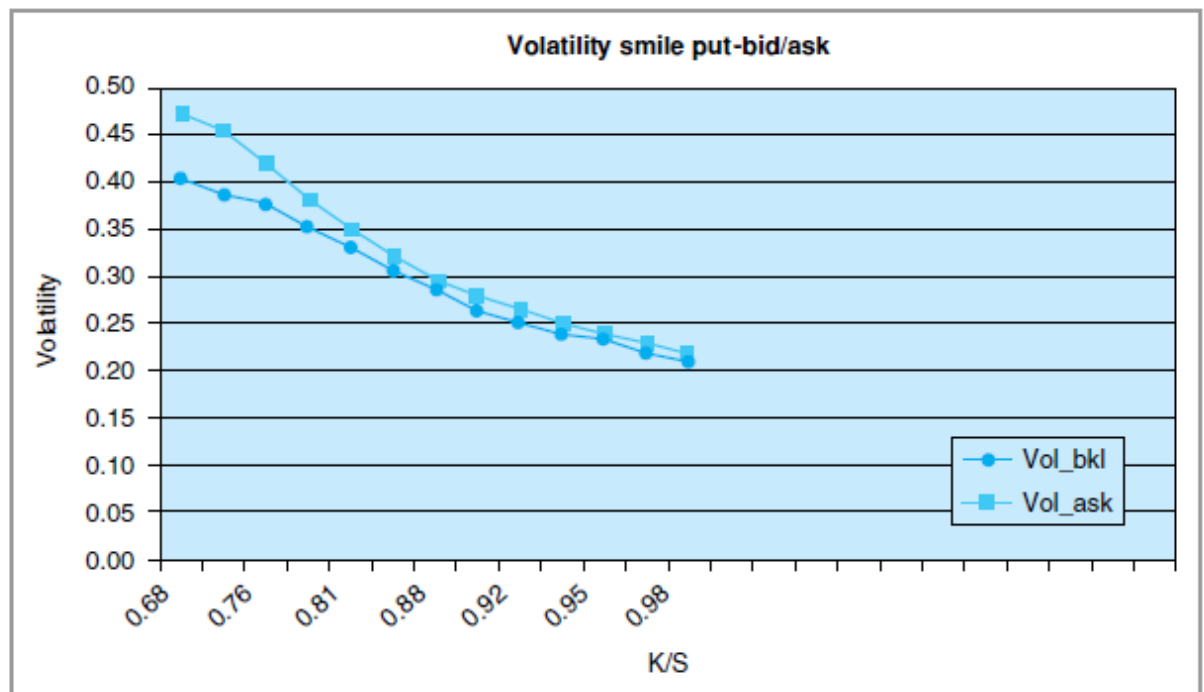# CHAPTER 16

Solutions

Exercise 1

1. In this question the implied volatilities are computed by using a risk- free interest rate of 2%. The calculations are summarized by the following figure.

Volatility smile put-bid/ask

2. The calculations yield the following results in the Figure shown below. The bid-ask spread on the implied volatilities can be seen in the following Figure. Clearly, these spreads are not constant across options with different moneyness.

Call volatility smile-ask
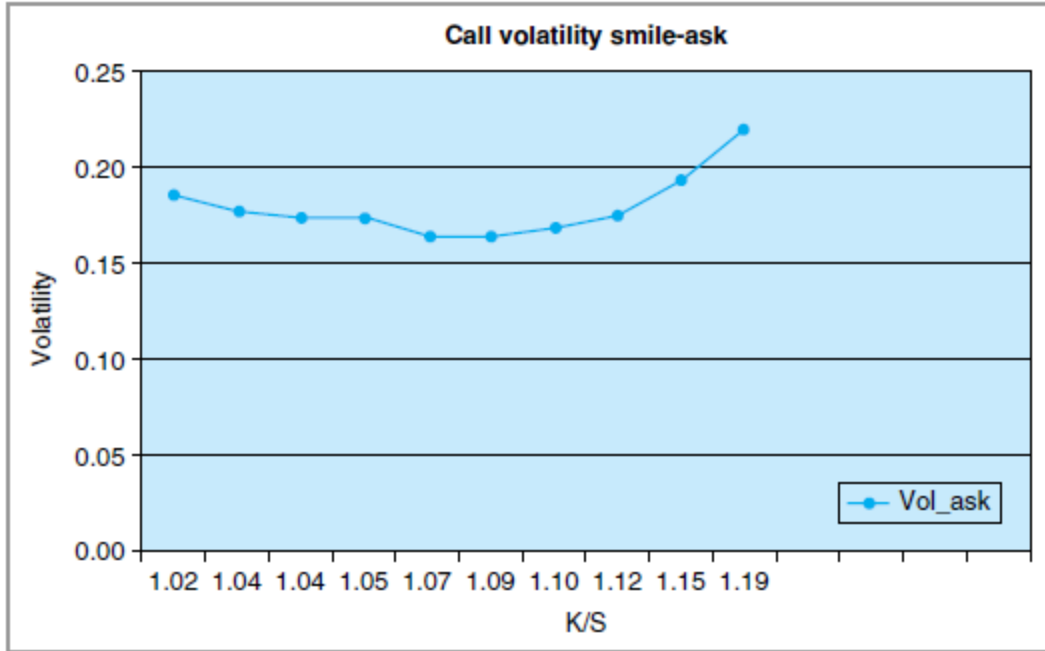
3. When the two Figure are put together the result will be more like a skew.

Vol_ask (put-call)
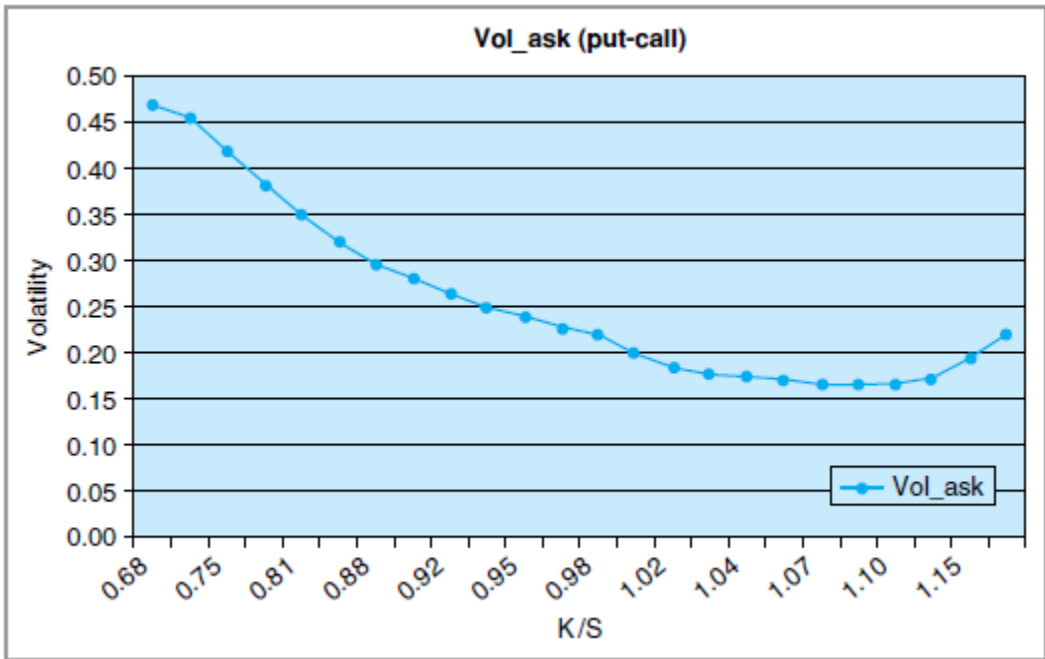
Exercise 2

(Stop-loss hedge)

(For detailed calculation see also Excel file 'Exercise 15.3 Solution Excel Calculation' on book webpage.)

Calculation (Assuming N = 1)

At time $t_0 = 0$

1. The dealer buys a call with maturity $T = 1\ yrs$.

2. He borrows the amount $C(S_0, t_0)$ amount of money at the risk free rate r for 1 year.

3. Short the $\Delta_0 = 1$ amount of shares and deposit the amount S_0 for $\delta = 1/M$ time period.

At time $t_1 = \delta$

1. Change the delta value to $\Delta = 1$ if $S_T > K$ or $\Delta = 0$ if $S_T < K$.

   a) If change is positive means that $\Delta_1 - \Delta_0$ no. of more shares needs to be shorted.

   b) If change is negative implies that $\Delta_1 - \Delta_0$ to be bought back from the market.

2. Now add the resultant cash flow $(\Delta_1 - \Delta_0) * S_1$ due of the above portfolio adjustment to the existing cash position.

Repeat this portfolio adjustment until expiration.

At the time of expiration T

1. Change the delta value to $\Delta = 1$ if $S_T > K$ or $\Delta = 0$ if $S_T < K$.

2. Add the resultant cash flow $(\Delta_T - \Delta_{T-1}) * S_T$ to the existing cash position.

3. Close the short stock position by buying the stock after the payment of $\Delta_T * S_T$.

4. Close the loan position by the payment of $C(S_0, t_0) * (1 + r)$.

5. Obtain the net position after adding the payoff from the call option $MAX[(S_T - K), 0]$.

Exercise 3

(Stop loss hedged long call)

(For detailed calculation see also Matlab file 'Exercise 16.3 Solution Matlab Calculation' on book webpage.)

Calculation

Assuming N = 1 and M = 10

At time $t_0 = 0$

1. The dealer buys a call with maturity $T = 1\ yrs$.

2. He borrows the amount $C(S_0, t_0)$ amount of money at the risk free rate r for 1 year.

3. Short the $\Delta_0 = 1$ amount of shares and deposit the amount S_0 for $\delta = 1/M$ time period.

At time $t_1 = \delta$

1. Change the delta value to $\Delta = 1$ if $S_T > K$ or $\Delta = 0$ if $S_T < K$.

    a) If change is positive means that $\Delta_1 - \Delta_0$ no. of more shares needs to be shorted.

    b) If change is negative implies that $\Delta_1 - \Delta_0$ to be bought back from the market.

2. Now add the resultant cash flow $(\Delta_1 - \Delta_0) * S_1$ due of the above portfolio adjustment to the existing cash position.

Repeat this portfolio adjustment until expiration.
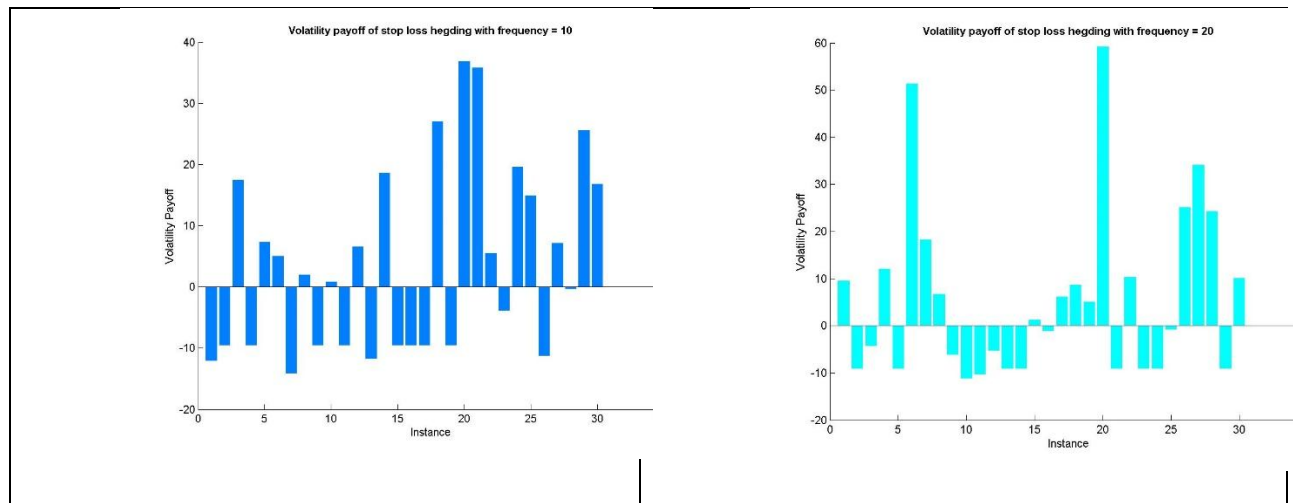
At the time of expiration T

1. Change the delta value to $\Delta = 1$ if $S_T > K$ or $\Delta = 0$ if $S_T < K$.
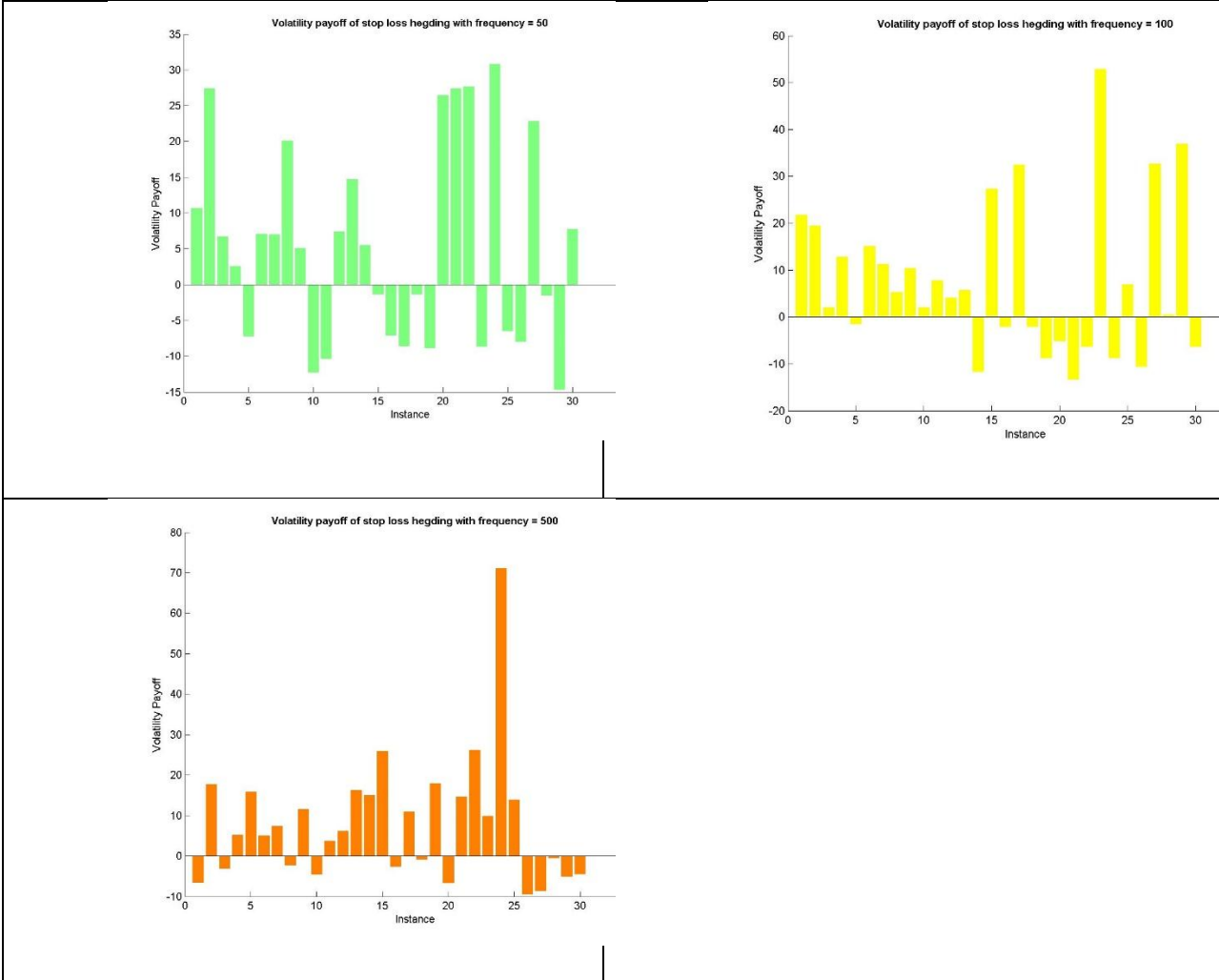
2. Add the resultant cash flow $(\Delta_T - \Delta_{T-1}) * S_T$ to the existing cash position.

3. Close the short stock position by buying the stock after the payment of $\Delta_T * S_T$.

4. Close the loan position by the payment of $C(S_0, t_0) * (1 + r)$.

5. Obtain the net position after adding the payoff from the call option $MAX[(S_T - K), 0]$.

Obtain the volatility payoff from this delta hedge position for 30 different instances for each value of M and report the payoff in the bar diagram.
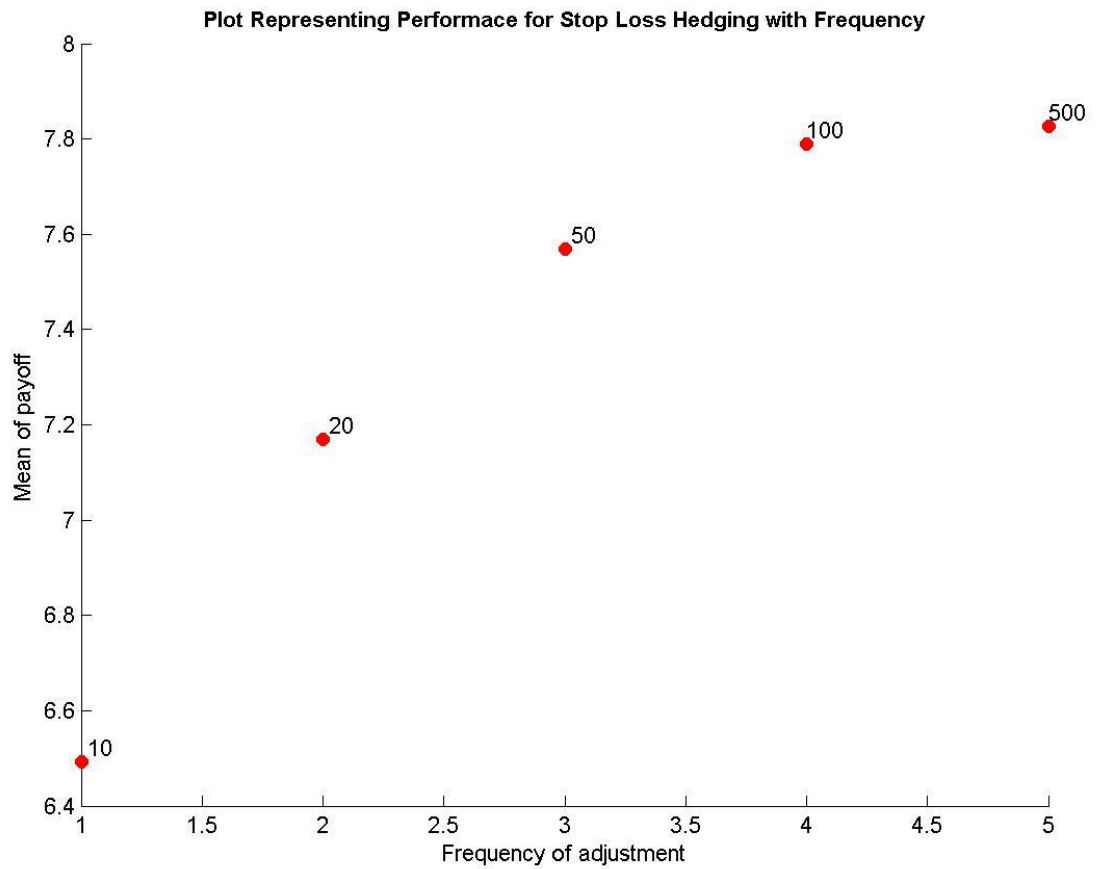
Plots

Volatility payoff for 30 instances different adjustment frequency

Volatility payoff of stop loss hegding with frequency = 50

Volatility payoff of stop loss hegding with frequency = 100

Volatility payoff of stop loss hegding with frequency = 500

Mean return of the stop loss hedging strategy v/s frequency of adjustment

Plot Representing Performace for Stop Loss Hedging with Frequency

Exercise 4

(Stochastic volatility)

(For detailed calculation see also Matlab file 'Exercise 16.4 Solution Matlab Calculation' on book webpage.)

Simulation

For various values of strike price K use following the dynamics to simulate stock prices and employing the stochastic volatility and find out the payoff of out-of-money options.

$$\sigma_{t+1} - \sigma_t = r_\sigma \sigma_t \Delta + \sigma_\sigma \sigma_t (\Delta Z_t)$$

$$S_{t+1} - S_t = r S_t \Delta + \sigma_{t+1} S_t (\Delta W_t)$$

Calculation

Calculate the payoff of the option at time T and discount it to obtain the present value.

$$C_t = e^{-r(T-t)} E^{\tilde{P}}[\max(S_T - K, 0)]$$

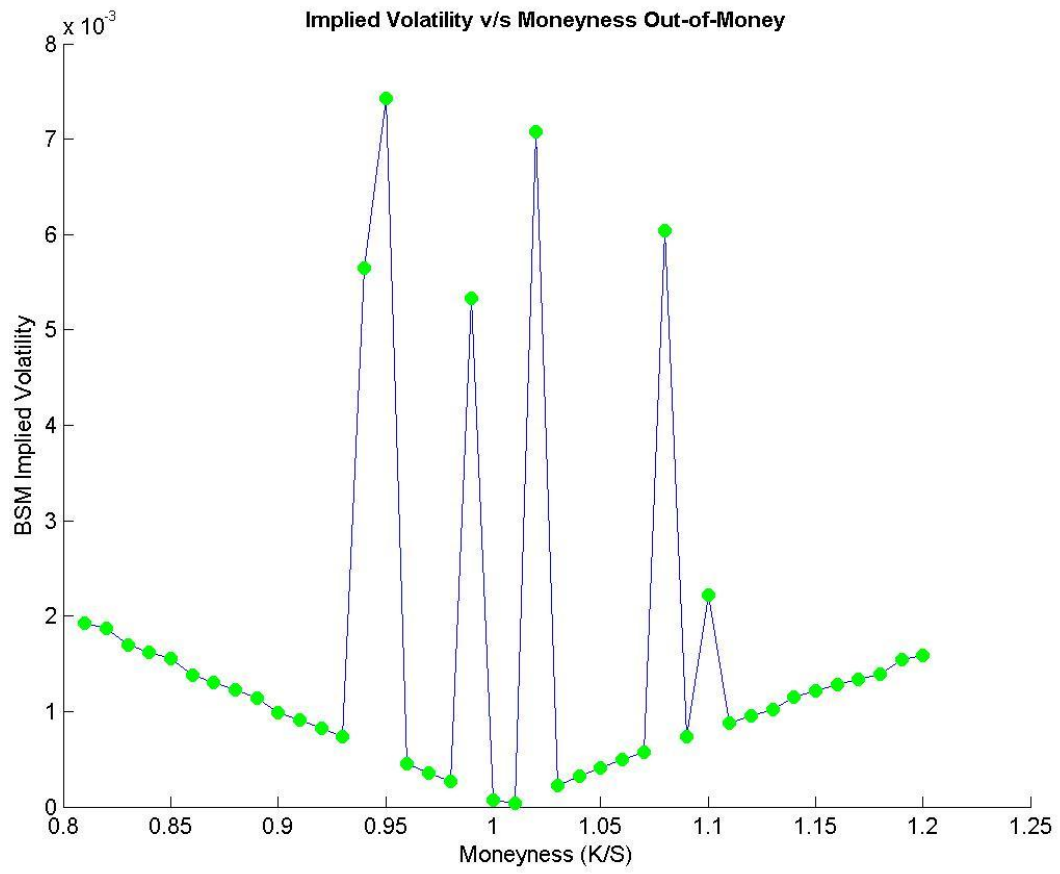$$P_t = e^{-r(T-t)} E^{\tilde{P}}[\max(K - S_T, 0)]$$

Take the average of payoffs for all simulations and report the obtained value as the option price estimated from the simulation.

Now if $K > S_0$ calculate the implied volatility of the out-of-money *call* option using the reverse engineering technique for the BSM option price formula. Similarly when $K < S_0$ calculate the implied volatility for out-of-money *put* option and report them in the graph of implied volatility versus the moneyness K/S_0 of the option.
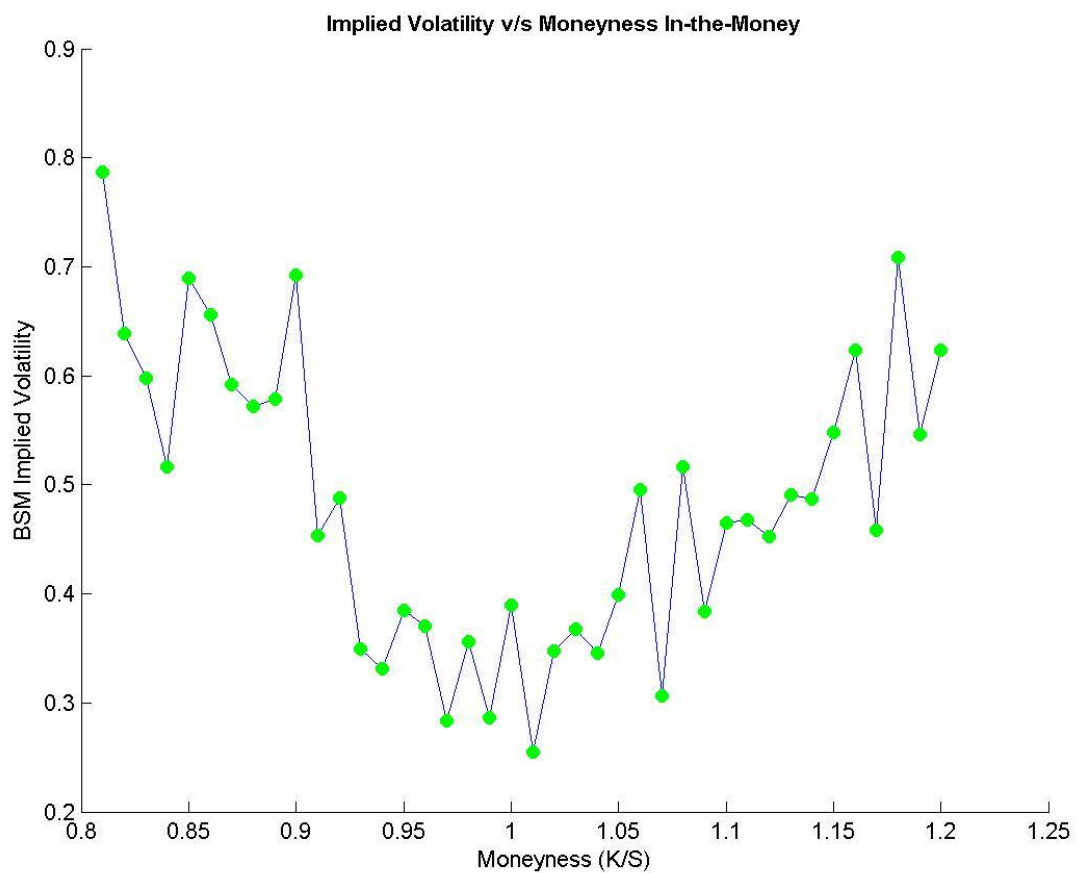
Plots

Plot of the implied volatility against the moneyness of the option derived from out-of-money

options

Implied Volatility v/s Moneyness Out-of-Money

Plot of the implied volatility against the moneyness of the option derived from in-the-money

options

Implied Volatility v/s Moneyness In-the-Money

Exercise 5

(Nongeometric Brownian Motion)

(For detailed calculation see also Matlab file 'Exercise 16.5 Solution Matlab Calculation' on book webpage.)

Simulation

For various values of strike price K use following the dynamics to simulate stock prices which follows non-geometric Brownian motion and find out the payoff of out-of-money options.

$$S_{t+1} - S_t = rS_t\Delta + \sigma S_t{}^\alpha(\Delta W_t)$$

Calculation

Calculate the payoff of the option at time T and discount it to obtain the present value.
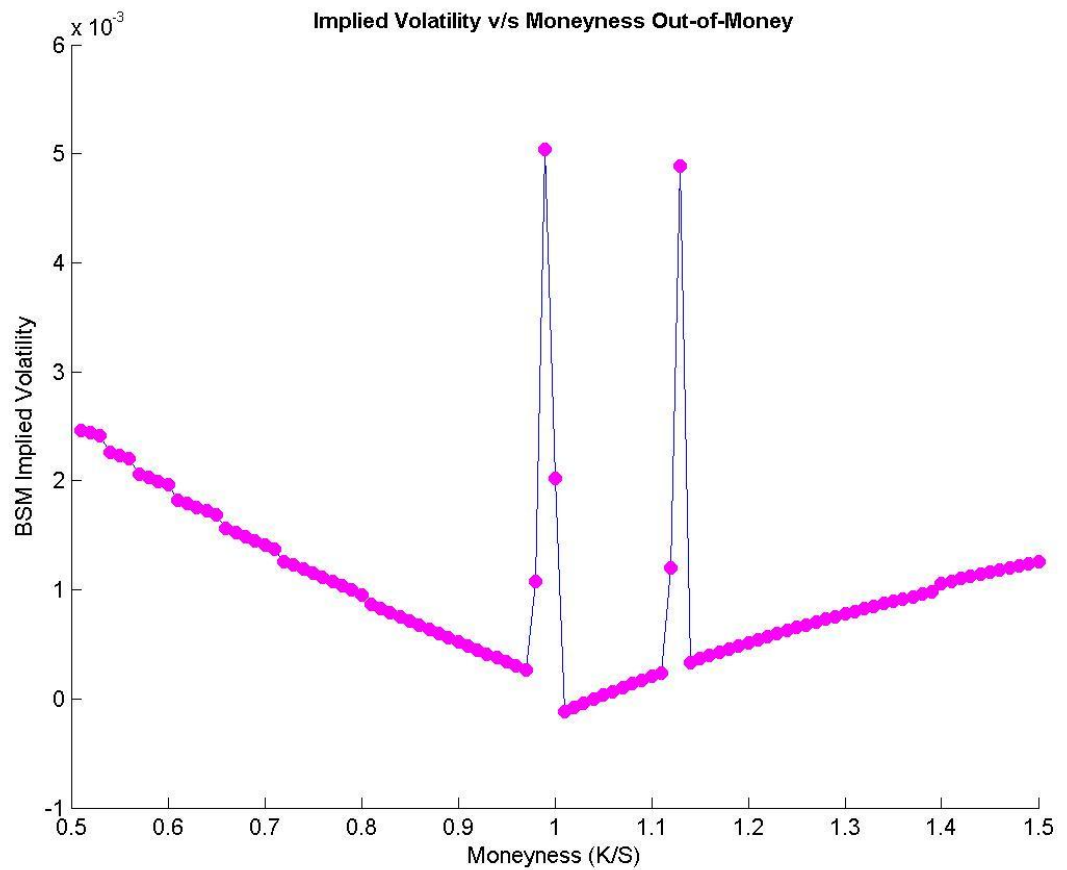
$$C_t = e^{-r(T-t)}E^{\tilde{P}}[\max(S_T - K, 0)]$$

$$P_t = e^{-r(T-t)}E^{\tilde{P}}[\max(K - S_T, 0)]$$

Take the average of payoffs for all simulations and report the obtained value as the option price estimated from the simulation.
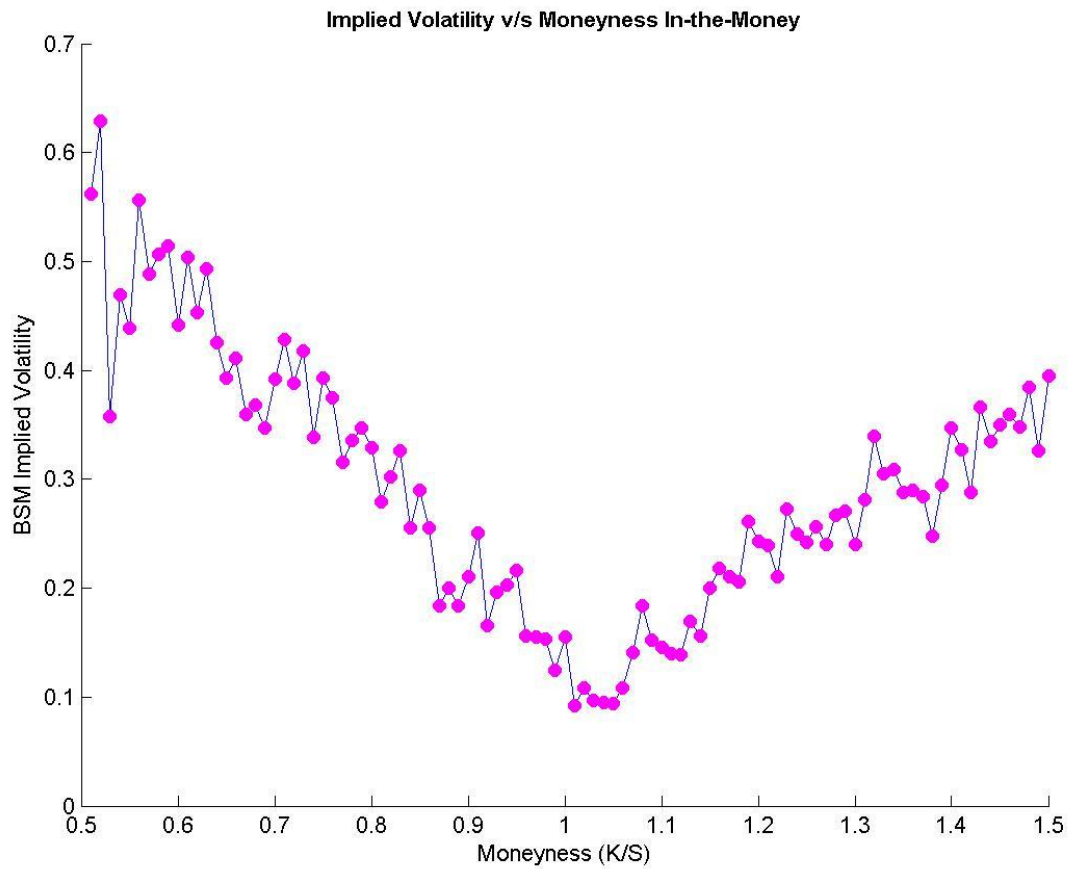
Now if $K > S_0$ calculate the implied volatility of the out-of-money *call* option using the reverse engineering technique for the BSM option price formula. Similarly when $K < S_0$ calculate the implied volatility for out-of-money *put* option and report them in the graph of implied volatility versus the moneyness K/S_0 of the option.

Plots

Plot of the implied volatility against the moneyness of the option derived from out-of-money options

Plot of the implied volatility against the moneyness of the option derived from in-the-money options

Implied Volatility v/s Moneyness In-the-Money

Exercise 6

(Stock price crashes and jumps)

TBC

(For detailed calculation see also Matlab file 'Exercise 16.6 Solution Matlab Calculation' on

book webpage.)

Simulation

For various values of strike price K follow the steps given below for the stock price simulation

and at maturity calculate the payoff of out-of-money option.

1. Select m from the Poisson distribution; $\quad\quad\quad$ m = poissrnd(λ)

2. Generate m random numbers having normal distribution $N(0, \Delta)$

3. Calculate $k = e^{\alpha_J} - 1$

$$S_{t+1} = S_t e^{(r-\lambda k-0.5\sigma^2)\Delta + \sigma\sqrt{\Delta}W_\Delta} e^{m(\alpha_J - 0.5\sigma_J^2) + \sigma_J \sum_{i=0}^{m} W_i}$$

Reference: Robert McDonald | Chapter 19 Section 8 Pg. no. 595 Equation 19.16

Calculation

Calculate the payoff of the option at time T and discount it to obtain the present value.

$$C_t = e^{-r(T-t)} E^{\tilde{P}}[\max(S_T - K, 0)]$$

$$P_t = e^{-r(T-t)} E^{\tilde{P}}[\max(K - S_T, 0)]$$

Take the average of payoffs for all simulations and report the obtained value as the option price

estimated from the simulation.

Now if $K > S_0$ calculate the implied volatility of the out-of-money *call* option using the reverse

engineering technique for the BSM option price formula. Similarly when $K < S_0$ calculate the

implied volatility for out-of-money *put* option and report them in the graph of implied volatility
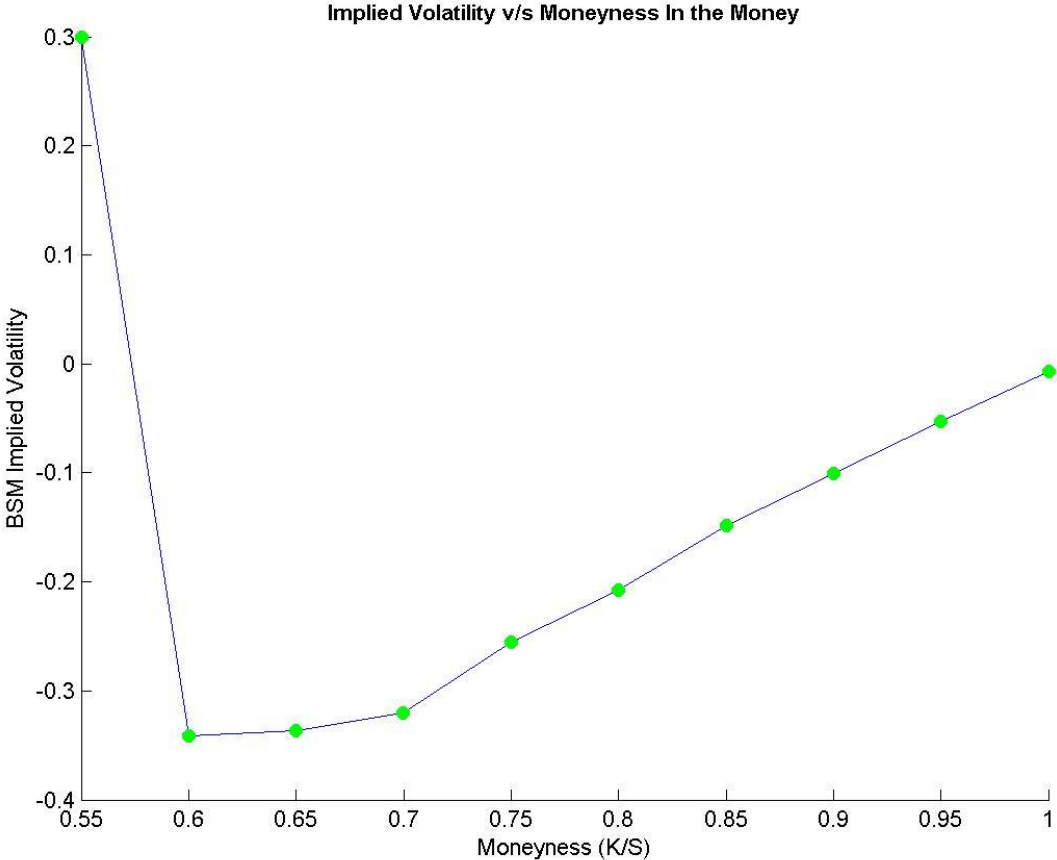
versus the moneyness K/S_0 of the option.

Sadly, smile is not evident in this case.

Plots

Plot of the implied volatility against the moneyness of the option derived from out-of-money
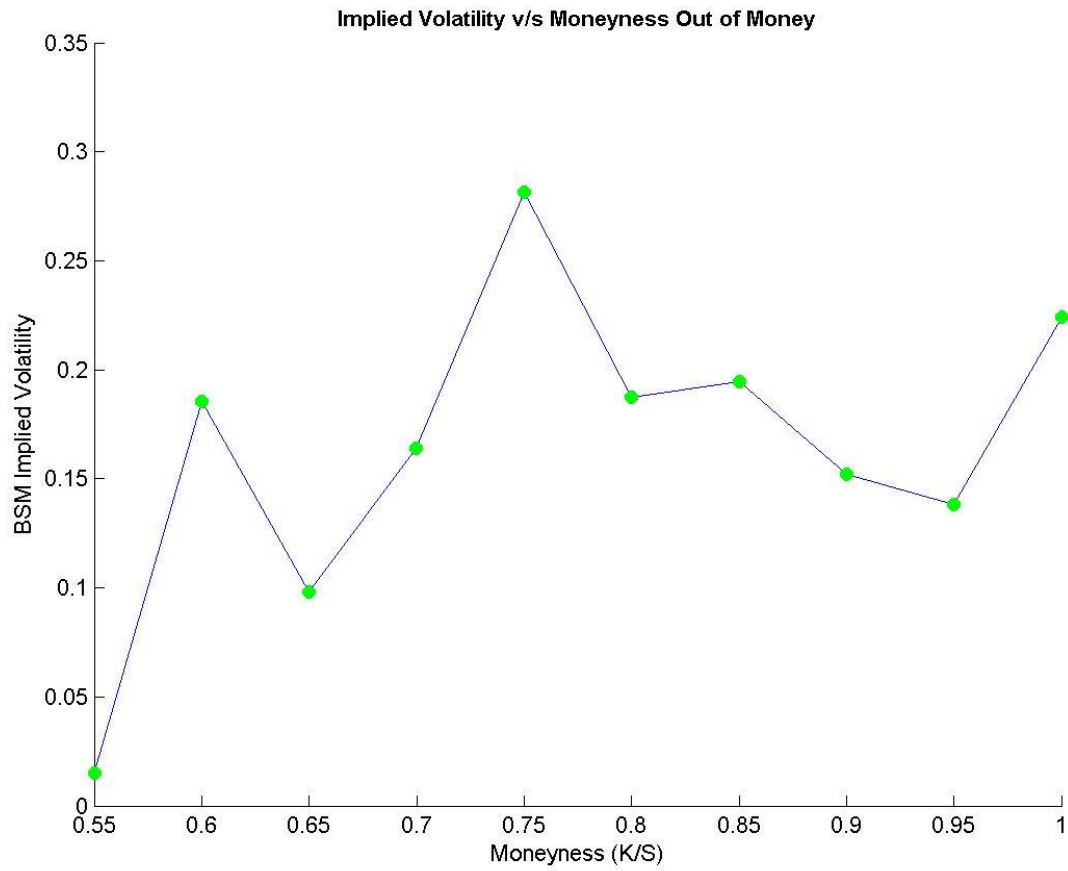
options

Plot of the implied volatility against the moneyness of the option derived from in-the-money

options

Exercise 7

(VIX Calculation; this exercises was listed under the category MATLAB exercises, but here we provide the Excel solution)

(For detailed calculation see also Excel file 'Exercise 16.7 Solution Excel Calculation VIX Calculation' on book webpage)

Calculation

The generalized formula for the VIX calculation is:

$$\sigma^2 = \frac{2}{T} \sum_i \frac{\Delta K_i}{K_i} e^{rT} Q(K_i) - \frac{1}{T} \left[\frac{F}{K_0} - 1 \right]$$

For strike price = 1635 we have minimum absolute difference between call and put price so the forward price will be equal to

$$F = 1635 - 1.7 * e^{5*21/365} = 1632.733$$

Now form the excel calculation we obtained the sum of the contribution of options by strike hence $\sum_i \frac{\Delta K_i}{K_i} e^{rT} Q(K_i) = 5.53e^{-4}$

Putting this value in the VIX equation we have

$$\sigma^2 = \frac{2 * 365}{21} * 5.53e^{-4} - \frac{365}{21} \left[\frac{1632.73}{1635} - 1 \right] = 4.33e^{-2}$$

$$\boldsymbol{VIX = 100 * \sigma = 20.816}$$