

# CHAPTER 14

## Solutions

### Exercise 1

1. In order to compute the bond prices from FRA rates, we can simply use the equation (14.23) in the chapter.

$$B(t_0, t_i) = \frac{100}{\prod_{j=0}^{i-1} (1 + \delta F(t_0, t_j))}$$

where

$$\delta = 0.25 \text{ years}$$

and  $F(t_0, t_i)$  is the  $i \times (i + 1)$  FRA rate.

Using the equation we will get the following discount bond prices:

<u>Term</u>	<u>Bid</u>	<u>Ask</u>
3	99.01	99.01
6	97.88	97.91
9	96.72	96.77
12	95.51	95.58
15	94.16	94.28
18	92.70	92.86

2. We use the following relationship between discount bond prices and the yield:

$$B(t_0, t_i) = \frac{100}{(1 + \delta_i r_i)}$$

where  $\delta i$  is the time to maturity as a fraction of a year, and  $r_i$  is the corresponding interest rate (yield) for that maturity.

Rewriting the equation in terms of  $r_i$ , we get

$$r_i = \frac{1}{\delta_i} \left( \frac{100}{B(t_0, t_i)} - 1 \right)$$

Using this equation we obtain the yields shown in the first Table below.

	Yield Curve	
<i>Term</i>	<i>Bid</i>	<i>Ask</i>
<i>3</i>	<i>4.00</i>	<i>4.00</i>
<i>6</i>	<i>4.27</i>	<i>4.32</i>
<i>9</i>	<i>4.45</i>	<i>4.52</i>
<i>12</i>	<i>4.63</i>	<i>4.71</i>
<i>15</i>	<i>4.85</i>	<i>4.96</i>
<i>18</i>	<i>5.12</i>	<i>5.25</i>

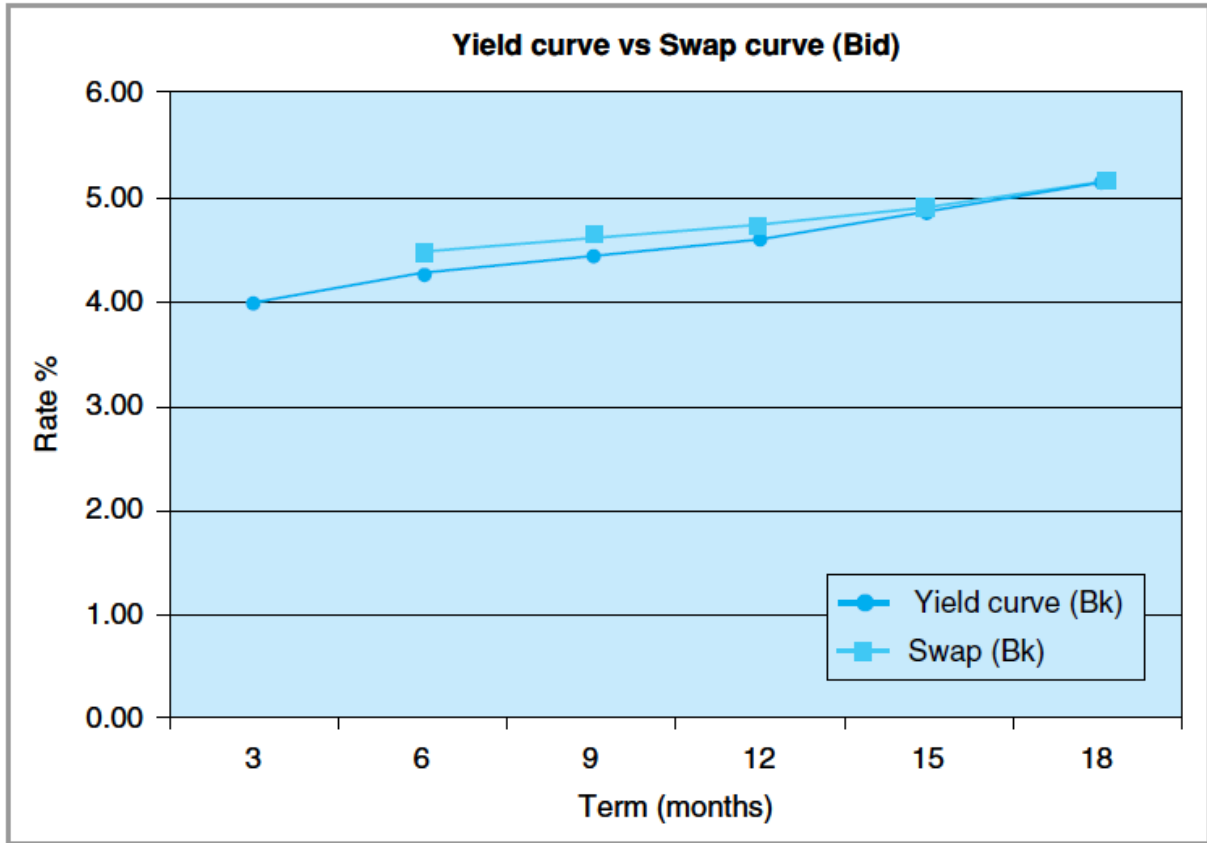
3. Computation of swap rates is explicitly shown in the text. We can apply equations (14.25)–(14.31). This would yield the swap curve which is shown in the second Table below

	Swap Curve	
<i>Term</i>	<i>Bid</i>	<i>Ask</i>
<i>6</i>	<i>4.50</i>	<i>4.60</i>
<i>9</i>	<i>4.60</i>	<i>4.70</i>
<i>12</i>	<i>4.73</i>	<i>4.83</i>
<i>15</i>	<i>4.92</i>	<i>5.04</i>
<i>18</i>	<i>5.15</i>	<i>5.29</i>

4. We can easily see that curves computed in part (1) and (3) are not the same. The following graph shows this:

5. See appendix for the calculation of the par yield curve. We can derive

$$y_n = \frac{1}{\delta} \left( \frac{100 - B(0, n)}{\sum_{i=1}^n B(0, i)} \right)$$



Par Yield Curve		
<i>Term</i>	<i>Bid</i>	<i>Ask</i>
<i>3</i>	<i>4.00</i>	<i>4.00</i>
<i>6</i>	<i>4.25</i>	<i>4.30</i>
<i>9</i>	<i>4.40</i>	<i>4.46</i>
<i>12</i>	<i>4.55</i>	<i>4.62</i>
<i>15</i>	<i>4.73</i>	<i>4.83</i>
<i>18</i>	<i>4.95</i>	<i>5.07</i>

from the equation provided in the appendix ( $\delta = 0.25$  years). This leads to:

- The computation of zero-coupon yield curve is very similar to computation of yield curve from discount bond except that the yield this time corresponds to a formula that uses Bond-equivalent yields:

$$B(t_0, t_i) = \frac{100}{(1 + y_i)^{\delta^i}}$$

where  $\delta^i$  is the same as in part (2). Solving this equation for  $y_i$ , we obtain:

$$y_i = \left( \left( \frac{100}{B(t_0, t_i)} \right)^{\frac{1}{\delta^i}} - 1 \right).$$

By using the formula just derived, the zero coupon rates are computed as in the Table below.

(You may, alternatively, follow the method described in the Appendix of Chapter 3 to calculate the zero-coupon yield curve from par yield curve).

<b>Zero-Coupon Yield Curve</b>		
<i>Term</i>	<i>Bid</i>	<i>Ask</i>
<i>3</i>	<i>4.06</i>	<i>4.06</i>
<i>6</i>	<i>4.37</i>	<i>4.32</i>
<i>9</i>	<i>4.54</i>	<i>4.47</i>
<i>12</i>	<i>4.71</i>	<i>4.63</i>
<i>15</i>	<i>4.93</i>	<i>4.82</i>
<i>18</i>	<i>5.18</i>	<i>5.06</i>

## Exercise 2

1. Let's start by considering the following two portfolios:

Portfolio A: Nominal = \$100. Long position in  $(0 \times 1)$  and  $(1 \times 2)$  FRA's.

Portfolio B: Nominal = \$100. Long position in plain vanilla swap.

Portfolio A:

Since we start at time zero, we already know the current Libor rate  $L_0$  and  $(0 \times 1)$

FRA rate. They are both equal to 5%. But, we do not know  $L_1$  and the  $(1 \times 2)$

FRA rate.

<table border="1"> <tr><td><math>L_0</math></td></tr> <tr><td>5%</td></tr> </table>	$L_0$	5%	<table border="1"> <tr><td><math>L_1</math></td></tr> <tr><td>?</td></tr> </table>	$L_1$	?
$L_0$					
5%					
$L_1$					
?					
5%	$F_{1 \times 2}$				

Portfolio B:

<table border="1"> <tr><td><math>L_0</math></td></tr> <tr><td>5%</td></tr> </table>	$L_0$	5%	<table border="1"> <tr><td><math>L_1</math></td></tr> <tr><td>?</td></tr> </table>	$L_1$	?
$L_0$					
5%					
$L_1$					
?					
5%	$F_{1 \times 2}$				

From the text we know that both structures are very similar to each other. We can use that similarity to derive the (1×2) FRA rate. Indeed, we have the following relationship between FRA and swap rate: (See equation (14. 17) and explain why we can't use it)

$$\$100 \left( \frac{5\%}{1.05} + \frac{F_{1 \times 2}}{1.05(1 + F_{1 \times 2})} \right) = \$100 \left( \frac{6.2\%}{1.05} + \frac{6.2\%}{1.05(1 + F_{1 \times 2})} \right)$$

Simplifying and rearranging

$$F_{1 \times 2} = \frac{6.2\% + (6.2\% - 5\%)}{1 - (6.2\% - 5\%)} = 7.49\%$$

In order to write a general formula, it would be helpful to write the same equation for  $F_{2 \times 3}$ .

$$F_{2 \times 3} = \frac{6.4\% + (6.4\% - 5\%) \times (1.0749)}{1 - ((6.4\% - 5\%) \times (1.0749) + (6.4\% - 7.49\%))} = 6.48\%$$

Then a general formula would be in the form of

$$F_{(n-1) \times (n)} = \frac{S_n + \sum_{i=1}^{n-1} [(S_n - F_{(i-1) \times i}) \prod_{j=i+1}^{n-1} (1 + F_{(j-1) \times j})]}{1 - \sum_{i=1}^{n-1} [(S_n - F_{(i-1) \times i}) \prod_{j=i+1}^{n-1} (1 + F_{(j-1) \times j})]}$$

Using the general formula just derived, we obtain the following FRA rates (We used bid quotations for the corresponding swap rate):

<i>Term</i>	<i>FRA</i>
<i>0X1</i>	<i>5.00</i>
<i>1X2</i>	<i>7.49</i>
<i>2X3</i>	<i>6.84</i>
<i>3X4</i>	<i>9.11</i>
<i>4X5</i>	<i>9.98</i>
<i>5X6</i>	<i>12.11</i>

After computing the FRA rates from swap curve, the rest is relatively easy to handle.

<i>Term</i>	<i>Bond Price</i>
<i>1</i>	<i>95.24</i>
<i>2</i>	<i>88.60</i>
<i>3</i>	<i>82.93</i>
<i>4</i>	<i>76.01</i>
<i>5</i>	<i>69.11</i>
<i>6</i>	<i>61.64</i>

2. We can follow the same method used in the previous question part (1). As a result we get the answers shown in the Table below.

3. See question one, part (2) for the details:

<i>Term</i>	<i>Yield</i>
1	5.00
2	6.43
3	6.86
4	7.89
5	8.94
6	10.37

4. See the previous question, part (5):

<i>Term</i>	<i>Par Yield Curve</i>
1	5.00
2	6.20
3	6.40
4	7.00
5	7.50
6	8.10

### Exercise 3

1. The bid-ask spread on a forward start swap is equal to  $S^{ask} - S^{bid}$ . We can use the equations

(14.25)–(14.31) to compute  $S^{ask}$  and  $S^{bid}$ .

$$\begin{aligned}
 S^{bid} &= \frac{B(0,3) \times F_{2 \times 3} + B(0,4)F_{3 \times 4} + B(0,5)F_{4 \times 5}}{B(0,3) + B(0,4) + B(0,5)} \\
 &= \frac{82.93 \times 6.84\% + 76.01 \times 9.11\% + 69.11 \times 9.98\%}{82.93 + 76.01 + 69.11} = 8.55\%
 \end{aligned}$$

The  $S^{ask}$  can be computed in a similar way.

2. The price of the coupon bond at time zero is given by

$$P(0, 4) = (\sum_{i=1}^4 B(0, i) \times \$7) + B(0,4) \times \$100 = \$100$$

A trader who wants to (forward) sell this coupon bond at a price of  $F$ , to be delivered at time 2, may want to hedge himself. So this trader can buy this coupon bond in the spot market today and “store” it. Since its price is \$100, he needs to borrow \$100 for funding purposes. When the trader receives coupon payments at time 1 and 2 and the forward price amount,  $F$  of the coupon bond, the trader will repay the initial \$100. This means that the trader needs to borrow:

$$B(0, 1) \$7 + B(0, 2) \times (\$7 + F) = \$100. \text{ Solving this equation we find, } F = \$98.34$$

#### **Exercise 4**

(For detailed calculation see also Excel file ‘Exercise 14.4 Solution Excel Calculation’ on book webpage.)

#### **Calculation**

In order to compute the bond prices from FRA rate, we can simply use the following formula

$$B(t_0, t_i) = \frac{100}{\prod_{j=0}^{i-1} (1 + \delta F(t_0, t_j))}$$

Now to determine the yield of the yield from discount bond prices we can use the following formula

$$r_i = \frac{1}{\delta_i} \left( \frac{100}{B(t_0, t_1)} - 1 \right)$$

For the computation of the (forward) swap rate use the following relationship



$$s_{t_0} = \frac{\sum_{i=1}^n B(t_0, t_{i+1})F(t_0, t_i)}{\sum_{i=1}^n B(t_0, t_{i+1})}$$

We can find derive par yield and the zero yield from the following formula

$$y_n = \frac{1}{\delta} \left( \frac{100 - B(t_0, t_n)}{\sum_{i=1}^n B(t_0, t_i)} \right)$$

Zero Coupon Yield

$$y_i = \left( \left( \frac{100}{B(t_0, t_i)} \right)^{\frac{1}{\delta_i}} - 1 \right)$$

### Exercise 5

(For detailed calculation see also Excel file ‘Exercise 14.5 Solution Excel Calculation’ on book webpage.)

### Calculation

First we calculate the forward rate from the given swap rate use the following formula

$$F_{(n-1)xn} = \frac{S_n + \sum_{i=1}^{n-1} [(S_n - F_{(i-1)xi}) \prod_{j=i+1}^{n-1} (1 + F_{(j-1)xj})]}{1 - \sum_{i=1}^{n-1} [(S_n - F_{(i-1)xi}) \prod_{j=i+1}^{n-1} (1 + F_{(j-1)xj})]}$$

Then we proceed by calculating bond prices and then the yield curves. The bond prices

from FRA rate, we can simply use the following formula

$$B(t_0, t_i) = \frac{100}{\prod_{j=0}^{i-1} (1 + \delta F(t_0, t_j))}$$

Now to determine the yield of the yield from discount bond prices we can use the following formula

$$r_i = \frac{1}{\delta_i} \left( \frac{100}{B(t_0, t_1)} - 1 \right)$$

We can find derive par yield and the zero yield from the following formula

$$y_n = \frac{1}{\delta} \left( \frac{100 - B(t_0, t_n)}{\sum_{i=1}^n B(t_0, t_i)} \right)$$

Zero Coupon Yield

$$y_i = \left( \left( \frac{100}{B(t_0, t_i)} \right)^{\frac{1}{\delta_i}} - 1 \right)$$

### Exercise 6

(For detailed calculation see also Excel file 'Exercise 14.6 Solution Excel Calculation' on book webpage.)

### Calculation

In order to compute the FRA rate from bond prices we can simply use the following formula

$$F(t_0, t_i) = \frac{1}{\delta} \left( \frac{B(t_0, t_{i+1})}{B(t_0, t_i)} - 1 \right)$$

Now to determine the yield of the yield from discount bond prices we can use the following formula

$$r_i = \frac{1}{\delta_i} \left( \frac{100}{B(t_0, t_1)} - 1 \right)$$

For the computation of the (forward) swap rate use the following relationship

$$s_{t_0} = \frac{\sum_{i=1}^n B(t_0, t_{i+1}) F(t_0, t_i)}{\sum_{i=1}^n B(t_0, t_{i+1})}$$

We can find derive par yield and the zero yield from the following formula

$$y_n = \frac{1}{\delta} \left( \frac{100 - B(t_0, t_n)}{\sum_{i=1}^n B(t_0, t_i)} \right)$$

Zero Coupon Yield

$$y_i = \left( \left( \frac{100}{B(t_0, t_i)} \right)^{\frac{1}{\delta_i}} - 1 \right)$$

### **Exercise 7**

(For detailed calculation see also Excel file ‘Exercise 14.7 Solution Excel Calculation’ on book webpage.)

### **Calculation**

Price of the interest rate swap is given by  $\frac{\sum_{i=1}^n B(t_0, t_{i+1})(F(t_0, t_i) - R) * N}{\sum_{i=1}^n B(t_0, t_{i+1})}$

where,  $B(t_0, t_i) = \frac{100}{\prod_{j=0}^{i-1} (1 + \delta F(t_0, t_j))}$

Hence the price = \$ **302.56**

### **Exercise 8**

(For detailed calculation see also Excel file ‘Exercise 14.8 Solution Excel Calculation’ on book webpage.)

### **Calculation**

Unlike the interest rate swap the cross currency swap gets you the exposure of the foreign currency rate and also leads to the exposure of the spot exchange rate risk.

Now equating the discounted value of fixed payment made at the rate of  $F_{t_0}$  with the present value of payment of floating payment made at the GBP LIBOR rate and exchanged at the spot rate of  $e_{t_i}$ . The initial exchange of payment at  $t_0$  cancels out each other hence we have the following equality:

$$\frac{\sum_{i=1}^n B(t_0, t_i) * F_{t_0} * N\$}{\sum_{i=1}^n B(t_0, t_i)} + \frac{N\$}{B(t_0, t_n)} = \frac{\sum_{i=1}^n B^*(t_0, t_i) * N\text{£} * L^*(t_{i-1}) * e_{t_i}}{\sum_{i=1}^n B^*(t_0, t_i)} + \frac{N\text{£}}{B^*(t_0, t_n)}$$

where

$$B(t_0, t_i) = \frac{100}{\prod_{j=0}^{i-1} (1 + \delta F(t_0, t_j))}$$

$$B^*(t_0, t_i) = \frac{100}{\prod_{j=0}^{i-1} (1 + \delta L^*(t_j))}$$

Cross Currency Swap Rate  $F_{t_0} = 4.305\%$

### Exercise 9

(For detailed calculation see also MATLAB file ‘Exercise 14.9 Solution MATLAB Calculation’ on book webpage.)

(a) (bond price and swap rate from FRA rates)

Exercise 14.9a Solution Matlab Calculation Forward

### Calculation

In order to compute the bond prices from FRA rate, we can simply use the following formula

$$B(t_0, t_i) = \frac{100}{\prod_{j=0}^{i-1} (1 + \delta F(t_0, t_j))}$$

Now to determine the yield of the yield from discount bond prices we can use the following formula

$$r_i = \frac{1}{\delta_i} \left( \frac{100}{B(t_0, t_1)} - 1 \right)$$

For the computation of the (forward) swap rate use the following relationship

$$s_{t_0} = \frac{\sum_{i=1}^n B(t_0, t_{i+1})F(t_0, t_i)}{\sum_{i=1}^n B(t_0, t_{i+1})}$$

We can find derive par yield and the zero yield from the following formula

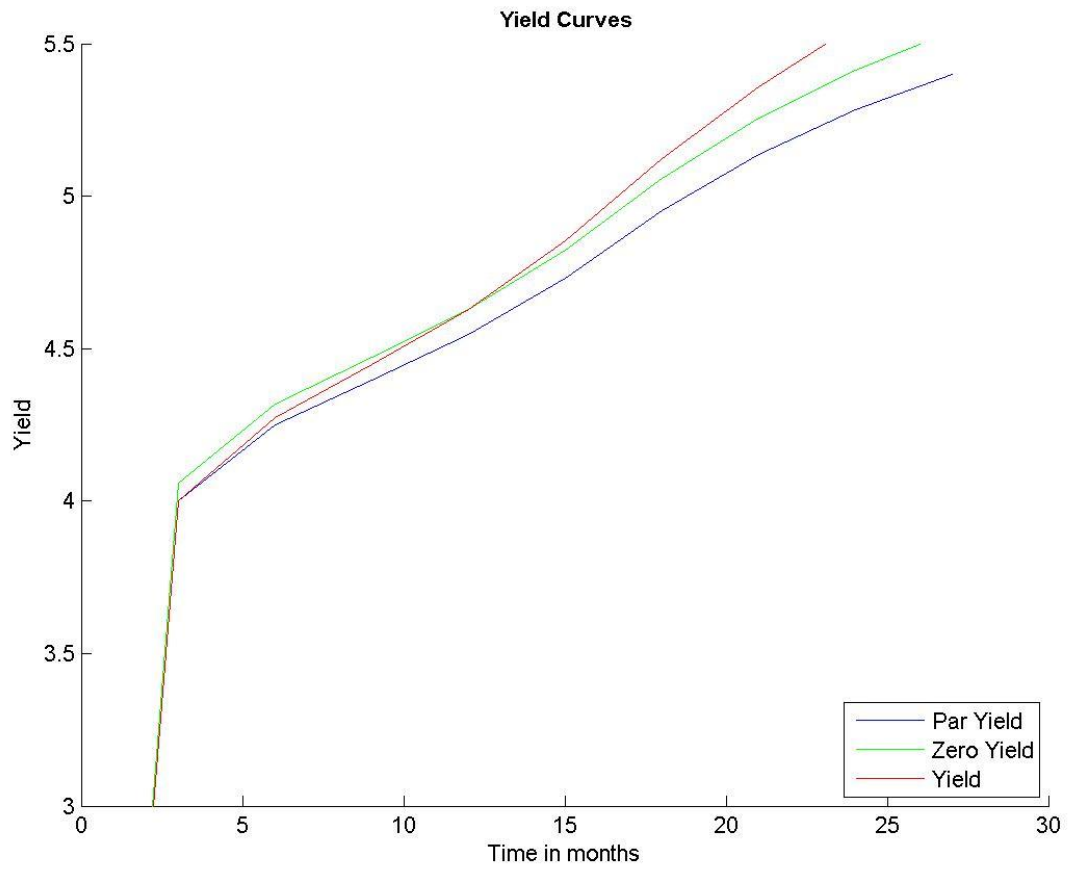
$$y_n = \frac{1}{\delta} \left( \frac{100 - B(t_0, t_n)}{\sum_{i=1}^n B(t_0, t_i)} \right)$$

Zero Coupon Yield

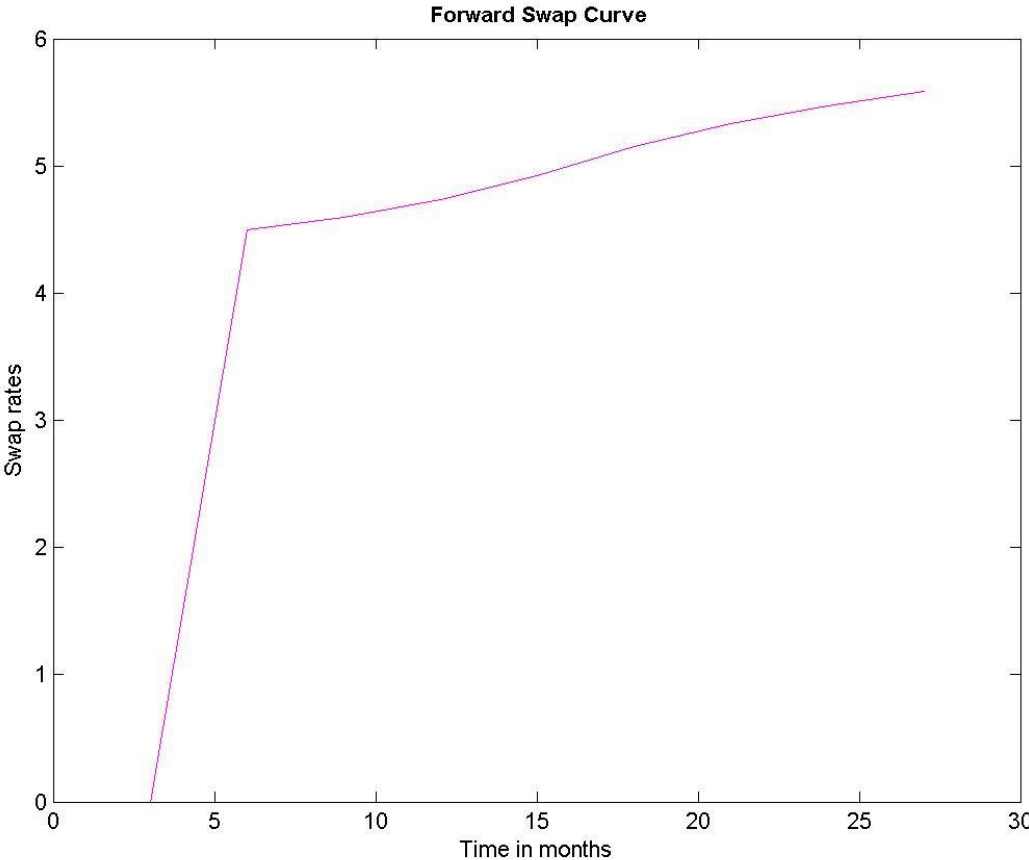
$$y_i = \left( \left( \frac{100}{B(t_0, t_i)} \right)^{\frac{1}{\delta_i}} - 1 \right)$$

## **Plots**

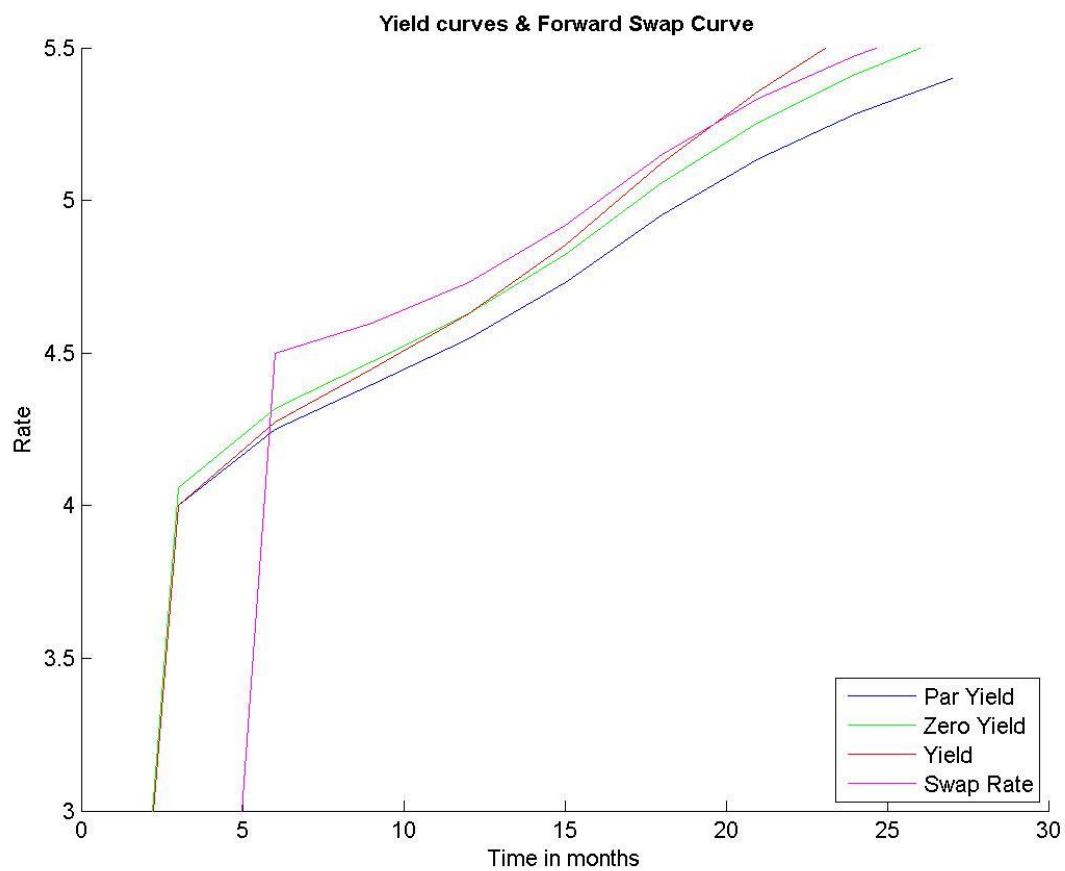
Bond Yield Curves



# Forward Swap Curve



All the curves together



(b) (bond price and FRA rate from forward swap rate)

(Exercise 14.9b Solution Matlab Calculation swap)

**Calculation from Forward Swap Rates**



Write a MATLAB program to determine the bond price and FRA rate from the given set of forward swap rates and calculate the yield, par yield and zero-coupon yield from the input data which is to be read from the 'Swap' worksheet of 'Data.xlsx' file. Now plot the all the curves simultaneously to observe their characteristics.

### **Calculation**

First we calculate the forward rate from the given swap rate use the following formula

$$F_{(n-1)xn} = \frac{S_n + \sum_{i=1}^{n-1} [(S_n - F_{(i-1)xi}) \prod_{j=i+1}^{n-1} (1 + F_{(j-1)xj})]}{1 - \sum_{i=1}^{n-1} [(S_n - F_{(i-1)xi}) \prod_{j=i+1}^{n-1} (1 + F_{(j-1)xj})]}$$

Then we proceed by calculating bond prices and then the yield curves. The bond prices from FRA rate, we can simply use the following formula

$$B(t_0, t_i) = \frac{100}{\prod_{j=0}^{i-1} (1 + \delta F(t_0, t_j))}$$

Now to determine the yield of the yield from discount bond prices we can use the following formula

$$r_i = \frac{1}{\delta_i} \left( \frac{100}{B(t_0, t_1)} - 1 \right)$$

We can find derive par yield and the zero yield from the following formula

$$y_n = \frac{1}{\delta} \left( \frac{100 - B(t_0, t_n)}{\sum_{i=1}^n B(t_0, t_i)} \right)$$

Zero Coupon Yield

$$y_i = \left( \left( \frac{100}{B(t_0, t_i)} \right)^{\frac{1}{\delta_i}} - 1 \right)$$

## Calculation from Forward Swap Rates

Write a MATLAB program to determine the bond price and FRA rate from the given set of forward swap rates and calculate the yield, par yield and zero-coupon yield from the input data which is to be read from the 'Swap' worksheet of 'Data.xlsx' file. Now plot the all the curves simultaneously to observe their characteristics.

### Calculation

First we calculate the forward rate from the given swap rate use the following formula

$$F_{(n-1)xn} = \frac{S_n + \sum_{i=1}^{n-1} [(S_n - F_{(i-1)xi}) \prod_{j=i+1}^{n-1} (1 + F_{(j-1)xj})]}{1 - \sum_{i=1}^{n-1} [(S_n - F_{(i-1)xi}) \prod_{j=i+1}^{n-1} (1 + F_{(j-1)xj})]}$$

Then we proceed by calculating bond prices and then the yield curves. The bond prices from FRA rate, we can simply use the following formula

$$B(t_0, t_i) = \frac{100}{\prod_{j=0}^{i-1} (1 + \delta F(t_0, t_j))}$$

Now to determine the yield of the yield from discount bond prices we can use the following formula

$$r_i = \frac{1}{\delta_i} \left( \frac{100}{B(t_0, t_1)} - 1 \right)$$

We can find derive par yield and the zero yield from the following formula

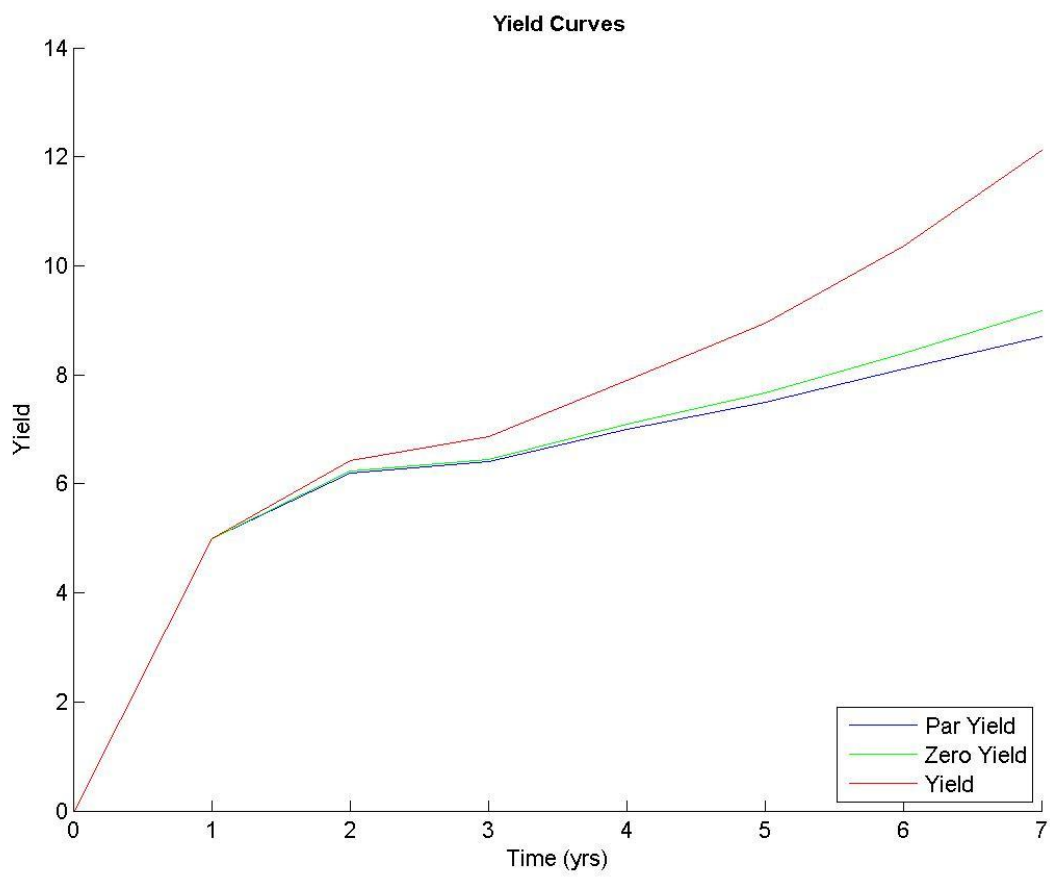
$$y_n = \frac{1}{\delta} \left( \frac{100 - B(t_0, t_n)}{\sum_{i=1}^n B(t_0, t_i)} \right)$$

Zero Coupon Yield

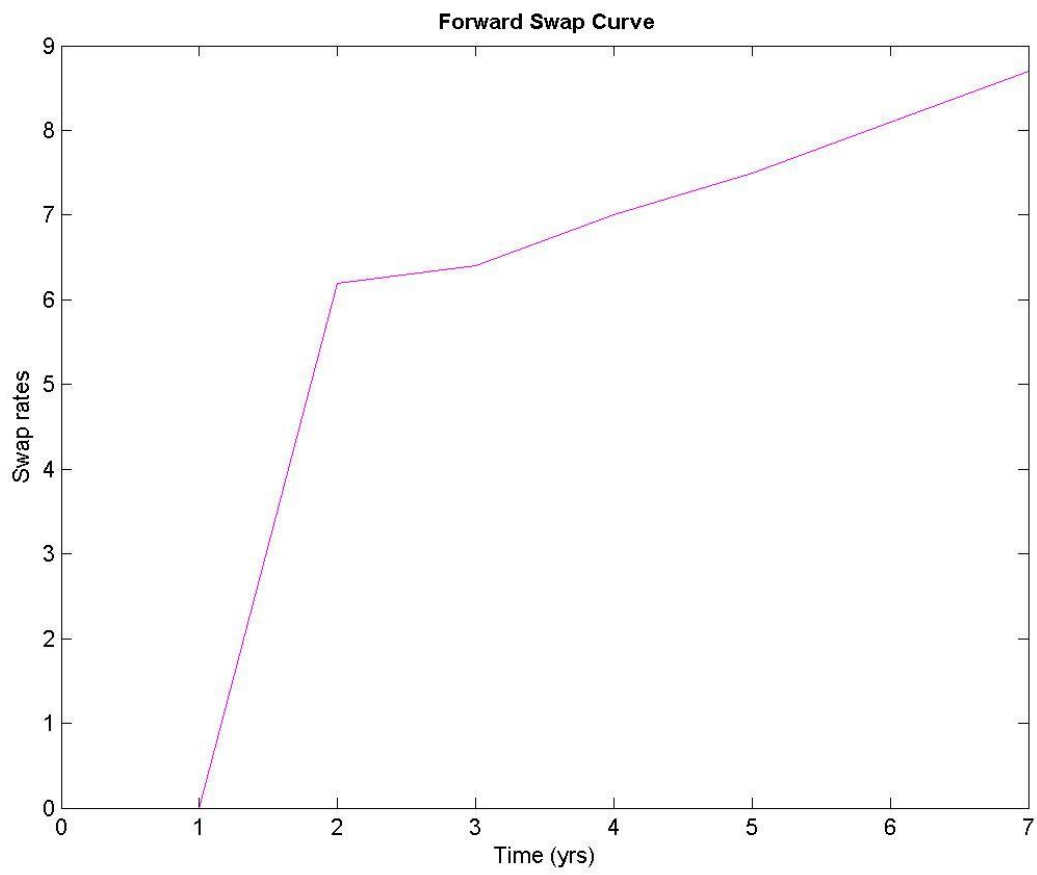
$$y_i = \left( \left( \frac{100}{B(t_0, t_i)} \right)^{\frac{1}{\delta_i}} - 1 \right)$$

## Plots

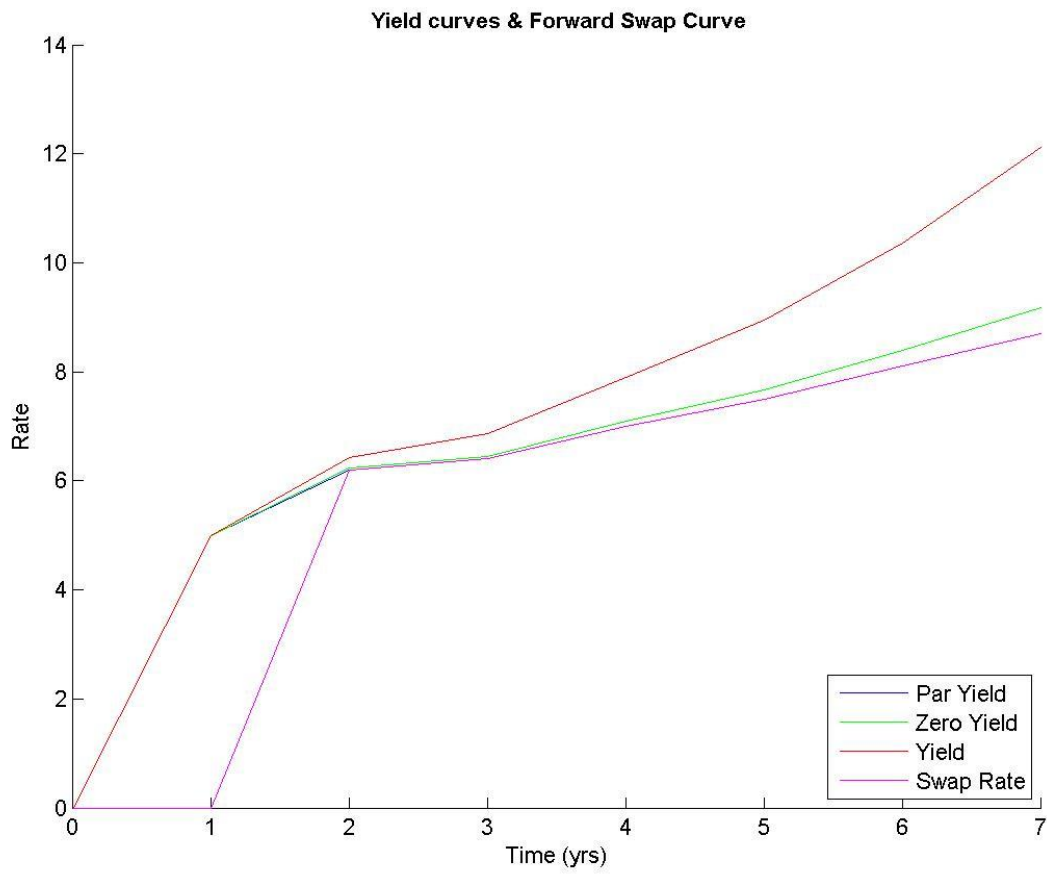
### Bond Yield Curves



Forward Swap Curve



All the curves together



(c)

### Calculation from Bond Price

(Exercise 14.9c Solution Matlab Calculation Bond price)

Write a MATLAB program to determine the FRA price and swap rate from the given set of bond prices and calculate the yield, par yield and zero-coupon yield from the input data which is to be read from the 'Bond' worksheet of 'Data.xlsx' file. Now plot the all the curves simultaneously to observe their characteristics.

### **Calculation**

In order to compute the FRA rate from bond prices we can simply use the following formula

$$F(t_0, t_i) = \frac{1}{\delta} \left( \frac{B(t_0, t_{i+1})}{B(t_0, t_i)} - 1 \right)$$

Now to determine the yield of the yield from discount bond prices we can use the following formula

$$r_i = \frac{1}{\delta_i} \left( \frac{100}{B(t_0, t_i)} - 1 \right)$$

For the computation of the (forward) swap rate use the following relationship

$$s_{t_0} = \frac{\sum_{i=1}^n B(t_0, t_{i+1}) F(t_0, t_i)}{\sum_{i=1}^n B(t_0, t_{i+1})}$$

We can find derive par yield and the zero yield from the following formula

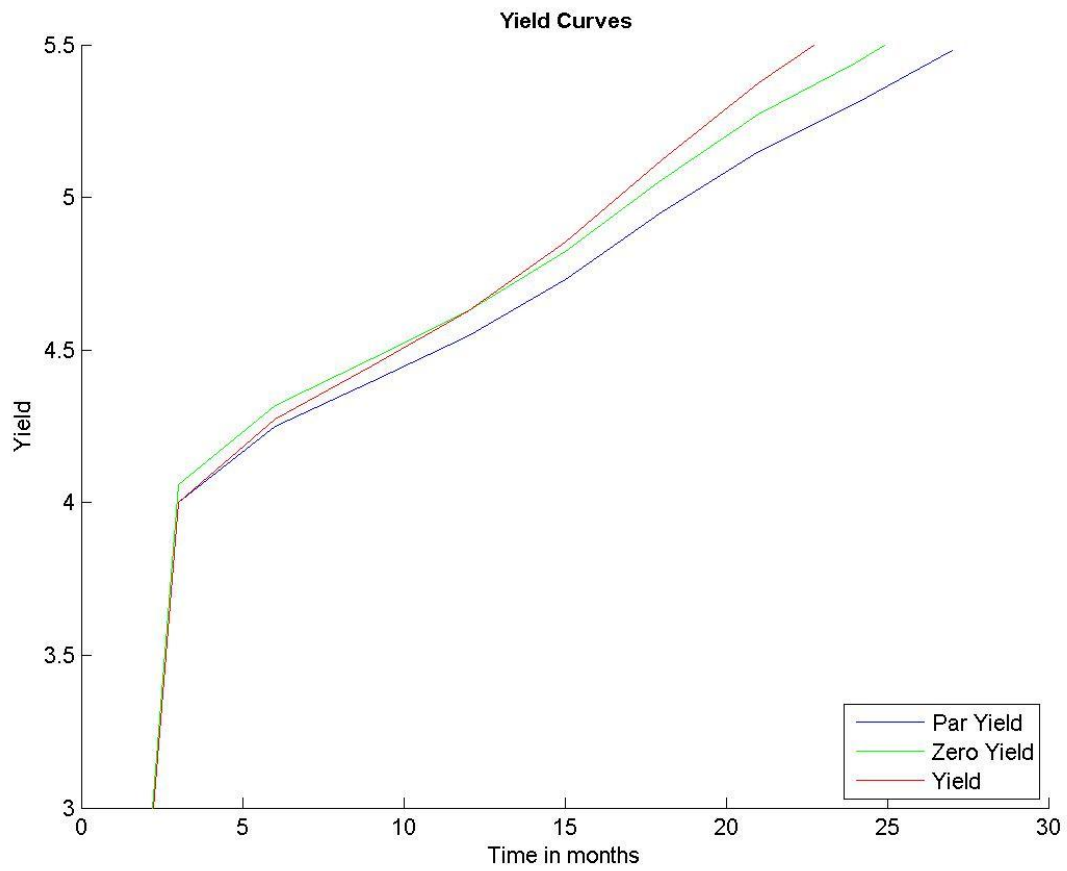
$$y_n = \frac{1}{\delta} \left( \frac{100 - B(t_0, t_n)}{\sum_{i=1}^n B(t_0, t_i)} \right)$$

Zero Coupon Yield

$$y_i = \left( \left( \frac{100}{B(t_0, t_i)} \right)^{\frac{1}{\delta_i}} - 1 \right)$$

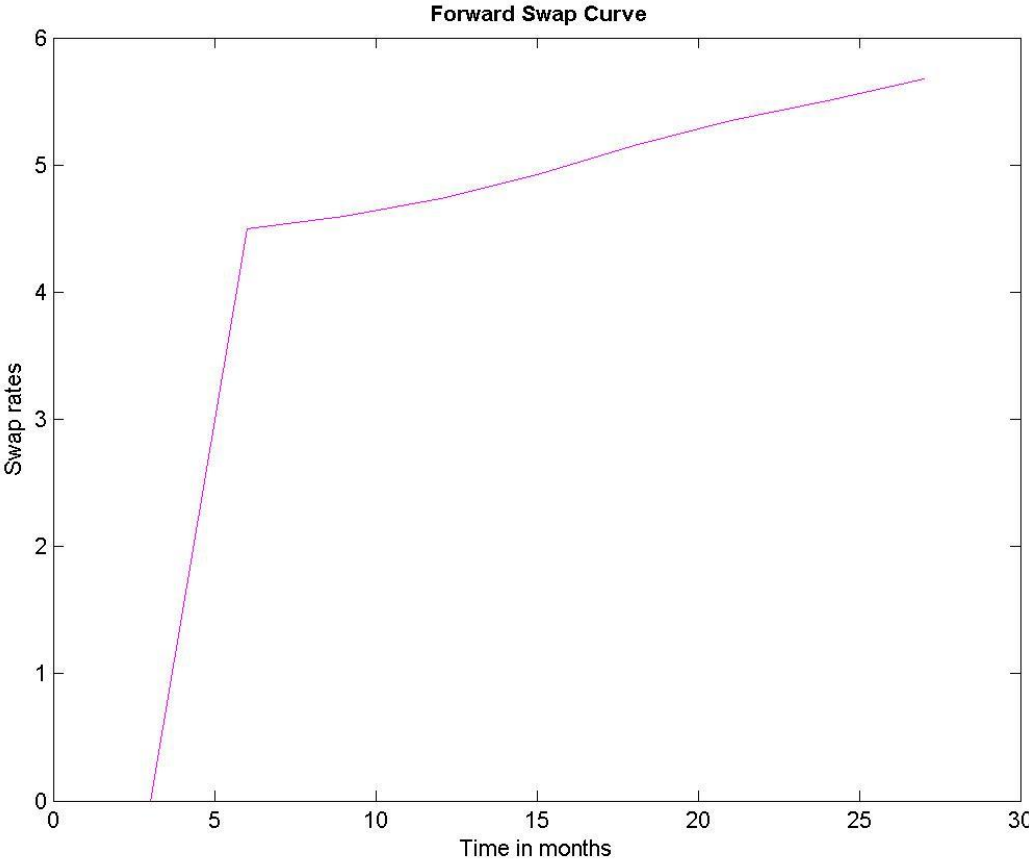
## Plots

### Bond Yield Curves





Forward Swap Curve



All the curves together

Yield curves & Forward Swap Curve

