CHAPTER 22

Solutions

Exercise 1

The distinction between cash and synthetic instruments is that while in a cash CDO assets are physically removed from a bank's balance sheet, in a synthetic transaction the risk is transferred to third-party investors, with the originating bank retaining the assets. The attraction of synthetic CDOs for many banks is the ability to pass the risk to third parties whilst retaining ownership of the assets.

Exercise 2

In the Gaussian copula model the implied correlation is that level of ρ which yields a calculated tranche spread that equals the observed spread in the markets. One of the issues is that the implied (compound) correlations can sometimes not be unique or may not even exist in mezzanine tranches. To address the issue of the nonmonotonicity of the mezzanine tranche, it became market practice to quote in another correlation, the base correlation, in addition to the compound correlation. The approach consists of transforming tranche quotes into quotes for equity tranches with increasing attachment points The procedure to calculate base correlations in this way is referred to as a bootstrapping process as explained in the chapter.

Exercise 3

The $1-$ year CDS rates in terms of basis points for the three IG names in the portfolio are given by:

$$c(1) = 116,\ c(2) = 193,\ c(3) = 140.$$

The recovery rate is uniformly 40%, i.e., $R = 0.4$. In every tranche a notional amount of \$1.50 is invested.

(a) The default probabilities are given by:

$p_1 = [c(1)/10000]/(1 - R)=0.0116/(1 - 0.4) = 0.0193333,$

$p_2 = [c(2)/10000]/(1 - R)= 0.0193/(1 - 0.4) = 0.0321667,$

$p_3 = [c(3)/10000]/(1 - R)=0.0140/(1 - 0.4) = 0.0233333.$

(b) To obtain the default distribution we need to get the default correlations between the three names first. If the correlations are all zero or all one (not possible in this case as the default probabilities are different) we can calculate the default distribution using standard probabilistic techniques. If they are different all zero or all one, then we can use the latent variable technique described in this chapter to generate dependent zero-one-valued random variables and simulate the default distribution from that.

(c) The defaults are uncorrelated. If D denoted the number of defaults,

$\mathbf{P}(D = 0) = (1 - p_1)(1 - p_2)(1 - p_3) = 0.9269757,$

$\mathbf{P}(D = 1) = p_1(1 - p_2)(1 - p_3) + (1 - p_1)p_2(1 - p_3) + (1 - p_1)(1 - p_2) * p_3 = 0.07122975,$

$\mathbf{P}(D = 2) = p_1p_2(1 - p_3) + (1 - p_1)p_2p_3 + p_1(1 - p_2) * p_3 = 0.00178002,$

$\mathbf{P}(D = 3) = p_1 * p_2 * p_3 = 0.00001451.$

(d) A $0 - 66\%$ tranche provides protection on the first two defaults. The expected loss (in dollars) on this tranche is given by:

$L_{0-66} = 0\times \mathbf{P}(D = 0)+ [(1/2) \times \mathbf{P}(D = 1) + 1 \times \mathbf{P}(D \geq 2)]\times 1.50\times (1 - R) = 0.03366846.$

(e) The pay over (in dollars) for the 0−50% and 50−100% tranches are:

$L_{0-50} = 0\times \mathbf{P}(D = 0)+1.50\times\mathbf{P}(D \geq 1)\times(1 - R)$=0.06572185,

$L_{50-100} = 0\times [\mathbf{P}(D \leq 1)] + [2/3\times \mathbf{P}(D = 2) + 1 \times \mathbf{P}(D = 3)]\times 1.50 \times (1 - R) =$ 0.001081071.

(f) We have $c(1) = c(2) = c(3) = 100$. Hence $p = p1 = p2 = p3$ =[100/10000]/((1−0.4) = 0.01667. The default distribution is given by:

$\mathbf{P}(D = 0) = (1 - p) = 0.98333$,

$\mathbf{P}(D = 3) = 0.01667$.

There is only one tranche in this case which has spread: $\mathbf{P}(D = 3)\times0.6 = 0.01$, i.e., 100bp, whose expected loss is \$1.50×$\mathbf{P}(D = 3) \times 0.6$ = \$0.015.

Exercise 4

The 1− year CDS rates in terms of basis points for the four IG names in the portfolio are given by: $c(1) = 14$, $c(2) = 7$, $c(3) = 895$, $c(4) = 33$.

The recovery rate is uniformly 30%, i.e., $R = 0.3$. In every tranche a notional amount of \$1.00 is invested.

(a) The default probabilities are given by:

$p_1 = [c(1)/10000]/(1 - R)=0.0014/(1 - 0.3)= 0.002$,

$p_2 = [c(2)/10000]/(1 - R)=0.0007/(1 - 0.3)= 0.001$,

$p_3 = [c(3)/10000]/(1 - R)=0.0895/(1 - 0.3)= 0.1278571$

$p_4 = [c(4)/10000]/(1 - R)= 0.0033/(1 - 0.3) = 0.004714286$.

(b) The defaults are uncorrelated. Let D denote the number of defaults. Then

$\mathbf{P}(D = 0) = (1 - p_1)(1 - p_2)(1 - p_3)(1 - p_4) = 0.865429$,

$\mathbf{P}(D = 1) = p_1(1-p_2)(1-p_3)(1-p_4) + (1-p_1)p_2(1-p_3)(1-p_4) + (1-p_1)(1-p_2)p_3(1-p_4) + (1-p_1)(1-p_2)(1-p_3)p_4 = 0.1335727$,

$\mathbf{P}(D = 2) = p_1p_2(1 - p_3)(1 - p_4) + \cdots + (1 - p_1)(1 - p_2)p_3p_4 = 0.00099626$

$\mathbf{P}(D = 3) = p_1p_2p_3(1 - p_4) + p_1p_2p_4(1 - p_3) + p_1p_3p_4(1 - p_2) + p_2p_3p_1(1 - p_1) = 0.000002069$,

$\mathbf{P}(D = 4) = p_1p_2p_3p_4 = 0.000000001205$.

(c) The payment over Libor in an year (in dollars) for the $0 - 50\%$, $50 - 75\%$ and $75 - 100\%$ tranches are:

$L_{0-50} = 0\times \mathbf{P}(D = 0) + (1/2 \times \mathbf{P}(D = 1) + 1 \times \mathbf{P}(D \geq 2)) \times (1 - R) = \$0.04744927$,

$L_{50-75} = 0\times [\mathbf{P}(D \leq 2)] + 1 \times \mathbf{P}(D \geq 3) \times (1 - R) = \$0.0000011449$,

$L_{75-100} = 0\times [\mathbf{P}(D \leq 3)] + 1 \times \mathbf{P}(D = 4) \times (1 - R) = \$0.00000000084$.

(d) We have $c(1) = c(2) = c(3) = c(4) = 60$. Hence $p = p_1 = p_2 = p_3 = p_4 = [60/10000]/(1-0.3) = 0.008571429$. The default distribution is given by:

$\mathbf{P}(D = 0) = (1 - p) = 0.9914286$,

$\mathbf{P}(D = 3) = 0.008571429$.

There is only one tranche in this case which has expected pay over a year given by:

$\$(\mathbf{P}(D = 3) \times (1 - R)) = \$0.006$.

Exercise 5

1. The 1− year CDS rates in terms of basis points for the three IG names in the portfolio are given by:

$c(1) = 15$, $c(2) = 11$, $c(3) = 330$.

The recovery rate is uniformly 40%, i.e., $R = 0.4$. In every tranche a notional amount of \$1.50 is invested.

(a) The default probabilities are given by:

$p_1 = [c(1)/10000]/(1 - R) = 0.0015/(1 - 0.4) = 0.0025,$

$p_2 = [c(2)/10000]/(1 - R) = 0.0011 /(1 - 0.4) = 0.001833,$

$p_3 = [c(3)/10000]/(1 - R) = 0.0330 / (1 - 0.4) = 0.055.$

(b) To obtain the default distribution we need to get the default correlations between the three names first. If the correlations are all zero or all one (not possible in this case as the default probabilities are different) we can calculate the default distribution using standard probabilistic techniques. If they are different from all zero or all one, then we can use the latent variable technique described in this chapter to generate dependent zero-one-valued random variables and simulate the default distribution from that.

(c) The defaults are uncorrelated. Let D denote the number of defaults. Then

$\mathbf{P}(D = 0) = (1 - p_1)(1 - p_2)(1 - p_3) = 0.9409093,$

$\mathbf{P}(D = 1) = p_1(1 - p_2)(1 - p_3) + (1 - p_1)p_2(1 - p_3) + (1 - p_1)(1 - p_2) * p_3 = 0.05884826,$

$\mathbf{P}(D = 2) = p_1p_2(1 - p_3) + (1 - p_1)p_2p_3 + p_1(1 - p_2) * p_3 = 0.00024216,$

$\mathbf{P}(D = 3) = p_1p_2p_3 = 0.000000252.$

(d) A 0 − 66% tranche provides protection on the first two defaults.

The expected loss (in dollars) on this tranche is given by:

$L_{0-66} = 0\times \mathbf{P}(D = 0)+ [(1/2)\times \mathbf{P}(D = 1) + 1 \times \mathbf{P}(D \geq 2)]\times 1.50 \times (1 - R) = 0.02669989.$

(e) The pay over (in dollars) for the 0−50% and 50−100% tranches are:

$L_{0-50} = 0\times \mathbf{P}(D = 0) + 1.50\times\mathbf{P}(D \geq 1) * (1 - R)$=0.05318163,

$L_{50-100} = 0\times \mathbf{P}(D \leq 1) + [(2/3)\times \mathbf{P}(D = 2) + 1 \times \mathbf{P}(D = 3)]\times 1.50 \times (1 - R) = 0.0001455$.

(f) We have $c(1) = c(2) = c(3) = 100$. Hence $p = p_1 = p_2 = p_3 = [100/10000]/(1-0.4) = 0.01667$. The default distribution is given by:

$\mathbf{P}(D = 0) = (1 - p) = 0.98333$,

$\mathbf{P}(D = 3) = 0.01667$.

There is only one tranche in this case which has spread:

$\mathbf{P}(D = 3)\times 0.6 = 0.01$, i.e., 100bp, whose expected loss is \$1.50× $\mathbf{P}(D = 3) \times 0.6$ = \$0.015.

Exercise 6

A *barbell* is a strategy of maintaining a portfolio of securities concentrated at two extremes in terms of maturity date: “very” short-term and “very” long term.

The investor has sold 7−year protection on the equity tranche and bought a barbell of 5-year and 10−year protection on the equity tranche. Since equity correlation has tightened for 7−year, the spread has widened. Look at the following graph showing spread vs. time for the equity tranche.

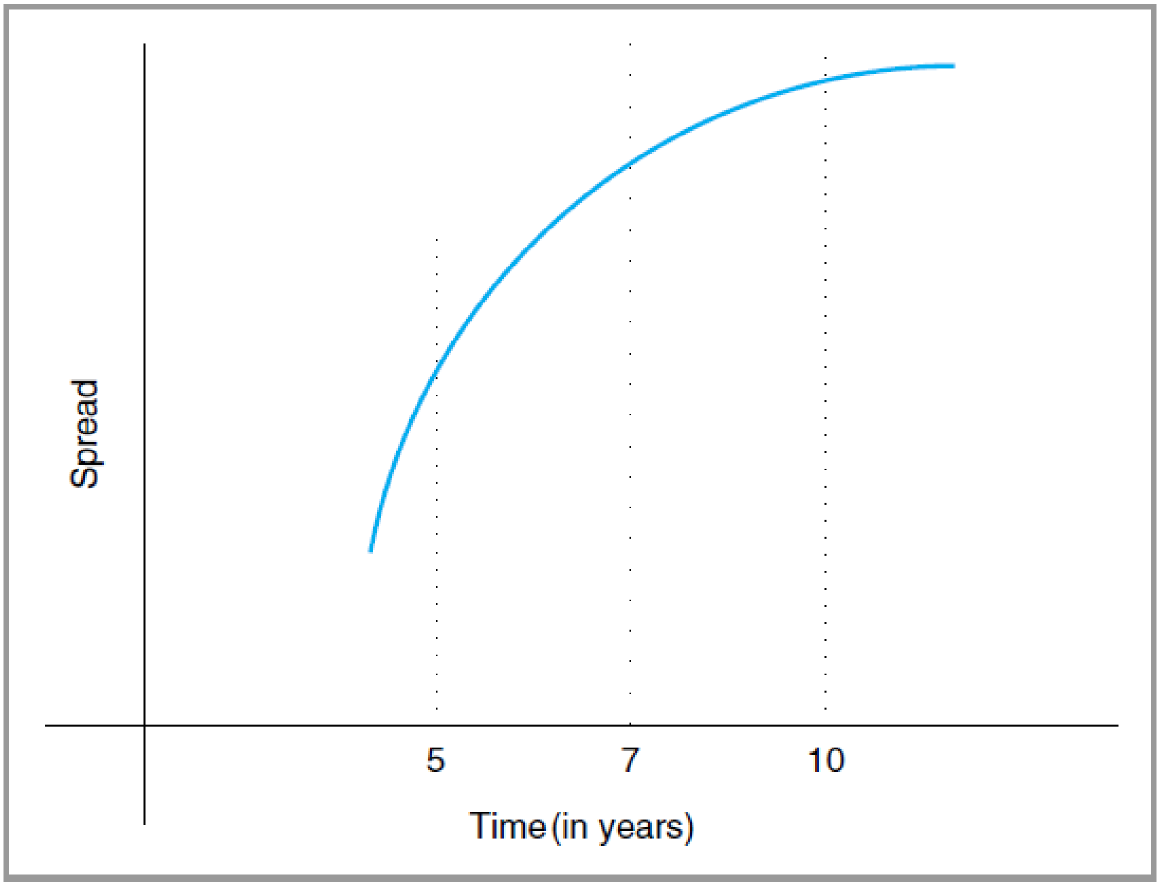

Figure : Spread of equity tranche$(0 - 3\%)$ vs. time

The convexity position comes from the fact that we are receiving from the high volatility in the 7-year protection and paying less from the barbell strategy. Refer to the figure.

Exercise 7

TBC

Exercise 8

TBC

Exercise 9

TBC