CHAPTER 3

Solution to Exercise 3.1

(a) The cash flows of this coupon bond can be separated into 8 separate payments. The first 7 of these will pay \$4 at ti, i = 1, .., 7. The last payment will be of size \$104. This can be used to create the synthetic:

$$Coupon\ Bond = \sum_{i=1}^{7} 4B(t_0, t_i) + 104B(t_0, t_8)$$

where the $B(t_0, t_i)$ are the time t_0 value of the default-free discount bonds maturing at t_i. These bonds pay \$1 at maturity. Hence the price of the coupon bond should equal the value on the right hand side plus a profit margin.

(b) In this question the B_i are measured at annual frequencies. However, the underlying cash flows are semi-annual. Hence some sort of interpolation of B_i is needed. Using a linear interpolation, and then applying the above equality twice we can get the bid-ask prices. For example, the Bid price of the coupon bond can be calculated as:

$$Bid = 4(.95 + .90 + .885 + .87 + .845 + .82 + .81) + 104(.80) = 107.52$$

According to this the coupon bond sells at a premium. This is not very surprising since, the 8% coupon is significantly higher than the annual rate implied by the term structure.

(c) Here one can use either the interpolated data or the original term structure, depending on how one interprets the numbers in 1x2 FRA. Taking these as the FRA rate on an 12-month loan that will be made in one year we get the equation:

$$1 + f(t_0, t_2, t_4) = \frac{B(t_0, t_2)}{B(t_0, t_4)}$$

Replacing from the term structure we obtain:

$$f(t_0, t_2, t_4)^{Bid} = \frac{0.9}{0.88} - 1 = 2.27\%$$

Solution to Exercise 3.2

(a) These are just the differences of the two prices. So, the mark to market losses are given by $\{Q_1 - Q_0, Q_2 - Q_1, Q_3 - Q_2, Q_4 - Q_3, Q_5 - Q_4\}$. Of course, negative losses are gains.

(b) You just calculate the interest accrued after multiplying by 1/360 for every day and,

(c) then adding the gains and losses.

Solution to Exercise 3.3

(a) Treasurer has risks for three months starting in three months. So a 3×6 FRA is needed.

(b) To get the break even rate we need:

$$\left(1 + 0.0673\left(\frac{1}{4}\right)\right)\left(1 + f\left(\frac{1}{4}\right)\right) = \left(1 + 0.0787\left(\frac{1}{2}\right)\right)$$

(c) Lowest offered rate. (6.87%)

(d) (FRA settlement) (0.0687 − 0.0609)(38 million)(1/4)

Solution to Exercise 3.4

(a) The futures price has moved by 34 ticks. (It moved from $Q_{t_0} = \$94.90$ to $Q_{t_0} = \$94.56$.

(b) The current implied forward rate is given by

$$\tilde{F}_{t_0} = \frac{100 - 94.90}{100} = 0.0510$$

which means the buyer of the contract needs to deposit

$$100\left(1 - \frac{0.0510}{4}\right) = 98.725$$

dollars per \$100 dollars on expiry (which is in three months in this case)

(c) In three months the futures price moves to $Qt1$ = \$94.56 giving a implied

forward rate of

$$\tilde{F}_{t_{10}} = \frac{100 - 94.56}{100} = 0.0544$$

and a settlement of

$$100\left(1 - \frac{0.0544}{4}\right) = 98.64$$

So the buyer of the original contract receives a compensation as if she were making a deposit of

\$98.725 and receiving a loan of \$98.64, making a loss of

$$98.64 - 98.725 = -0.085 \text{ per \$100 dollars} \Rightarrow \text{Loss of \$595000}$$

since the sum involved is \$7 million.

Solution to Exercise 3.5

(a) The trader will buy (sell) the LIBOR-based FRA, and sell (buy) TIBOR-based FRA. This way the market risk inherent in the LIBOR positions will be eliminated to a large degree. However, TIBOR and LIBOR fixings occur at different times, so there still some risk in this position.

(b) Use two cash flow diagrams, one for LIBOR FRA the other for the TIBOR FRA. In one case the trader is paying fixed and receiving floating. The other cash flow diagram will display the reserve situation. In this setting, the two fixed rates are known and their difference will remain fixed. The trader will have exposure to the difference between the floating rates.

(c) If LIBOR panel is made of better-rated banks, then the LIBOR fixings will be lower everything else being the same. This means that the spread between LIBOR and TIBOR will widen. According to this, traders need to *buy* the spread if they decide to take such a speculative position.

Solution to Exercise 3.6

(a) This can be done by taking a cash loan at time t_0, pay the Libor rate L_{t_0}, and buy a FRA strip made of two sequential FRA contracts – a (3×6) FRA and a (6×9) FRA. The cash flow diagrams are left as an exercise.

(b) Let *N* be the sum to be borrowed. To find the fixed borrowing cost, simply add the costs incurred by:

- The (3 × 6) FRA, since 3.4 > 3.2, so the floating rate is higher.
- The (6 × 9) FRA, since 3.7 > 3.2, so the floating rate is higher.
- The cost from the three month fixed rate loan.

Solution to Exercise 3.7

To rank the instruments we need to recall the conventions from Chapter 3. We review Section 3.5 from Chapter 3, and Table 3-1 in particular.

(a) According to the formula given there, we first calculate present day values of these instruments.

- **30-day US T-bill: Day count convention:** ACT/360. Yield is quoted at discount rate, so we have

$$B(t,T) = 100 - R^T \left(\frac{T-t}{360}\right) 100 = 100 - 5.5\left(\frac{30}{365}\right) = 99.54167$$

• **30-day UK T-bill:** Day count convention: ACT/365. Yield is quoted at discount rate, so we have

$$B(t,T) = 100 - R^T \left(\frac{T-t}{365}\right) 100 = 100 - 5.4\left(\frac{30}{365}\right) = 99.5561$$

• **30-day ECP:** Day count convention: ACT/360. Yield is quoted at the money market yield, so we have

$$B(t,T) = \frac{100}{\left(1 + R^T \left(\frac{T-t}{360}\right)\right)} = \frac{100}{\left(1 + 0.052\left(\frac{T-t}{360}\right)\right)} = 99.56854$$

• **30-day interbank deposit USD:** Day count convention: ACT/360. Yield is quoted at the money market yield, so we have

$$B(t,T) = \frac{100}{\left(1 + R^T \left(\frac{T-t}{360}\right)\right)} = \frac{100}{\left(1 + 0.055\left(\frac{T-t}{360}\right)\right)} = 99.54376$$

• **30-day US CP:** Day count convention: ACT/360. Yield is quoted at the discount rate, so we have

$$B(t,T) = 100 - R^T \left(\frac{T-t}{360}\right) 100 = 100 - 5.6\left(\frac{30}{360}\right) = 99.53973$$

Yields on these instruments = *100−B(t, T),* so to arrange these instruments in increasing order of their yields, we simply arrange them in decreasing order of their present day values.

(b) Since we are dealing with an ECP (Euro), the day count convention used is ACT/360. So there are 62 days till maturity. Also, we have to use the money market yield rate to compute the present day value. (We have again used conventions from Chapter 3, Table 3-1).

$$B(t,T) = 100 - R^T\left(\frac{T-t}{360}\right)100 = 100 - 3.2\left(\frac{30}{360}\right) = 99.45644$$

is the present day of a bond that would yield 100 USD. So, we have to make a payment of 99.45644× 10, 000, 000/100= 9945644 US Dollars for this ECP.

Solution to Exercise 3.8

a) If the settlement occurs on the date of loan initiation

$$\frac{(L_{t_3} - F_{t_0})\delta N}{1 + L_{t_3}\delta} = \frac{(0.0402 - 0.0438) * 1/4 * 300{,}000}{1 + 0.0402 * 1/4} = -\$3{,}252.27$$

b) If the settlement occurs on the date of loan repayment

$$(L_{t_3} - F_{t_0})\delta N = (0.0402 - 0.0438) * 1/4 * 300{,}000 = -\$3{,}284.96$$

Solution to Exercise 3.9

a) The 3-month implied forward rate is $F_{t_0} = \frac{100-93.83}{100} = 0.0617$

b) At the end of three month this contract involves a delivery of

$100 \times \left(1 - 0.0617 \times \frac{1}{4}\right) = 98.45$ dollars per contract.

Total repayment amount $= 5\ millions * 98.45 = \$492.25\ millions$

Solution to Exercise 3.10

Using the formula $1 + F(t_0, t_i, t_j) = \frac{B(t_0,t_i)}{B(t_0,t_j)}$ we will calculate the implied forward rate

3 x 6 FRA rate $F(t_0, t_1, t_2) = \frac{B(t_0,t_1)}{B(t_0,t_2)} - 1 = \frac{98.79}{97.21} - 1 = 0.01625$

6 x 9 FRA rate $F(t_0, t_2, t_3) = \frac{B(t_0,t_2)}{B(t_0,t_3)} - 1 = \frac{97.21}{95.84} - 1 = 0.0143$

3 x 9 FRA rate $F(t_0, t_1, t_3) = \frac{B(t_0,t_1)}{B(t_0,t_3)} - 1 = \frac{98.79}{95.84} - 1 = 0.0307$

6 x 12 FRA rate $F(t_0, t_2, t_4) = \frac{B(t_0,t_2)}{B(t_0,t_4)} - 1 = \frac{97.21}{93.21} - 1 = 0.0429$

Solution to Exercise 3.11

We abbreviated the value of the 2-year and the 10-year bond in the barbell portfolio by V_2 and V_{10}. We can write the condition that the barbell portfolio should have the same value as the bullet as

$$V_2 + V_{10} = \$990{,}468.75 \qquad (1)$$

The requirement that the duration of the barbell portfolio an the bullet should be the same can be written as

$$\frac{V_2}{\$990{,}468.75} \times 1.943 + \frac{V_{10}}{\$990{,}468.75} \times 8.701 = 4.748 \qquad (2)$$

Solving equations (1) and (2) leads to the solutions V_2 =579361.6 and V_{10} = 411107.2 or in other words 58.5% of the portfolio is to be invested in the 2 year and 41.5% to be invested in the 10 year bond.

We can also use these weights to calculate the convexity of the barbell portfolio:

$$Convexity\ of\ barbell = 58.5\% \times 0.048 + 41.5\% \times 0.859 = 0.384565 \quad (3)$$

The barbell portfolio has greater convexity than the bullet which is 0.254. The reason for this is that duration increases linearly with maturity while conversity increases with the square root of maturity as shown in Chapter 3. We can deduce from this fact that if a combination of short and long durations (which are related to maturities), equals the duration of the bullet portfolio, that same combination of the two convexities (which can be viewed as maturities squared) must be greater than the convexity of the bullet.

From the point of view of the fund manager in the example, the barbell portfolio is attractive since it has the same duration but a higher convexity. In other words, for the same amount of duration risk, the barbell portfolio has greater convexity, which implies that its value will increase more than the value of the bullet when rates rise or fall.

However, the disadvantage of the barbell portfolio lies in its lower yield which is

$$0.457\% \times 58.5\% + 2.598\% \times 41.5\% = 13.457\% \quad (4)$$

Therefore there is a tradeoff. The barbell portfolio can be expected to perform worse than the bullet portfolio if yields remain unchanged or near current levels. However, if yields were to move significantly, the barbell portfolio can be expected to outperform. Therefore the choice depends on the fund managers view about interest rate volatility. A fund manager with a view that rates will be particularly volatility will prefer the barbell portfolio while a manager with a view that rates will not be particularly volatility will prefer the bullet portfolio.