CHAPTER 13

Solutions

Exercise 1

1. In this exercise, we can easily employ the equations (13.66)–(13.70), (13.79)–(13.80) and (13.82)–(13.86). Also, remember that BDT model will yield a recombining binomial tree.

2. Using the equation,

$$E^P[L_i] = \frac{B(0,i)}{B(0,i+1)} - 1$$

we get:

$L_0 = 5.26\%$

$E^P\,[L_1] = 2.15\%$

$E^P\,[L_2] = 2.20\%$

$E^P\,[L_3] = 2.25\%$.

Also,

$E^P\,[L_1] = 0.5x(L_1^u + L_1^d\ )$

and

$L_1^u = e^{2\sigma_1} L_1^d$

Solving these two equations, we find

$L_1^u = 2.58\%$

and

$L_1^d = 1.73\%$

We can use the same logic to determine all possible values of the Libor rates at each node. This would lead to the following values:

At node 2:

L_2^{uu} = 3.42%

$L_2^{ud} = L_2^{du}$ = 2.07%

L_2^{dd} = 1.26%

At node 3:

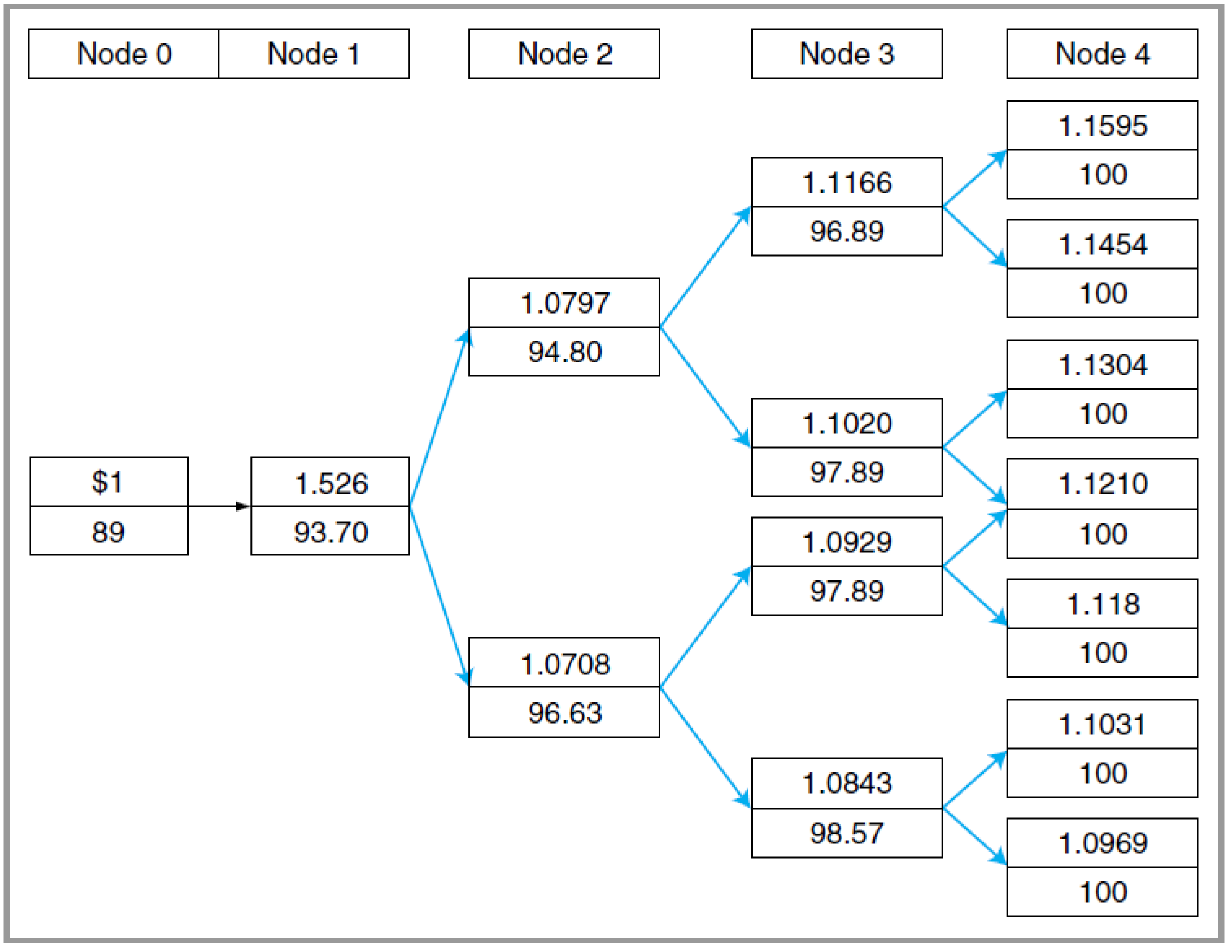
L_3^{uuu} = 3.84%

$L_3^{uud} = L_3^{udu} = L_3^{duu}3$ = 2.58%

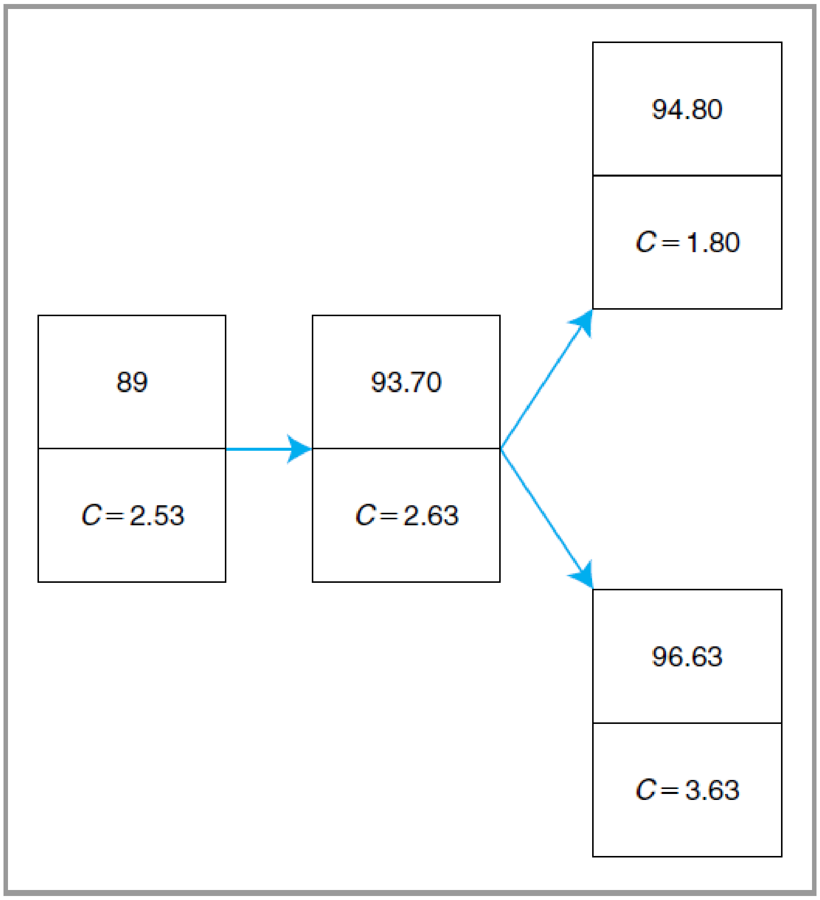
L_3^{udd}=L_3^{dud} =L_3^{ddu} $=$ 1.73%

L_3^{ddd} = 1.16%

BDT tree for bond price of B(0,4) can be found by using the libor rates determined in the previous part (a): In the following figure, at each node, the numbers in the top box refer to money market account, and the numbers in the second box refer to price of bond at this node. Forward price of bond which expires at time 4, at node 2 is the expected price of this bond at node 2. So forward price would be 0.5(94.80 + 96.63) = 95.72.



2. Price of the call option which expires at time can be calculated from the binomial tree presented below:



Exercise 2

1.

$$L_{US} = \frac{100}{B(t,t+1)^{US}} - 1 = 1.08\%$$

and

$$L_{Euro} = \frac{100}{B(t,t+1)^{Euro}} - 1 = 1.29\%$$

2.

$$r_t^{US} = \log(1.108) = 1.074\%$$

and

$$r_t^{Euro} = \log(1.0129) = 1.28\%$$

3. We need to use continuously compounded rates.

4. Forward Libor model uses Libor rates.

Exercise 3

1. Consider the Stochastic Differential Equation,

$$S_{t+\Delta} = S_t + (r - r_f)S_t\Delta + \sigma S_{t+\Delta}\sqrt{\Delta}e_{t+\Delta}$$

and the time interval,

Δ = 1

2. Assume that the following sets of random numbers are given:

{−0.4326,−1.6656, 0.1253, 0.28779,−1.1465}

{1.1909, 1.1892,−0.0376, 0.3273, 0.1746}

{−0.1867, 0.7258,−0.583, 2.1832,−0.1364}

{0.1139, 1.0668, 0.0593,−0.0956,−0.8323}

{0.2944,−1.3362, 0.7143, 1.6236,−0.6918}.

These five trajectories are risk free since, i) random variables in each set have a mean of zero and ii) the equation given in part (a) has a known mean that equals interest rate differentials. We can compute the exchange rate, S_i^j, where j is the path index and i is the node (time) index. (Remember that the Δ = 1 otherwise, the drift of the equation below will involve a Δ as well.)

$$S_i^j = S_{i-1}^j + (r - r_f)S_{i-1}^j + \sigma S_{i-1}^j e_i^j.$$

For example,

$$S_1^1 = S_0\ (1 + r + r_f + \sigma e_1^1) = 1.1015(1 + 0.01074 - 0.0128 + 0.15(-0.4326) = 1.0278$$

Similarly we can compute the remaining values for path one:

S_2^1 = 0.7689, S_3^1 = 0.7818, S_4^1 = 0.8140, S_5^1= 0.6724

Path 2:

S_1^2 = 1.2961, S_2^2 = 1.5247, S_3^2= 1.5130, S_4^2 = 1.5843, S_5^2 = 1.6226

Path 3:

…

Path 4:

S_1^4 = 1.1181, S_2^4 = 1.2948, S_3^4 = 1.3037, S_4^4 = 1.2824, S_5^4 = 1.1198

Path 5:

S_1^5 = 1.1479, S_2^5 = 0.9156, S_3^5 = 1.0118, S_4^5 = 1.2562, S_5^5 = 1.1234

3. Assume that we are dealing with a put option. Then, by simply checking the third number in each path, we can determine the value of the option at expiration.

Finding the expected value of the option at expiration and discounting it we get

$$P = 0.0319$$

(Remember that each path has a probability of 0.2.)

Exercise 4

1. By using the parameters provided in the exercise and the method employed in the previous question, we proceed as follows:

(a) We assume three time steps which implies that Δ = 90 *days*. We generated the following 5 set of random numbers:

Set 1: {0.9501, 0.7621, 0.6154},

Set 2: {0.2311, −0.4565, 0.7919},

Set 3: {0.6068, 0.0185, −0.9218}

Set 4: $\{-0.4860, -0.8214, 0.7382\}$,

Set 5: $\{0.8913, 0.4447, -0.1763\}$.

The values of exchange rate, on each of 5 paths, will be:

Path 1:

S_1^1 = 4.1532, S_2^1 = 4.5216, S_3^1 = 4.8564

Path 2:

S_2^2= 3.8835, S_2^2= 3.7548, S_2^2 = 4.0991

Path 3:

S_1^3= 4.02144, S_2^3= 4.0822, S_2^3== 3.7569

Path 4:

S_1^4 = 3.6146, S_2^4= = 3.3629, S_3^4= = 3.6532

Path 5:

S_1^5 = 4.1311, S_2^5= = 4.3665, S_3^5= = 4.3441

Let's assume that the option under consideration is a call option with an exercise price of 4.10Peso/$. It is easy to see that call option expires in the money only in paths 1 and 5. So, we first compute the value of the option on each path, determine its expected value and then discount it with Mexican interest rate. Hence, the call premium is 0.1884.

2. This information is important in a sense that it is an additional risk factor which affects the exchange rate process.

3. Let's first have look at the distribution of the reserves. Let R_t be the level of reserves. Then the distribution of $\log(R_t)$ is

$$\varphi[\log(R_0) + (\mu - \frac{o^2}{2})T, \sigma\sqrt{T}]$$

So probability that R_t <6 (probability of experiencing a one shot devaluation) is less than 2%.

Even though the information is important in pricing the option, it is very hard to determine its effect when we use only 5 paths with only 3 steps. So we need to increase the number of paths and steps to allow for such an event with very low probability. We can proceed in two ways. Either we can use the same random variables generated for the exchange rate process to compute the reserves. Here we assume that there is a perfect correlation between two processes. Or, alternatively, we can generate a separate set of random variables which implies that there is no correlation between the two.

Exercise 5

(For detailed calculation see also Excel file 'Exercise 13.5 Solution Excel Calculation' on book webpage.)

Simulation

Using the dynamics given below simulate M no. of stock prices and calculate the payoff of the option at time T.

$$S_{t+1} - S_t = rS_t\Delta + \sigma S_t(\Delta W_t)$$

Take the average of all the payoff and discount it to obtain the present value of that average payoff and report the obtained value as the option price obtained from the simulation.

$$C_t = e^{-r(T-t)}E^{\tilde{P}}[\max(S_T - K, 0)]$$

$$P_t = e^{-r(T-t)}E^{\tilde{P}}[\max(K - S_T, 0)]$$

Observation

As you increase the value of M the option price calculated from the Monte Carlo simulation gets closer to the actual value calculated from the *BSM* formula

European Call Price = 21.01 European Put Price = 17.93

Exercise 6

(For detailed calculation see also Excel file 'Exercise 13.6 Solution Excel Calculation' on book webpage.)

<u>Simulation</u>

Using the dynamics given below simulate M no. of stock prices and calculate the payoff of the option at time T depending on whether $S_T > K$ or $S_T < K$.

$$S_{t+1} - S_t = (r - r_f)S_t\Delta + \sigma S_t(\Delta W_t)$$

Take the average of all the payoff and discount it to obtain the present value of that average payoff and report the obtained value as the option price obtained from the simulation and compare it with BSM formula price calculated from

$$C_t = Re^{-r(T-t)}N(d_2)$$

$$P_t = Re^{-r(T-t)}N(-d_2)$$

Observation

As you increase the value of M the option price calculated from the binomial model gets closer to the actual value calculated from the *BSM* formula

European Call Price = 3.996 European Put Price = 5.234

Exercise 7

(For detailed calculation see also Excel file 'Exercise 13.7 Solution Excel Calculation' on book webpage.)

<u>Simulation</u>

Using the dynamics given below simulate M no. of stock prices and calculate the payoff of the option at time T depending on whether $S_{t_i} > H$ or $S_{t_i} < H \; \forall \, i$

$$S_{t+1} - S_t = rS_t\Delta + \sigma S_t(\Delta W_t)$$

Take the average of all the payoff and discount it to obtain the present value of that average payoff and report the obtained value as the option price obtained from the simulation and compare it with BSM formula price calculated from

$$C_T^{out} = C_T - C_T^{in}$$

$$C_T = S_t * N(d_1) - Ke^{-r(T-t)} * N(d_2)$$

$$C_T^{in} = S_t * \left(\frac{H}{S_t}\right)^{\frac{2\left(r-\frac{\sigma^2}{2}\right)}{\sigma^2}+2} N(c_1) - Ke^{-r(T-t)} * \left(\frac{H}{S_t}\right)^{\frac{2\left(r-\frac{\sigma^2}{2}\right)}{\sigma^2}} N(c_2)$$

Observation

As you increase the value of M the option price calculated from the Monte Carlo simulation gets closer to the actual value calculated from the *BSM* formula

$BSM\_C_T^{out} \; = \; 9.253 \qquad\qquad BSM\_C_T^{in} = 9.877$

NOTE The practical method of pricing barrier options is more rigorous and tactical.

Exercise 8

(European Options)

(For detailed calculation see also Excel file 'Exercise 13.8 Solution Excel Calculation' on book webpage.)

Simulation

Using the dynamics given below simulate M no. of stock prices and calculate the payoff of the option at time T.

$$S_{t+1} - S_t = rS_t\Delta + \sigma S_t(\Delta W_t)$$

Take the average of all the payoff and discount it to obtain the present value of that average payoff and report the obtained value as the option price obtained from the simulation.

$$C_t = e^{-r(T-t)}E^{\tilde{P}}[\max(S_T - K, 0)]$$

$$P_t = e^{-r(T-t)}E^{\tilde{P}}[\max(K - S_T, 0)]$$

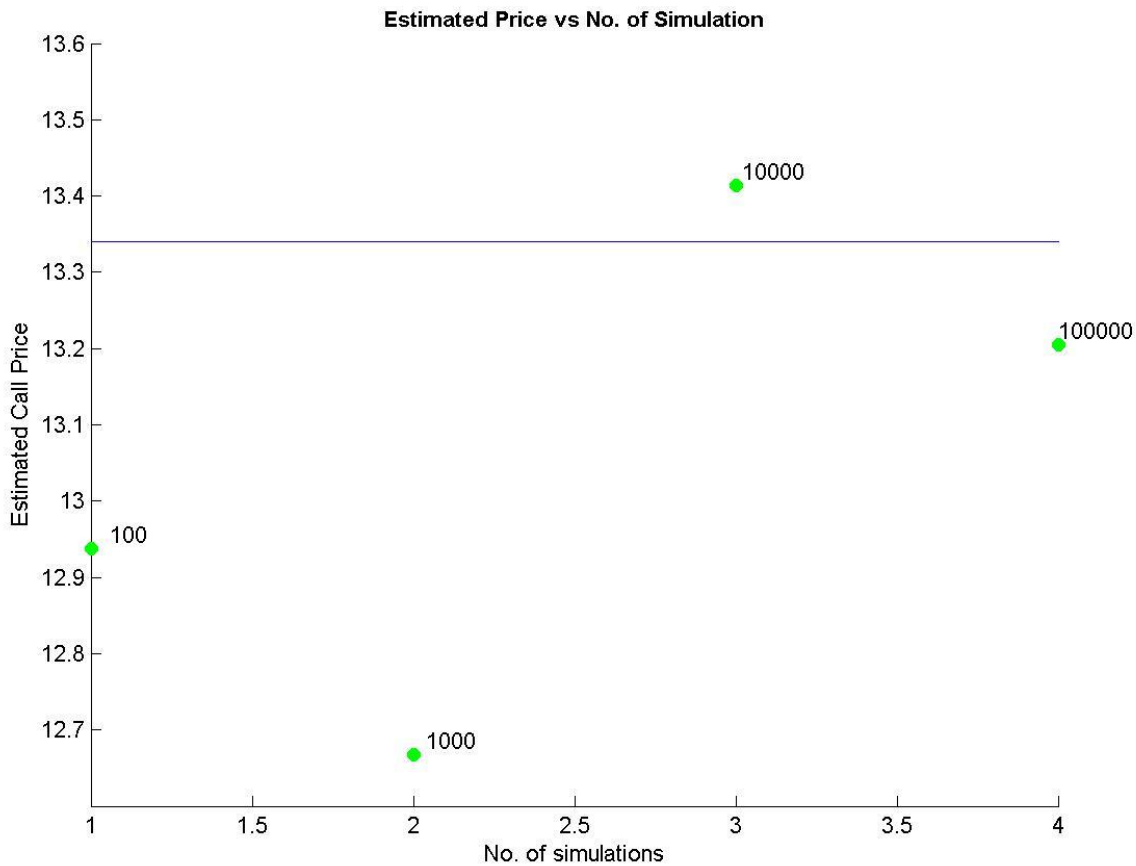
Observation

As you increase the value of M the option price calculated from the binomial model gets closer to the actual value calculated from the *BSM* formula

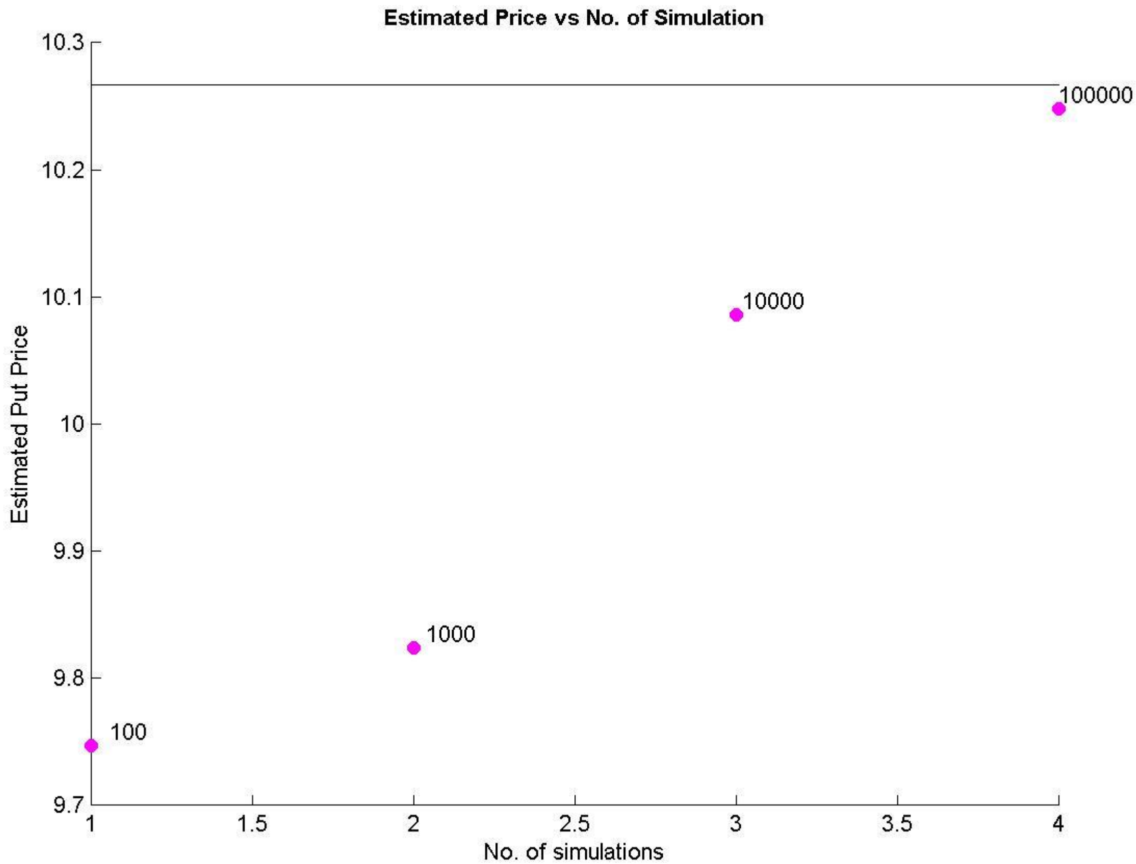
European Call Price = 13.34 European Put Price = 10.26

Plots

Estimated Price of European Call from Monte Carlo Simulation



Estimated Price of European Put from Monte Carlo Simulation



Exercise 9

(Barrier Options)

(For detailed calculation see also Excel file 'Exercise 13.9 Solution Matlab Calculation' on book webpage.)

<u>**Simulation**</u>

Using the dynamics given below simulate M no. of stock prices and calculate the payoff of the option at time T depending on whether $S_{t_i} > H$ or $S_{t_i} < H \; \forall \, i$

$$S_{t+1} - S_t = rS_t\Delta + \sigma S_t(\Delta W_t)$$

Take the average of all the payoff and discount it to obtain the present value of that average payoff and report the obtained value as the option price obtained from the simulation and compare it with BSM formula price calculated from

$$C_T^{out} = C_T - C_T^{in}$$

$$C_T = S_t * N(d_1) - Ke^{-r(T-t)} * N(d_2)$$

$$C_T^{in} = S_t * \left(\frac{H}{S_t}\right)^{\frac{2\left(r-\frac{\sigma^2}{2}\right)}{\sigma^2}+2} N(c_1) - Ke^{-r(T-t)} * \left(\frac{H}{S_t}\right)^{\frac{2\left(r-\frac{\sigma^2}{2}\right)}{\sigma^2}} N(c_2)$$

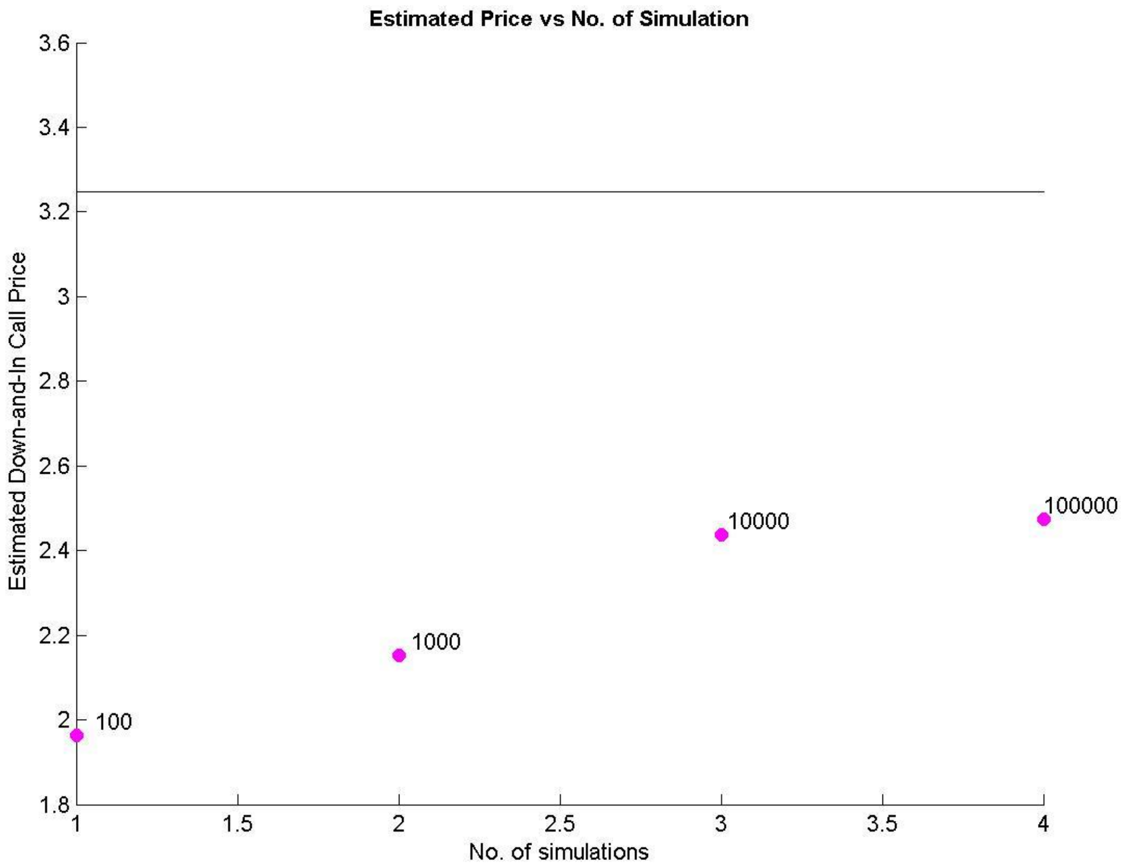
Observation

As you increase the value of M the option price calculated from the Monte Carlo simulation gets closer to the actual value calculated from the *BSM* formula

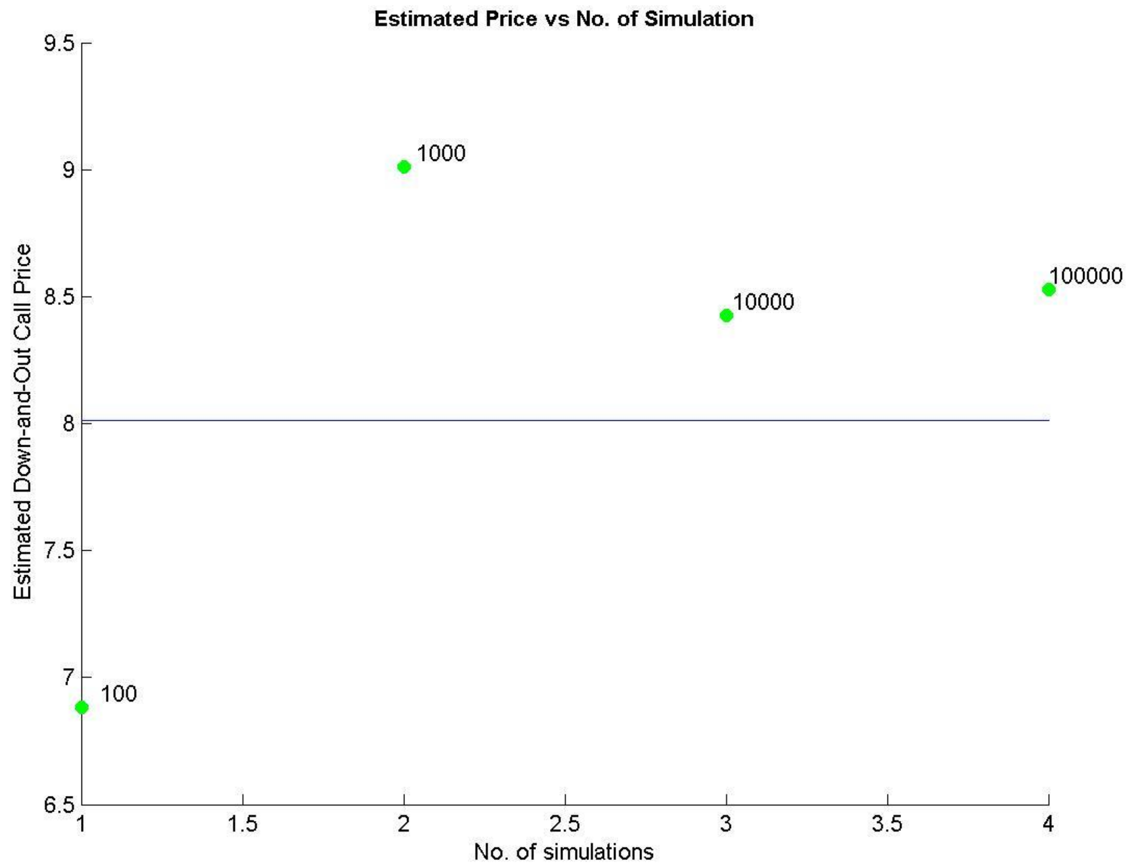
$BSM\_C_T^{out} = 8.01$ $BSM\_C_T^{in} = 3.24$

Plots

Estimated Price of Barrier Down-and-In Call from Monte Carlo Simulation



Estimated Price of Barrier Down-and-Out Call from Monte Carlo Simulation



Exercise 10

(Digital Currency Options)

(For detailed calculation see also Excel file 'Exercise 13.10 Solution Matlab Calculation' on book webpage.)

<u>Simulation</u>

Using the dynamics given below simulate M no. of stock prices and calculate the payoff of the option at time T depending on whether $S_T > K$ or $S_T < K$.

$$S_{t+1} - S_t = (r - r_f)S_t\Delta + \sigma S_t(\Delta W_t)$$

Take the average of all the payoff and discount it to obtain the present value of that average payoff and report the obtained value as the option price obtained from the simulation and compare it with BSM formula price calculated from

$$C_t = Re^{-r(T-t)}N(d_2)$$

$$P_t = Re^{-r(T-t)}N(-d_2)$$

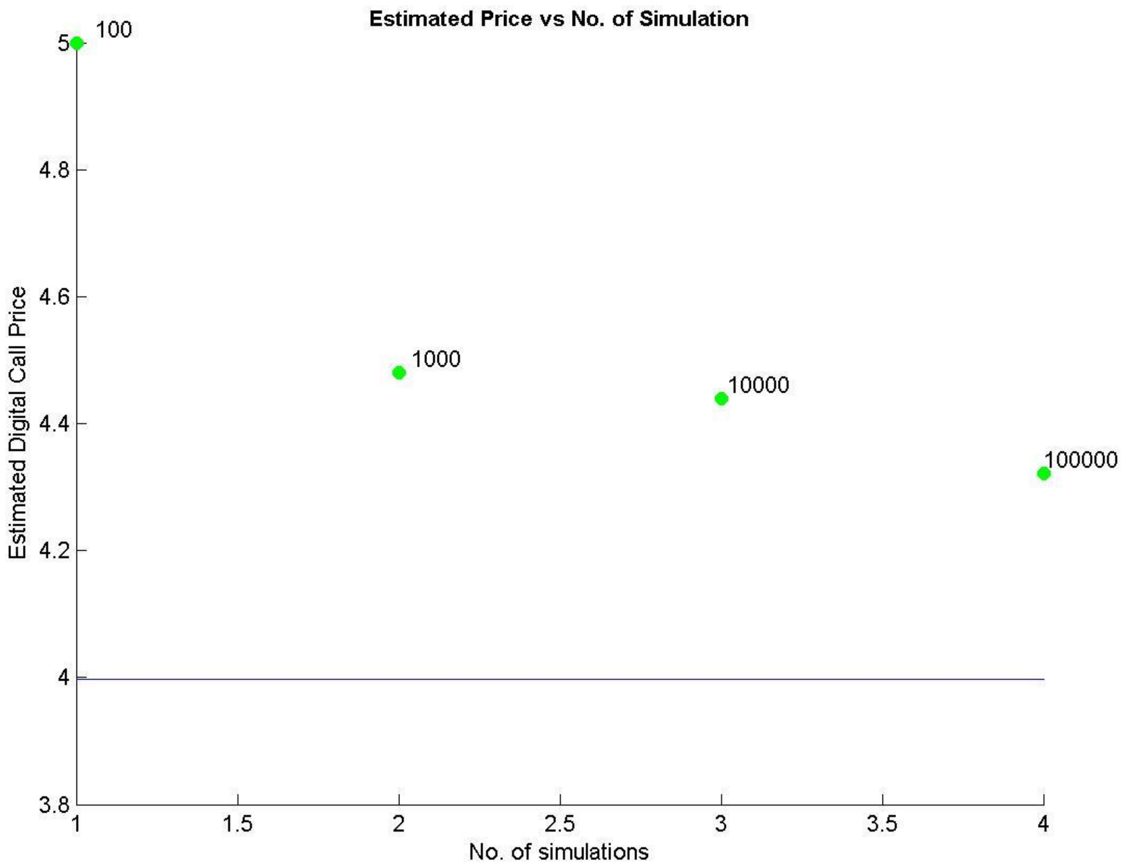
Observation

As you increase the value of M the option price calculated from the binomial model gets closer to the actual value calculated from the *BSM* formula

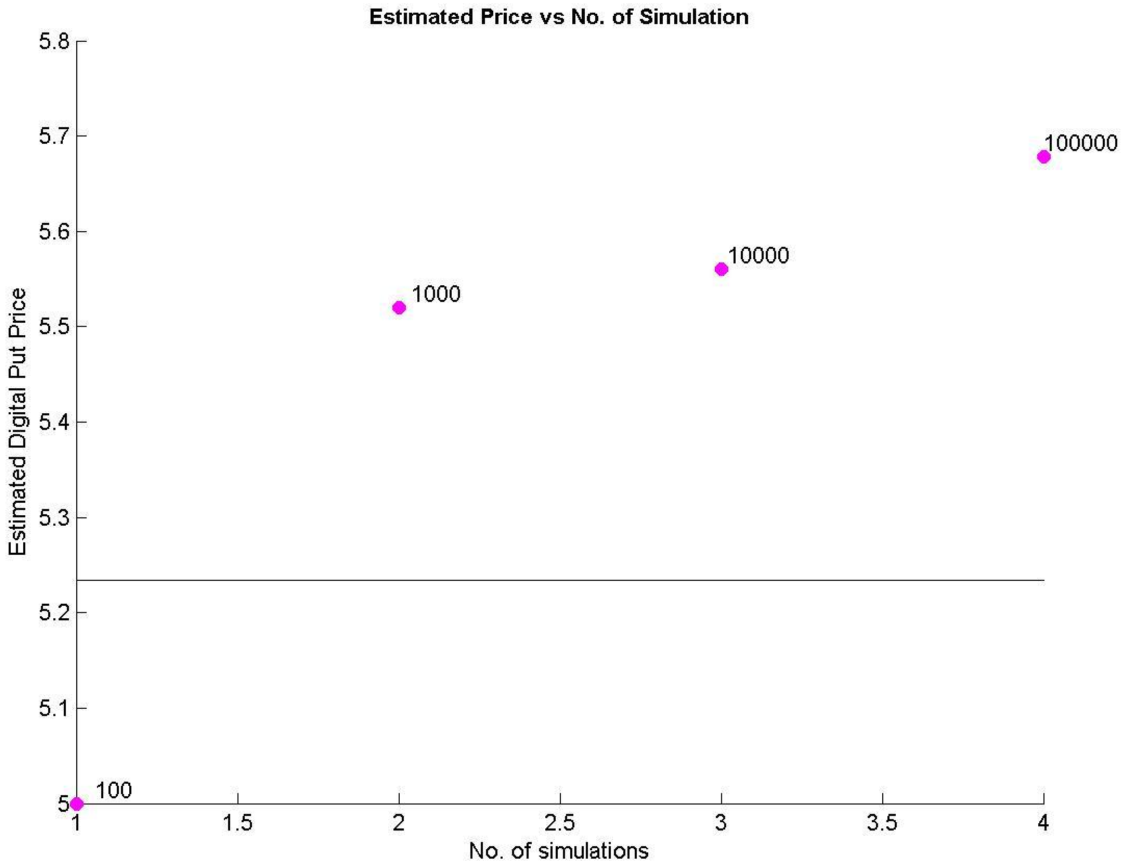
Digital FX Call Price = 3.996 Digital FX Put Price = 5.234

Plots

Estimated Price of Digital FX Call from Monte Carlo Simulation


Estimated Price vs No. of Simulation
Estimated Digital Call Price
No. of simulations
100
1000
10000
100000
5
4.8
4.6
4.4
4.2
4
3.8
1
1.5
2
2.5
3
3.5
4

Estimated Price of Digital FX Put from Monte Carlo Simulation



Exercise 11

(BDT Model Calibration)

(For detailed calculation see also Excel file ‘Exercise 13.11 Solution Matlab Calculation’ on book webpage.)

Solution

Obtain the value of L_0 from the 1-year zero coupon bond price as follows:

$$B(t_0, t_1) = \frac{1}{(1 + L_0)}$$

Using the value of L_0 and the price of 2-year zero coupon bond price further estimate the value of L_1^d and L_1^u

$$B(t_0, t_2) = \left[\frac{1}{2(1 + L_1^u)(1 + L_0)} + \frac{1}{2(1 + L_1^d)(1 + L_0)}\right]$$

$$\frac{1}{2} ln \left[\frac{L_1^u}{L_1^d}\right] = \sigma(0,1)$$

Iteratively solve for remaining value of LIBOR rates at time t_2 and t_3.

Output

L = 5.26%

L_u = 2.57% L_d = 1.73%

L_{uu} = 3.42% L_{ud} = 2.07% L_{dd} = 1.26%

L_{uuu} = 3.88% L_{uud} = 2.6% L_{udd} = 1.74% L_{ddd} = 1.17%