CHAPTER 20

Solutions

Exercise 1

(a) A convertible bond contains a call option. The investor has in a sense purchased an embedded call. If the price of the equity exceeds the conversion price then the investor will "call" the stocks. In a reverse-convertible bond it is the issuer who has purchased an option. In fact this is a put option. The issuer determines if and when the to convert the bond into stock. The investor on the other hand is short an embedded put. The investor will accept the delivery of a bond or a stock at a pre-determined price if the issuer chooses to "convert". For this added risk, the investor will receive a higher coupon.

(b) When volatility increases, it gives the following opportunities to dealers. High volatility implies high option prices. Thus reverse convertibles can be structured with higher coupons. This attracts investors. At the same time, the issuing company will be long an option. By hedging this the company can isolate the gamma. Hence, if the option is purchased at a "reasonable" price from the investor (which is quite likely in such cases...) then the gamma gains can very well exceed the premium paid for the option. The structurers gain two ways. From higher volatility and from selling new instruments.

(c) Chapter 19 shows how to construct synthetic convertibles. Chapter 20 discusses the construction of reverse convertibles. Figure 20.4 shows the general principle.

(d) Chapter 20 discusses other ways of selling volatility.

(e) Regulators may worry that such instruments are making investors sell options. Many investors may not realize how to price options given a certain volatility structure. Under such conditions they may sell options below the fair price.

Exercise 2

1. (a) Let r_m denote the *m* month swap rate (or the Libor rate). Then the 3 x n month forward rate $f_{(3\times n)}$is calculated thus:

$$\left(1 + r_3 \frac{3}{12}\right)(1 + f_{(3\times n)} \frac{n-3}{12}) = (1 + r_n \times \frac{n}{12}) .$$

The discount curve is calculated thus : the *n*- month discount rate $B(t_0, t_n)$ is

$$B(t_0, t_n) = \frac{1}{\prod_{i=0}^{n-1}(1+r_i/12)}.$$

(b) The 24 × *n* month forward rate $f_{(3\times n)}$ is calculated thus:

$$\left(1 + r_3 \frac{24}{12}\right)(1 + f_{(24\times n)} \frac{n - 24}{12}) = \left(1 + r_n \times \frac{n}{12}\right)$$

(c) The components for this note is a discount curve, a 2-year forward curve, a market for CMS swaps and Bermuda swaptions (since the note is callable).

(d) Let $cms_i^{j,k}$ denote *j*-year CMS bought for *k* years evaluated at the *i*-th year. Let c_{t_0} be the premium for a 2-year Bermuda swaption. Calculate $R_1 = L_0 + \alpha_0$, where L_0 is the current 1-year Libor rate and α_0 is calculated as:

$$c_{t_0} = \alpha_0 + B(t_0, t_1)\alpha_0 + B(t_0, t_2)\alpha_0$$

Year 2 coupon is: $\alpha_1(cms_2^{2,3}) + L_0 + \alpha_0$. We know $cms_2^{2,3}$ and hence α_1 is calculated by equating:

$$\alpha_1(cms_2^{2,3}) + L_0 = cms_2^{2,3}$$

Giving $\alpha_1 = 1 - \frac{L_0}{cms_2^{2,3}}$.

Similarly$\alpha_2$is calculated from

$$\alpha_1(cms_2^{2,3}) + \alpha_2(cms_3^{2,3}) + L_0 = cms_3^{2,3}$$

Type equation here.

Giving $\alpha_2 = 1 - \frac{cms_2^{2,3}}{cms_3^{2,3}}$.

(e) The components for this note is a discount curve, a 3-year and a 2-year forward curve, a market for CMS swaps and Bermuda swaptions(since the note is callable).

(f) Let c_{t_0} be the premium for a 2-year Bermuda swaption. Calculate $R_1 = L_0 + \alpha_0$, where L_0 is the current 1-year Libor rate and α_0 is calculated as:

$c_{t_0} = \alpha_0 + B(t_0, t_1)\beta_1 + B(t_0, t_2)\beta_2$.

Here c_{t_0} , $B(t_0, t_1)$, $B(t_0, t_2)$ are known and the rest α_0, β_1, β_2 has to be determined from that. α is calculated as

$\alpha_0 = \frac{cms_0^{3,3} - cms_0^{2,3}}{s_0^3}$where s_0^3 is the 3-year swap rate.

(g) $\beta_1 = \beta_2$ can be chosen appropriately to satisfy

$c_{t_0} = \alpha_0 + B(t_0, t_1)\beta_1 + B(t_0, t_2)\beta_2$

Exercise 3

(a) The note can be engineered thus:

- 1-year Libor deposit.
- Get into a receiver interest rate swap paying *Libor* and getting

5.23%.

- Buy digital cap (for *Libor* > 6.13%) and digital floor (for *Libor* > 6.13%).
- Pay *CMS*10 for first 2 years and 8 × *CMS*10 for the next three years.
- Receive *CMS*30 for first 2 years and 8×*CMS*30 for the next three years.

(b) An investor who expects the yield curve to steepen in the long run and expects Libor to remain quite high would demand this product. She/he would expect Libor to increase and also the CMS spread to increase leading to an increase in the difference $CMS30 - CMS10$.

Exercise 4

(a) The investor expects that the CMS10 rate would gradually increase.

(b) If the CMS10 curve flattens or the rate does not increase significantly (slow increase) then the coupons will go on decreasing and might become negligible.

The other risk is that the issuer might not call the note in such a situation and the investment would be stuck having sub-optimal gains.

(c) (*typo in problem, should be CMS10 instead of Libor, and we have an additional rate 10%*)

The following table give the coupons, given that we know the CMS rates as given in the problem:

$Year\ 1, Q1 : 9.00\%$.

$Year\ 1, Q2 : (9.00 + 5.00 - 4.65)\% = 9.35\%$

$Year\ 1, Q3 : (9.35 + 6.00 - 4.85)\% = 10.50\%$

$Year\ 1, Q4 : (10.50 + 6.50 - 5.25)\% = 11.75\%$

$Year\ 2, Q1 : (11.75 + 7.00 - 5.45)\% = 13.30\%$

$Year\ 2, Q2 : (13.30 + 8.0 - 5.65)\% = 15.65\%$

$Year\ 2, Q3: (15.65 + 9.0 - 5.65)\% = 19.00\%$

$Year\ 2, Q4: (19.00 + 10.0 - 5.65)\% = 23.35\%$

$Year\ 3: (23.35 + 10.0 - 5.65)\% = 27.70\%$

$Year\ 4 - 10: (23.35 + 10.0 - 5.65)\% = 27.70\%$

So, as the CMS10 rate increases the coupon payment snowballs. The following table give the coupons, given that we know the CMS rates are constant at 3.5% as given in the problem:

$Year\ 1, Q1: 9.00\%.$

$Year\ 1, Q2: (9.00 + 3.50 - 4.65)\% = 7.85\%$

$Year\ 1, Q3: (7.85 + 3.50 - 4.85)\% = 6.50\%$

$Year\ 1, Q4: (6.50 + 3.50 - 5.25)\% = 4.85\%$

$Year\ 2, Q1: (4.85 + 3.50 - 5.45)\% = 2.90\%$

$Year\ 2, Q2: (2.90 + 3.50 - 5.65)\% = 0.75\%$

$Year\ 2, Q3: max(0.75 + 3.50 - 5.65, 0)\% = 0\%$

$Year\ 2, Q4: 0\%$

$Year\ 3: 0\%$

$Year\ 4 - 10: 0\%$

Since the CMS10 does not increase, the coupon value decreases and reaches 0%.

(d) This coupon can be characterized by an interest rate swap. Note that the CMS10 rates are all known. Hence the coupons are all fixed in the beginning. The investor can pay the fixed coupons and receive *Libor* at every coupon payment date.

e) If the 8 FRAs are known, we can calculate the discount curve from that. Now if the coupons at each time t_1 is c_{t_1} , then the price of the coupon at t_i is

$P_i = B(t_0, t_i)(c_{t_i} - F_{t_i})$

where F_{t_i} is the forward rate at t_i.

(f) The first years coupon is generated from *Libor* and a part of the price of the Bermuda swaption sold.

(g) The coupons are floored as mentioned (“minimum of 0%”)

(h) CONTRACTUAL EQUATION:

Snowball Note = Libor deposit + Receiver interest rate swap (wrt CMS10) + Short Bermuda swaption

Exercise 5

The *CMS spread note* can be engineered thus:

- Deposit $10 million earning Libor.
- Get into a 10 year Receiver swap.
- Receive 16×*CMS*30 and Pay 16×*CMS*10 for 10 years with two CMS swaps.
- Sell a Bermuda swaption.
- Buy a CMS spread floor and a CMS cap.

(a) The investor expects the yield curve will steepen in the long run.

(b) The obvious risk is if the yield curve flattens then there will be very low yielding coupons. Also the coupon may not be called while a low yield is going on. Of less significance but still to be remembered is that in case of very steep yield curve, the gains will still be capped at 30%.