CHAPTER 15

Solutions

Exercise 1

1. Let's look at the cash flow of the volatility (variance) spread swap: $-\left(\sigma_{Nasdaq}^2 - \sigma_{S\&P500}^2\right)N2$

   It is clear from this expression that investor actually takes a long position on the S&P500 variance and a short position on the NASDAQ variance. This trade can be put in place by simultaneously entering into a long S&P500 variance swap and a short NASDAQ variance swap.

   Pricing here is to determine the initial variance swap *spread* which makes the initial value of the swap between the two indices have a value of zero. This price is given as 21% in the question.

2. Yes, we need the correlation between the two markets. If the correlation is high (close to one in absolute value) between these markets, then this implies that most of the time, volatility will move in both markets in the same direction which in return, indicates that volatility (variance) spread is relatively tight in the long run (This makes the position mentioned in the question reasonable). So the fixed leg of the spread has to be set (relatively) higher.

If the correlation is low (close to zero in absolute value), which implies that these two markets move more or less independently from each other, then there is no reason to believe that the volatility spread between the markets should get narrower.

It is less likely that the investor who holds a long position will end up with a positive payoff. In order to make the initial value of the swap equal to zero, the fixed leg of the spread needs to be set at a lower level.

3. The smile effect is important. However, in this present case the trade concerns realized volatility and not the Black-Scholes implied volatility. This means that the pricing of the swap will make no use of the smile in any direct way. Indirectly, the smile can be useful to calibrate a model, on the other hand.
4. If the position is taken by using volatility (variance) swaps, then this may be less risky compared to the other ways of taking the same position. Also the pricing of the instrument may be easier. (See the exercise for alternative ways of taking the same position and see the text for the risk of these positions). The main risk involved here is related to the assumption that the longrun dynamics of the volatility spread is stationary. If this underlying assumption is violated, then the position may lose.

Exercise 2

1. If an investor buys long-dated volatility, and sells short-dated volatility, then the investor is expecting a decrease in the short dated volatility and an increase in the long dated volatility. Of course, there is no guarantee that these expectations will be realized. If short run volatility goes up or / and long run volatility decreases, the pay off from the position would definitely be negative.
2. This is equivalent to an investor constructing a short straddle position by using knock-out options. Long straddle may be constructed by using options which have break-out clause to put a limit to unrestricted risk of loss that arises from the short (straddle) position in

the short run. If short dated volatility turns out to be high, an additional premium can be triggered on the options which are used for the long straddle. This additional payoff from the long volatility position off sets the losses from the short volatility position.

3. The relevant payoff function will shift upwards by the amount of the additional payoff.
4. If the additional premium is a fixed amount, this may cut potential losses, yet it may not be sufficient enough. However, considering that the short position is taken for one month, this risk may not be a very big risk.
5. It is clear from the text (see section 3.3) that volatility positions taken by using the straddles are not pure volatility positions.

Exercise 3

(For detailed calculation see also Matlab file 'Exercise 15.3 Solution Matlab Calculation' on book webpage.)

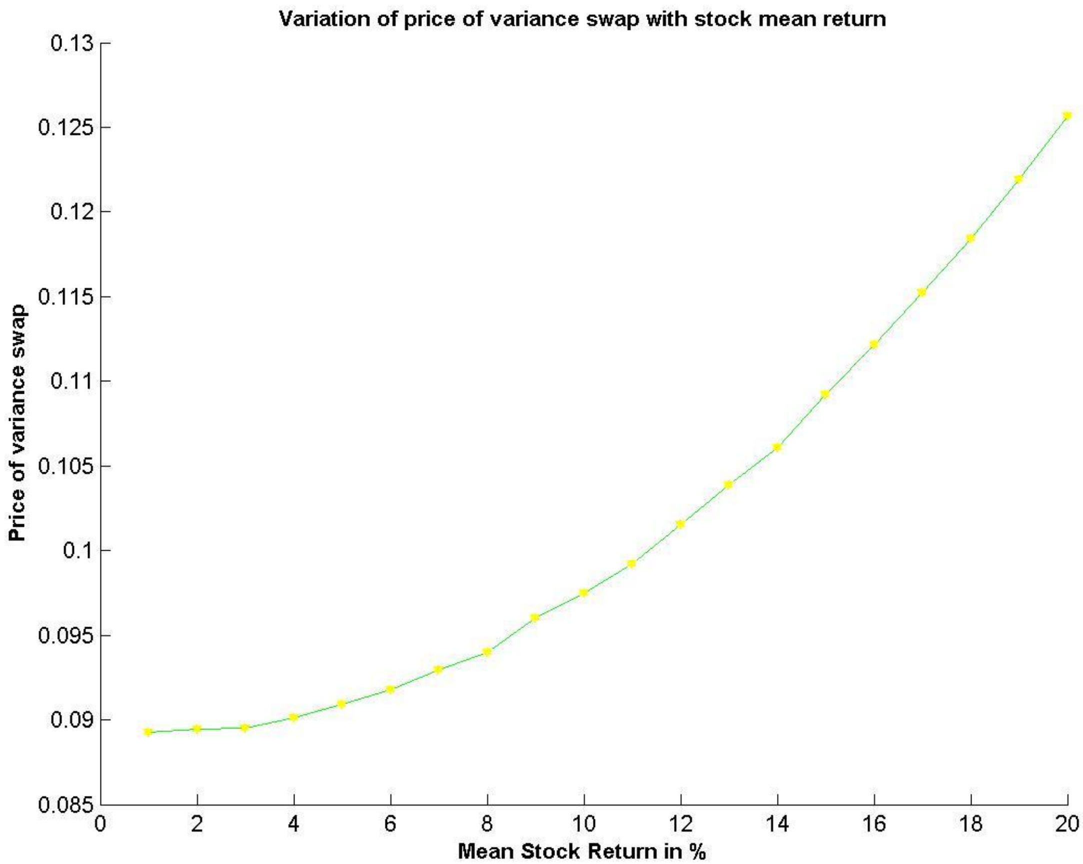
<u>Calculation:</u>

The fair price of the variance swap in the discrete time setting is given by the following formula

$$F_{t_0}^2 = \frac{1}{T_2 - T_1} E_{t_0}^{\tilde{P}}\left[\sum_{i=1}^{n}\left|\frac{S_{t_i} - S_{t_{i-1}}}{S_{t_{i-1}}} - \mu\delta\right|^2\right]$$

<u>Plot</u>

Variation of the variance swap price with the change in the mean return of the stock



Exercise 4

(For detailed calculation see also Matlab file 'Exercise 15.4 Solution Matlab Calculation' on book webpage.)

<u>Calculation</u> Le the frequency of delta hedge adjustment, M = 10

At time $t_0 = 0$

1. The dealer buys a call with maturity $T = 1\ yrs.$
2. He borrows the amount $C(S_0, t_0)$ amount of money at the risk free rate r for 1 year.
3. Short the Δ_0 amount of shares and deposit the amount $\Delta_0 S_0$ for $\delta = 1/M$ time period.

At time $t_1 = \delta$

1. Change in the delta value is observed i.e. $\Delta_1 - \Delta_0$
   a) If change is positive means that $\Delta_1 - \Delta_0$ no. of more shares needs to be shorted.
   b) If change is negative implies that $\Delta_1 - \Delta_0$ to be bought back from the market.
2. Now add the resultant cash flow $(\Delta_1 - \Delta_0) * S_1$ due of the above portfolio adjustment to the existing cash position.

Repeat this portfolio adjustment until expiration.

At the time of expiration T

1. Change the delta value to $\Delta = 1$ if $S_T > K$ or $\Delta = 0$ if $S_T < K$.
2. Add the resultant cash flow $(\Delta_T - \Delta_{T-1}) * S_T$ to the existing cash position.
3. Close the short stock position by buying the stock after the payment of $\Delta_T * S_T$.
4. Close the loan position by the payment of $C(S_0, t_0) * (1 + r)$.
5. Obtain the net position after adding the payoff from the call option $MAX[(S_T - K), 0]$.

Obtain the volatility payoff from this delta hedge position for 10 different instances for each value of M and report the payoff in the bar diagram.