21</td>
</tr>
<tr>
<td>00000110</td>
<td>6</td>
<td>00010110</td>
<td>22</td>
</tr>
<tr>
<td>00000111</td>
<td>7</td>
<td>00010111</td>
<td>23</td>
</tr>
<tr>
<td>00001000</td>
<td>8</td>
<td>00011000</td>
<td>24</td>
</tr>
<tr>
<td>00001001</td>
<td>9</td>
<td>00011001</td>
<td>25</td>
</tr>
<tr>
<td>00001010</td>
<td>10</td>
<td>00011010</td>
<td>26</td>
</tr>
<tr>
<td>00001011</td>
<td>11</td>
<td>00011011</td>
<td>27</td>
</tr>
<tr>
<td>00001100</td>
<td>12</td>
<td>00011100</td>
<td>28</td>
</tr>
<tr>
<td>00001101</td>
<td>13</td>
<td>00011101</td>
<td>29</td>
</tr>
<tr>
<td>00001110</td>
<td>14</td>
<td>00011110</td>
<td>30</td>
</tr>
<tr>
<td>00001111</td>
<td>15</td>
<td>00011111</td>
<td>31</td>
</tr>
</tbody>
</table>

Solution 1.3

Divide the number into 2 repeatedly and take the remainders:

\[
\begin{align*}
28/2 & \rightarrow 14 \text{ Remainder 0 (LSD)} \\
14/2 & \rightarrow 7 \text{ Remainder 0} \\
7/2 & \rightarrow 3 \text{ Remainder 1} \\
3/2 & \rightarrow 1 \text{ Remainder 1} \\
1/2 & \rightarrow 0 \text{ Remainder 1 (MSD)}
\end{align*}
\]

The binary number is \(11100_2\).
Example 1.4

Convert decimal number $65_{10}$ into binary.

Solution 1.4

Divide the number into 2 repeatedly and take the remainders:

\[
\begin{align*}
65/2 &\rightarrow 32 \text{ Remainder 1 (LSD)} \\
32/2 &\rightarrow 16 \text{ Remainder 0} \\
16/2 &\rightarrow 8 \text{ Remainder 0} \\
8/2 &\rightarrow 4 \text{ Remainder 0} \\
4/2 &\rightarrow 2 \text{ Remainder 0} \\
2/2 &\rightarrow 1 \text{ Remainder 0} \\
1/2 &\rightarrow 0 \text{ Remainder 1 (MSD)}
\end{align*}
\]

The binary number is $1000001_2$.

Example 1.5

Convert decimal number $122_{10}$ into binary.

Solution 1.5

Divide the number into 2 repeatedly and take the remainders:

\[
\begin{align*}
122/2 &\rightarrow 61 \text{ Remainder 0 (LSD)} \\
61/2 &\rightarrow 30 \text{ Remainder 1} \\
30/2 &\rightarrow 15 \text{ Remainder 0} \\
15/2 &\rightarrow 7 \text{ Remainder 1} \\
7/2 &\rightarrow 3 \text{ Remainder 1} \\
3/2 &\rightarrow 1 \text{ Remainder 1} \\
1/2 &\rightarrow 0 \text{ Remainder 1 (MSD)}
\end{align*}
\]

The binary number is $1111010_2$.

1.8 Converting Binary Numbers into Hexadecimal

To convert a binary number into hexadecimal, arrange the number in groups of four and find the hexadecimal equivalent of each group. If the number cannot be divided exactly into groups of four, insert zeros to the left of the number as needed so the number of digits are divisible by four.
Example 1.6

Convert binary number $10011111_2$ into hexadecimal.

Solution 1.6

First, divide the number into groups of four, then find the hexadecimal equivalent of each group:

$$10011111 = 1001\ 1111$$

$$9\ F$$

The hexadecimal number is $9F_{16}$.

Example 1.7

Convert binary number $1110111100001110_2$ into hexadecimal.

Solution 1.7

First, divide the number into groups of four, then find the hexadecimal equivalent of each group:

$$1110111100001110 = 1110\ 1111\ 0000\ 1110$$

$$E\ F\ 0\ E$$

The hexadecimal number is $EF0E_{16}$.

Example 1.8

Convert binary number $111110_2$ into hexadecimal.

Solution 1.8

Since the number cannot be divided exactly into groups of four, we have to insert, in this case, two zeros to the left of the number so the number of digits is divisible by four:

$$111110 = 0011\ 1110$$

$$3\ E$$

The hexadecimal number is $3E_{16}$.

Table 1.2 shows the hexadecimal equivalent of numbers 0 to 31.
1.9 Converting Hexadecimal Numbers into Binary

To convert a hexadecimal number into binary, write the 4-bit binary equivalent of each hexadecimal digit.

Example 1.9

Convert hexadecimal number A9\textsubscript{16} into binary.

Table 1.2: Hexadecimal equivalent of decimal numbers

<table>
<thead>
<tr>
<th>Decimal</th>
<th>Hexadecimal</th>
<th>Decimal</th>
<th>Hexadecimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>16</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>17</td>
<td>11</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>18</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>19</td>
<td>13</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>20</td>
<td>14</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>21</td>
<td>15</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>22</td>
<td>16</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>23</td>
<td>17</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>24</td>
<td>18</td>
</tr>
<tr>
<td>9</td>
<td>9</td>
<td>25</td>
<td>19</td>
</tr>
<tr>
<td>10</td>
<td>A</td>
<td>26</td>
<td>1A</td>
</tr>
<tr>
<td>11</td>
<td>B</td>
<td>27</td>
<td>1B</td>
</tr>
<tr>
<td>12</td>
<td>C</td>
<td>28</td>
<td>1C</td>
</tr>
<tr>
<td>13</td>
<td>D</td>
<td>29</td>
<td>1D</td>
</tr>
<tr>
<td>14</td>
<td>E</td>
<td>30</td>
<td>1E</td>
</tr>
<tr>
<td>15</td>
<td>F</td>
<td>31</td>
<td>1F</td>
</tr>
</tbody>
</table>
Solution 1.9

Writing the binary equivalent of each hexadecimal digit:

A = 1010₂  9 = 1001₂

The binary number is 10101001₂.

Example 1.10

Convert hexadecimal number FE3₁₆ into binary.

Solution 1.10

Writing the binary equivalent of each hexadecimal digit:

F = 1111₂  E = 1110₂  3 = 0011₂  C = 1100₂

The binary number is 1111111000111100₂.

1.10 Converting Hexadecimal Numbers into Decimal

To convert a hexadecimal number into decimal, calculate the sum of the powers of 16 of the number.

Example 1.11

Convert hexadecimal number 2AC₁₆ into decimal.

Solution 1.11

Calculating the sum of the powers of 16 of the number:

\[ 2AC_{16} = 2 \times 16^2 + 10 \times 16^1 + 12 \times 16^0 \]
\[ = 512 + 160 + 12 \]
\[ = 684 \]

The required decimal number is 684₁₀.

Example 1.12

Convert hexadecimal number EE₁₆ into decimal.
Solution 1.12

Calculating the sum of the powers of 16 of the number:

\[
EE_{16} = 14 \times 16^1 + 14 \times 16^0 \\
= 224 + 14 \\
= 238
\]

The decimal number is \(238_{10}\).

1.11 Converting Decimal Numbers into Hexadecimal

To convert a decimal number into hexadecimal, divide the number repeatedly by 16 and take the remainders. The first remainder is the LSD, and the last remainder is the MSD.

Example 1.13

Convert decimal number \(238_{10}\) into hexadecimal.

Solution 1.13

Dividing the number repeatedly by 16:

\[
\begin{align*}
238/16 & \rightarrow 14 \text{ Remainder } 14 \text{ (E) (LSD)} \\
14/16 & \rightarrow 0 \text{ Remainder } 14 \text{ (E) (MSD)}
\end{align*}
\]

The hexadecimal number is \(EE_{16}\).

Example 1.14

Convert decimal number \(684_{10}\) into hexadecimal.

Solution 1.14

Dividing the number repeatedly by 16:

\[
\begin{align*}
684/16 & \rightarrow 42 \text{ Remainder } 12 \text{ (C) (LSD)} \\
42/16 & \rightarrow 2 \text{ Remainder } 10 \text{ (A)} \\
2/16 & \rightarrow 0 \text{ Remainder } 2 \text{ (MSD)}
\end{align*}
\]

The hexadecimal number is \(2AC_{16}\).
1.12 Converting Octal Numbers into Decimal

To convert an octal number into decimal, calculate the sum of the powers of 8 of the number.

Example 1.15

Convert octal number 158 into decimal.

Solution 1.15

Calculating the sum of the powers of 8 of the number:

\[ 15_8 = 1 \times 8^1 + 5 \times 8^0 \]
\[ = 8 + 5 \]
\[ = 13 \]

The decimal number is 1310.

Example 1.16

Convert octal number 2378 into decimal.

Solution 1.16

Calculating the sum of the powers of 8 of the number:

\[ 237_8 = 2 \times 8^2 + 3 \times 8^1 + 7 \times 8^0 \]
\[ = 128 + 24 + 7 \]
\[ = 159 \]

The decimal number is 15910.

1.13 Converting Decimal Numbers into Octal

To convert a decimal number into octal, divide the number repeatedly by 8 and take the remainders. The first remainder is the LSD, and the last remainder is the MSD.

Example 1.17

Convert decimal number 15910 into octal.
Solution 1.17

Dividing the number repeatedly by 8:
\[
\begin{align*}
159/8 & \rightarrow 19 \text{ Remainder 7} \text{ (LSD)} \\
19/8 & \rightarrow 2 \text{ Remainder 3} \\
2/8 & \rightarrow 0 \text{ Remainder 2} \text{ (MSD)}
\end{align*}
\]

The octal number is \(237_8\).

Example 1.18

Convert decimal number \(460_{10}\) into octal.

Solution 1.18

Dividing the number repeatedly by 8:
\[
\begin{align*}
460/8 & \rightarrow 57 \text{ Remainder 4} \text{ (LSD)} \\
57/8 & \rightarrow 7 \text{ Remainder 1} \\
7/8 & \rightarrow 0 \text{ Remainder 7} \text{ (MSD)}
\end{align*}
\]

The octal number is \(714_8\).

Table 1.3 shows the octal equivalent of decimal numbers 0 to 31.

1.14 Converting Octal Numbers into Binary

To convert an octal number into binary, write the 3-bit binary equivalent of each octal digit.

Example 1.19

Convert octal number \(177_8\) into binary.

Solution 1.19

Write the binary equivalent of each octal digit:
\[
1 = 001_2 \quad 7 = 111_2 \quad 7 = 111_2
\]

The binary number is \(00111111_2\).
Example 1.20

Convert octal number $75_8$ into binary.

Solution 1.20

Write the binary equivalent of each octal digit:

$7 = 111_2$  $5 = 101_2$

The binary number is $111101_2$. 

<table>
<thead>
<tr>
<th>Decimal</th>
<th>Octal</th>
<th>Decimal</th>
<th>Octal</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>16</td>
<td>20</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>17</td>
<td>21</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>18</td>
<td>22</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>19</td>
<td>23</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>20</td>
<td>24</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>21</td>
<td>25</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>22</td>
<td>26</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>23</td>
<td>27</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>24</td>
<td>30</td>
</tr>
<tr>
<td>9</td>
<td>11</td>
<td>25</td>
<td>31</td>
</tr>
<tr>
<td>10</td>
<td>12</td>
<td>26</td>
<td>32</td>
</tr>
<tr>
<td>11</td>
<td>13</td>
<td>27</td>
<td>33</td>
</tr>
<tr>
<td>12</td>
<td>14</td>
<td>28</td>
<td>34</td>
</tr>
<tr>
<td>13</td>
<td>15</td>
<td>29</td>
<td>35</td>
</tr>
<tr>
<td>14</td>
<td>16</td>
<td>30</td>
<td>36</td>
</tr>
<tr>
<td>15</td>
<td>17</td>
<td>31</td>
<td>37</td>
</tr>
</tbody>
</table>
1.15 Converting Binary Numbers into Octal

To convert a binary number into octal, arrange the number in groups of three and write the octal equivalent of each digit.

Example 1.21

Convert binary number $110111001_2$ into octal.

Solution 1.21

Arranging in groups of three:

$110111001 = 110 \ 111 \ 001$

$6 \ 7 \ 1$

The octal number is $671_8$.

1.16 Negative Numbers

The most significant bit of a binary number is usually used as the sign bit. By convention, for positive numbers this bit is 0, and for negative numbers this bit is 1. Figure 1.5 shows the 4-bit positive and negative numbers. The largest positive and negative numbers are $+7$ and $-8$ respectively.

<table>
<thead>
<tr>
<th>Binary number</th>
<th>Decimal equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>0111</td>
<td>+7</td>
</tr>
<tr>
<td>0110</td>
<td>+6</td>
</tr>
<tr>
<td>0101</td>
<td>+5</td>
</tr>
<tr>
<td>0100</td>
<td>+4</td>
</tr>
<tr>
<td>0011</td>
<td>+3</td>
</tr>
<tr>
<td>0010</td>
<td>+2</td>
</tr>
<tr>
<td>0001</td>
<td>+1</td>
</tr>
<tr>
<td>0000</td>
<td>0</td>
</tr>
<tr>
<td>1111</td>
<td>-1</td>
</tr>
<tr>
<td>1110</td>
<td>-2</td>
</tr>
<tr>
<td>1101</td>
<td>-3</td>
</tr>
<tr>
<td>1100</td>
<td>-4</td>
</tr>
<tr>
<td>1011</td>
<td>-5</td>
</tr>
<tr>
<td>1010</td>
<td>-6</td>
</tr>
<tr>
<td>1001</td>
<td>-7</td>
</tr>
<tr>
<td>1000</td>
<td>-8</td>
</tr>
</tbody>
</table>

Figure 1.5: 4-bit positive and negative numbers
To convert a positive number to negative, take the complement of the number and add 1. This process is also called the 2’s complement of the number.

**Example 1.22**

Write decimal number $-6$ as a 4-bit number.

**Solution 1.22**

First, write the number as a positive number, then find the complement and add 1:

\[
\begin{array}{c}
0110 +6 \\
1001 \text{ complement} \\
1 \text{ add 1} \\
\hline
1010 \text{ which is } -6
\end{array}
\]

**Example 1.23**

Write decimal number $-25$ as an 8-bit number.

**Solution 1.23**

First, write the number as a positive number, then find the complement and add 1:

\[
\begin{array}{c}
00011001 +25 \\
11100110 \text{ complement} \\
1 \text{ add 1} \\
\hline
11100111 \text{ which is } -25
\end{array}
\]

### 1.17 Adding Binary Numbers

The addition of binary numbers is similar to the addition of decimal numbers. Numbers in each column are added together with a possible carry from a previous column. The primitive addition operations are:
\[0 + 0 = 0\]
\[0 + 1 = 1\]
\[1 + 0 = 1\]
\[1 + 1 = 10\] generate a carry bit
\[1 + 1 + 1 = 11\] generate a carry bit

Some examples follow.

**Example 1.24**

Find the sum of binary numbers 011 and 110.

**Solution 1.24**

We can add these numbers as in the addition of decimal numbers:

\[
\begin{array}{c}
011 \\
+ 110 \\
\hline
1001
\end{array}
\]

**Example 1.25**

Find the sum of binary numbers 01000011 and 00100010.

**Solution 1.25**

We can add these numbers as in the addition of decimal numbers:

\[
\begin{array}{c}
01000011 \\
+ 00100010 \\
\hline
01100101
\end{array}
\]

\[
\begin{array}{c}
01000011 \\
+ 00100010 \\
\hline
01100101
\end{array}
\]
1.18 Subtracting Binary Numbers

To subtract one binary number from another, convert the number to be subtracted into negative and then add the two numbers.

Example 1.26

Subtract binary number 0010 from 0110.

Solution 1.26

First, convert the number to be subtracted into negative:

\[
\begin{align*}
0010 & \quad \text{number to be subtracted} \\
1101 & \quad \text{complement} \\
1 & \quad \text{add 1} \\
\hline
1110 & \\
\end{align*}
\]

Now add the two numbers:

\[
\begin{align*}
0110 \\
+ 1110 \\
\hline
0100 & \\
\end{align*}
\]

Since we are using only 4 bits, we cannot show the carry bit.

1.19 Multiplication of Binary Numbers

Multiplication of two binary numbers is similar to the multiplication of two decimal numbers. The four possibilities are:

\[
\begin{align*}
0 \times 0 &= 0 \\
0 \times 1 &= 0 \\
1 \times 0 &= 0 \\
1 \times 1 &= 1 \\
\end{align*}
\]

Some examples follow.
Example 1.27

Multiply the two binary numbers 0110 and 0010.

Solution 1.27

Multiplying the numbers:

```
0110
0010
---
0000
0110
0000
0000
---
001100 or 1100
```

In this example 4 bits are needed to show the final result.

Example 1.28

Multiply binary numbers 1001 and 1010.

Solution 1.28

Multiplying the numbers:

```
1001
1010
---
0000
1001
0000
1001
---
1011010
```

In this example 7 bits are required to show the final result.
1.20 Division of Binary Numbers

Division with binary numbers is similar to division with decimal numbers. An example follows.

Example 1.29

Divide binary number 1110 into binary number 10.

Solution 1.29

Dividing the numbers:

\[
\begin{array}{c|c}
111 & \\
10 & 1110 \\
- & 10 \\
- & 11 \\
- & 10 \\
- & 10 \\
- & 00 \\
\end{array}
\]

gives the result 111₂.

1.21 Floating Point Numbers

Floating point numbers are used to represent noninteger fractional numbers, for example, 3.256, 2.1, 0.0036, and so forth. Floating point numbers are used in most engineering and technical calculations. The most common floating point standard is the IEEE standard, according to which floating point numbers are represented with 32 bits (single precision) or 64 bits (double precision).

In this section we are looking at the format of 32-bit floating point numbers only and seeing how mathematical operations can be performed with such numbers.
According to the IEEE standard, 32-bit floating point numbers are represented as:

\[
\begin{array}{cccccccc}
31 & 30 & 29 & 28 & 27 & 26 & 25 & 24 \\
& X & X & X & X & X & X & X \\
\uparrow & \uparrow & \uparrow & \uparrow \\
\text{sign} & \text{exponent} & \text{mantissa} \\
\end{array}
\]

The most significant bit indicates the sign of the number, where 0 indicates the number is positive, and 1 indicates it is negative.

The 8-bit exponent shows the power of the number. To make the calculations easy, the sign of the exponent is not shown; instead, the excess-128 numbering system is used. Thus, to find the real exponent we have to subtract 127 from the given exponent. For example, if the mantissa is “10000000,” the real value of the mantissa is \(128 - 127 = 1\).

The mantissa is 23 bits wide and represents the increasing negative powers of 2. For example, if we assume that the mantissa is “1110000000000000000000,” the value of this mantissa is calculated as \(2^{-1} + 2^{-2} + 2^{-3} = \frac{7}{8}\).

The decimal equivalent of a floating point number can be calculated using the formula:

\[
\text{Number} = (-1)^s \times 2^{e-127} \times f
\]

where

\[
\begin{align*}
\text{\(s\)} & = 0 \text{ for positive numbers, 1 for negative numbers} \\
\text{\(e\)} & = \text{exponent (between 0 and 255)} \\
\text{\(f\)} & = \text{mantissa}
\end{align*}
\]

As shown in this formula, there is a hidden 1 in front of the mantissa (i.e, the mantissa is shown as \(1.f\)).

The largest number in 32-bit floating point format is:

\[
0 \quad 11111110 \quad 111111111111111111111111
\]

This number is \((2 - 2^{-23}) \times 2^{127}\) or decimal \(3.403 \times 10^{38}\). The numbers keep their precision up to 6 digits after the decimal point.
The smallest number in 32-bit floating point format is:
0 00000001 00000000000000000000000

This number is $2^{-126}$ or decimal $1.175 \times 10^{-38}$.

### 1.22 Converting a Floating Point Number into Decimal

To convert a given floating point number into decimal, we have to find the mantissa and the exponent of the number and then convert into decimal as just shown.

Some examples are given here.

**Example 1.30**

Find the decimal equivalent of the floating point number: 0 10000001
10000000000000000000000

**Solution 1.30**

Here

- sign = positive
- exponent = $129 - 127 = 2$
- mantissa = $2^{-1} = 0.5$

The decimal equivalent of this number is $+1.5 \times 2^2 = +6.0$.

**Example 1.31**

Find the decimal equivalent of the floating point number: 0 10000010
11000000000000000000000

**Solution 1.31**

In this example,

- sign = positive
- exponent = $130 - 127 = 3$
- mantissa = $2^{-1} + 2^{-2} = 0.75$

The decimal equivalent of the number is $+1.75 \times 2^3 = 14.0$. 
1.22.1 Normalizing Floating Point Numbers

Floating point numbers are usually shown in normalized form. A normalized number has only one digit before the decimal point (a hidden number 1 is assumed before the decimal point).

To normalize a given floating point number, we have to move the decimal point repeatedly one digit to the left and increase the exponent after each move.

Some examples follow.

Example 1.32

Normalize the floating point number 123.56

Solution 1.32

If we write the number with a single digit before the decimal point we get:

1.2356 \times 10^2

Example 1.33

Normalize the binary number 1011.12

Solution 1.33

If we write the number with a single digit before the decimal point we get:

1.0111 \times 2^3

1.22.2 Converting a Decimal Number into Floating Point

To convert a given decimal number into floating point, carry out the following steps:

- Write the number in binary.
- Normalize the number.
- Find the mantissa and the exponent.
- Write the number as a floating point number.

Some examples follow:
Example 1.34

Convert decimal number 2.25\textsubscript{10} into floating point.

Solution 1.34

Write the number in binary:

\[ 2.25\textsubscript{10} = 10.01\textsubscript{2} \]

Normalize the number:

\[ 10.01\textsubscript{2} = 1.001\textsubscript{2} \times 2^{1} \]

Here, \( s = 0 \), \( e – 127 = 1 \) or \( e = 128 \), and \( f = 00100000000000000000000 \).

(Remember that a number 1 is assumed on the left side, even though it is not shown in the calculation). The required floating point number can be written as:

\[
\begin{array}{ccc}
  s & e & f \\
  0 & 10000000 & (1)00100000000000000000000000000000 \\
\end{array}
\]

or, the required 32-bit floating point number is:

\[ 010000000001000000000000000000000000 \]

Example 1.35

Convert the decimal number 134.0625\textsubscript{10} into floating point.

Solution 1.35

Write the number in binary:

\[ 134.0625\textsubscript{10} = 10000110.0001 \]

Normalize the number:

\[ 10000110.0001 = 1.00001100001 \times 2^{7} \]

Here, \( s = 0 \), \( e – 127 = 7 \) or \( e = 134 \), and \( f = 0000110000010000000000000000000000 \).
The required floating point number can be written as:

\[ s \quad e \quad f \]
\[ 0 \quad 10000110 \quad (1)00011000010000000000000 \]

or, the required 32-bit floating point number is:

\[ 01000011000001100001000000000000 \]

### 1.22.3 Multiplication and Division of Floating Point Numbers

Multiplication and division of floating point numbers are rather easy. Here are the steps:

- Add (or subtract) the exponents of the numbers.
- Multiply (or divide) the mantissa of the numbers.
- Correct the exponent.
- Normalize the number.
- The sign of the result is the EXOR of the signs of the two numbers.

Since the exponent is processed twice in the calculations, we have to subtract 127 from the exponent.

An example showing the multiplication of two floating point numbers follows.

**Example 1.36**

Show the decimal numbers 0.510 and 0.7510 in floating point and then calculate their multiplication.

**Solution 1.36**

Convert the numbers into floating point as:

\[ 0.5_{10} = 1.0000 \times 2^{-1} \]

Here, \( s = 0, \ e - 127 = -1 \) or \( e = 126 \) and \( f = 0000 \)

Or,

\[ 0.5_{10} = 0 \quad 01110110 \quad (1)000 \quad 0000 \quad 0000 \quad 0000 \quad 0000 \quad 0000 \]

Similarly,

\[ 0.75_{10} = 1.1000 \times 2^{-1} \]
here, \( s = 0, \ e = 126 \) and \( f = 1000 \)
or,
\[
0.75_{10} = 0 \ 01110110 \ (1)100 \ 0000 \ 0000 \ 0000 \ 0000 \ 0000
\]

Multiplying the mantissas results in “\((1)100 \ 0000 \ 0000 \ 0000 \ 0000 \ 0000\).” The sum of the exponents is \(126 + 126 = 252\). Subtracting 127 from the mantissa, we obtain \(252 - 127 = 125\). The EXOR of the signs of the numbers is 0. Thus, the result can be shown in floating point as:

\[
0 \ 01111011 \ (1)100 \ 0000 \ 0000 \ 0000 \ 0000 \ 0000
\]

This number is equivalent to decimal \(0.375\) \((0.5 \times 0.75 = 0.375)\), which is the correct result.

1.22.4 Addition and Subtraction of Floating Point Numbers

The exponents of floating point numbers must be the same before they can be added or subtracted. The steps to add or subtract floating point numbers are:

- Shift the smaller number to the right until the exponents of both numbers are the same. Increment the exponent of the smaller number after each shift.
- Add (or subtract) the mantissa of each number as an integer calculation, without considering the decimal points.
- Normalize the result.

An example follows.

Example 1.37

Show decimal numbers \(0.510\) and \(0.7510\) in floating point and then calculate the sum of these numbers.

Solution 1.37

As shown in Example 1.36, we can convert the numbers into floating point as:

\[
0.5_{10} = 0 \ 01110110 \ (1)000 \ 0000 \ 0000 \ 0000 \ 0000 \ 0000
\]
Similarly,

\[ 0.75_{10} = 0.01110110 \ (1)100\ 0000\ 0000\ 0000\ 0000\ 0000 \]

Since the exponents of both numbers are the same, there is no need to shift the smaller number. If we add the mantissa of the numbers without considering the decimal points, we get:

\[
\begin{array}{c}
(1)000\ 0000\ 0000\ 0000\ 0000\ 0000 \\
(1)100\ 0000\ 0000\ 0000\ 0000\ 0000 \\
\hline \\
(10)100\ 0000\ 0000\ 0000\ 0000\ 0000 \\
\end{array}
\]

To normalize the number, shift it right by one digit and then increment its exponent. The resulting number is:

\[ 0.01111111 (1)010\ 0000\ 0000\ 0000\ 0000\ 0000 \]

This floating point number is equal to decimal number 1.25, which is the sum of decimal numbers 0.5 and 0.75.

A program for converting floating point numbers into decimal, and decimal numbers into floating point, is available for free on the following web site:

http://babbage.cs.qc.edu/courses/cs341/IEEE-754.html

### 1.23 BCD Numbers

BCD (binary coded decimal) numbers are usually used in display systems such as LCDs and 7-segment displays to show numeric values. In BCD, each digit is a 4-bit number from 0 to 9. As an example, Table 1.4 shows the BCD numbers between 0 and 20.

**Example 1.38**

Write the decimal number 295 as a BCD number.

**Solution 1.38**

Write the 4-bit binary equivalent of each digit:

\[ 2 = 0010_2 \quad 9 = 1001_2 \quad 5 = 0101_2 \]

The BCD number is 0010 1001 0101_2.
Table 1.4: BCD numbers between 0 and 20

<table>
<thead>
<tr>
<th>Decimal</th>
<th>BCD</th>
<th>Binary</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0000</td>
<td>0000</td>
</tr>
<tr>
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<td>20</td>
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Example 1.39
Write the decimal equivalent of BCD number 1001 1001 0110 0001₂.

Solution 1.39
Writing the decimal equivalent of each group of 4-bit yields the decimal number: 9961

1.24 Summary
Chapter 1 has provided an introduction to the microprocessor and microcontroller systems. The basic building blocks of microcontrollers were described briefly. The chapter also provided an introduction to various number systems, and described how to convert a given number from one base into another. The important topics of floating point numbers and floating point arithmetic were also described with examples.

1.25 Exercises
1. What is a microcontroller? What is a microprocessor? Explain the main difference between a microprocessor and a microcontroller.
2. Identify some applications of microcontrollers around you.
3. Where would you use an EPROM memory?
4. Where would you use a RAM memory?
5. Explain the types of memory usually used in microcontrollers.
6. What is an input-output port?
7. What is an analog-to-digital converter? Give an example of how this converter is used.
8. Explain why a watchdog timer could be useful in a real-time system.
9. What is serial input-output? Where would you use serial communication?
10. Why is the current sink/source capability important in the specification of an output port pin?
11. What is an interrupt? Explain what happens when an interrupt is recognized by a microcontroller?

12. Why is brown-out detection important in real-time systems?

13. Explain the difference between an RISC-based microcontroller and a CISC-based microcontroller. What type of microcontroller is PIC?

14. Convert the following decimal numbers into binary:
   a) 23  b) 128  c) 255  d) 1023
   e) 120  f) 32000  g) 160  h) 250

15. Convert the following binary numbers into decimal:
   a) 1111  b) 0110  c) 11110000
   d) 00001111  e) 10101010  f) 10000000

16. Convert the following octal numbers into decimal:
   a) 177  b) 762  c) 777  d) 123
   e) 1777  f) 655  g) 17777  h) 207

17. Convert the following decimal numbers into octal:
   a) 255  b) 1024  c) 129  d) 2450
   e) 4096  f) 256  g) 180  h) 4096

18. Convert the following hexadecimal numbers into decimal:
   a) AA  b) EF  c) 1FF  d) FFFF
   e) 1AA  f) FEF  g) F0  h) CC

19. Convert the following binary numbers into hexadecimal:
   a) 0101  b) 11111111  c) 1111  d) 1010
   e) 1110  f) 10011111  g) 1001  h) 1100

20. Convert the following binary numbers into octal:
   a) 111000  b) 000111  c) 111111  d) 010111
   e) 110001  f) 11111111  g) 1000001  h) 110000
21. Convert the following octal numbers into binary:
   a) 177    b) 777    c) 555    d) 111
   e) 1777777  f) 55571  g) 171    h) 1777

22. Convert the following hexadecimal numbers into octal:
   a) AA    b) FF    c) FFFF   d) 1AC
   e) CC    f) EE    g) EEFF   h) AB

23. Convert the following octal numbers into hexadecimal:
   a) 177    b) 777    c) 123    d) 23
   e) 1111   f) 1777777  g) 349    h) 17

24. Convert the following decimal numbers into floating point:
   a) 23.45  b) 1.25   c) 45.86  d) 0.56

25. Convert the following decimal numbers into floating point and then calculate their sum:
   0.255 and 1.75

26. Convert the following decimal numbers into floating point and then calculate their product:
   2.125 and 3.75

27. Convert the following decimal numbers into BCD:
   a) 128    b) 970    c) 900    d) 125