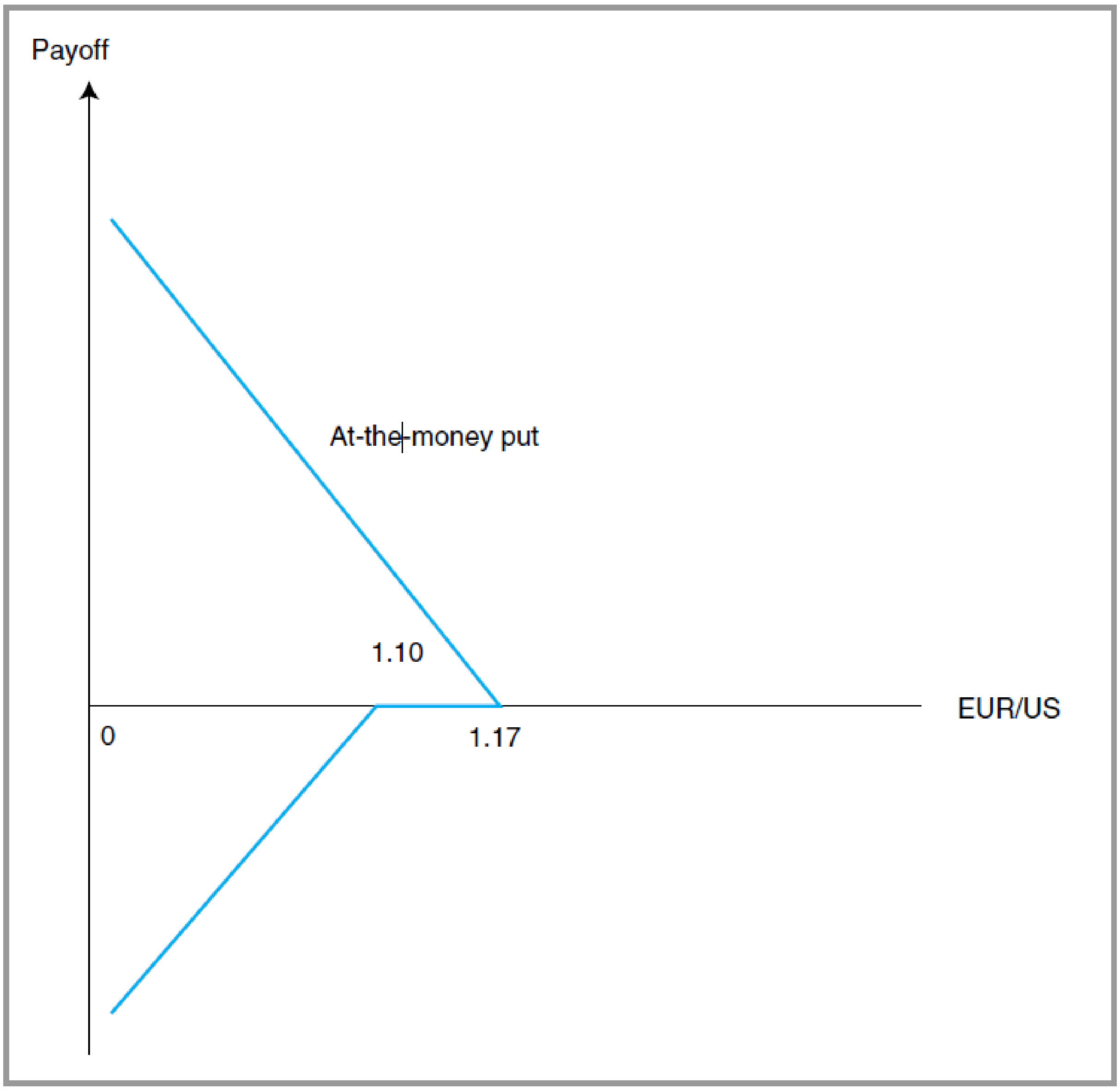
CHAPTER 9

Solutions

Exercise 1

1. The payoff diagrams will look as in the figure below.



2. Gross payoff at expiry will be:

$$P(T) = -\min[(1.23 - S_T), 0] + \min[(1.10 - S_T), 0]$$

where S_T is the EUR/USD exchange rate at expiration.

3. The net payoff will be given by:

$$P(T) - P^{1.23} + P^{1.10}$$

where $P^{1.23}$, $P^{1.10}$ are the premiums of the corresponding options.

4. If there is a volatility smile, then the implied volatility of the out-of-the-money put will be higher than the implied volatility of the ATM Put. The trader selling the out-of-the-money volatility and buying the ATM volatility. Hence if the "smile" flattens, the trade gains.

Exercise 2

(For detailed calculation see also Excel file 'Exercise 9.2 Solution Excel Calculation' on book webpage.)

<u>Calculation</u> (Assuming N = 1)

At time $t_0 = 0$

1. The dealer buys a call with maturity $T = 1\ yrs.$
2. He borrows the amount $C(S_0, t_0)$ amount of money at the risk free rate r for 1 year.
3. Short the Δ_0 amount of shares and deposit the amount $\Delta_0 S_0$ for $\delta = 1/M$ time period.

At time $t_1 = \delta$

1. Change in the delta value is observed i.e. $\Delta_1 - \Delta_0$
   a) If change is positive means that $\Delta_1 - \Delta_0$ no. of more shares needs to be shorted.
   b) If change is negative implies that $\Delta_1 - \Delta_0$ to be bought back from the market.

2. Now add the resultant cash flow $(\Delta_1 - \Delta_0) * S_1$ due of the above portfolio adjustment to the existing cash position.

   Repeat this portfolio adjustment until expiration.

   At the time of expiration T

1. Change the delta value to $\Delta = 1$ if $S_T > K$ or $\Delta = 0$ if $S_T < K$.
2. Add the resultant cash flow $(\Delta_T - \Delta_{T-1}) * S_T$ to the existing cash position.
3. Close the short stock position by buying the stock after the payment of $\Delta_T * S_T$.
4. Close the loan position by the payment of $C(S_0, t_0) * (1 + r)$.
5. Obtain the net position after adding the payoff from the call option $MAX[(S_T - K), 0]$.

Exercise 3

1. Long gamma means, buying related Puts and/or Calls and then delta hedging these positions with the reverse position in the underlying. The hedge ratio will be Delta. This isolates the convexity of option payoffs and benefits from increased volatility. If markets have not priced-in the increase in volatility that may result from (anticipations of) FED announcements, then the trade will benefit. Realized volatility will be higher than the volatility priced in the options. Gamma gains would exceed any interest expense and time decay during the 7 day period.
2. 2. Here we can calculate the gamma of at-the-money options. We can assume interest rate differentials around 3%. We can let the life of the option be 7 days. Such ATM options would have maximum gamma, since the price curve will be very close to the piecewise linear option payoff diagram. This means that the traders are maximizing their exposure to increased volatility trough Gamma.

3. Given the volatility, we can approximately calculate possible gains by letting

$$N\frac{\partial^2 C}{\partial e_t^2}e_t\frac{7}{365}(\sigma_{realized}^2-\sigma^2)$$

where *et* is the expected USD/NZD exchange rate and *N* is the notional amount which is said to be around USD10-20 millions. Calculating the Black-Scholes Gamma and then plugging in the relevant quantities in the above formula will give approximate size of expected gains for various realized volatilities.

Exercise 4

1. Buying sort-dated euro Puts and the implied Gamma means that traders will go long EUR/USD exchange rates. Thus they will buy Euro and sell Dollars.
2. Buying euro puts is a hedge for further drops in euro.
3. Triggering of barrier options may lead to relatively large movements. This may or may not increase the realized volatility. If it does, then buying Gamma will be the natural response.

Exercise 5

1. Two very crude approximations for Delta are,

$[C(S+\Delta S) - C(S)]/\Delta S$

$[-C(S-\Delta S) + C(S)]/\Delta S$

A better approximation is

$[C(S+\Delta S) - C(S-\Delta S)]/2\Delta S$

In fact, applying these to the data shown in the Table we see that only in the last case we obtain an ATM delta of around .5. The two other cases give very different ATM Deltas.

2. ATM delta is around .5 as the third method illustrates. We can similarly calculate the Delta for spot equal to 25. However, we cannot use the third method when S = 10, or when S = 30. For these the first and the second formula need to be used.

   Once these deltas are calculated, then we can calculate daily gains/losses as:

   $-r1.3\frac{1}{365}+\frac{1}{2}[Delta_t - Delta_{t-1}][S_t - S_{t-1}] + Time - decay$

   3. In this case the volatility is much higher and the Gamma gains will be higher as well.

Exercise 6

1. Volga is a Greek relevant for Vega hedging. It is the second derivative of the option price relative to the volatility parameter,

$$Volga = \frac{\partial^2 C}{\partial \sigma^2}$$

2. Vanna represent the derivative of the Vega with respect to the spot price,

$$Vanna = \frac{\partial^2 C}{\partial \sigma \partial S_t}$$

3. These Greeks can be considered as changes in Vega when volatility and the underlying spot price change. Hence they will be relevant for hedging and measuring Vega exposures.

4.

Exercise 7.

(a) The futures strategy is to buy CAD and sell CHF. Geopolitical uncertainty is likely to be bullish for energy producing countries such as Canada and their currencies. Canada is a major trading partner of the US and would therefore benefit from increases in US growth. The bed is therefore for the CHF to weaken against CAD. Due to the peg of the CHF against EUR it is likely that appreciation of the CHF against the USD and CAD is also likely to be capped.

(b) An alternative strategy to benefit from the peg of the CHF against EUR would be to buy call options on CAD/CHF to benefit from a weakening of the CHF. Writing put options on the CAD/CHF is another possible option strategy which would be based on the notion that an appreciation of CHF against CAD is unlikely and therefore the writer of the put option could pocket the premium.

Exercise 8.

(For detailed calculation see also Matlab file 'Exercise 9.8 Solution Matlab Calculation' on book webpage.)

Calculation

The BSM formula for European vanilla call and put option is given as:

$$C(t) = S_t * N(c_1) - Ke^{-r(T-t)} * N(c_2)$$

$$P(t) = Ke^{-r(T-t)} * N(-c_2) - S_t * N(-c_1)$$

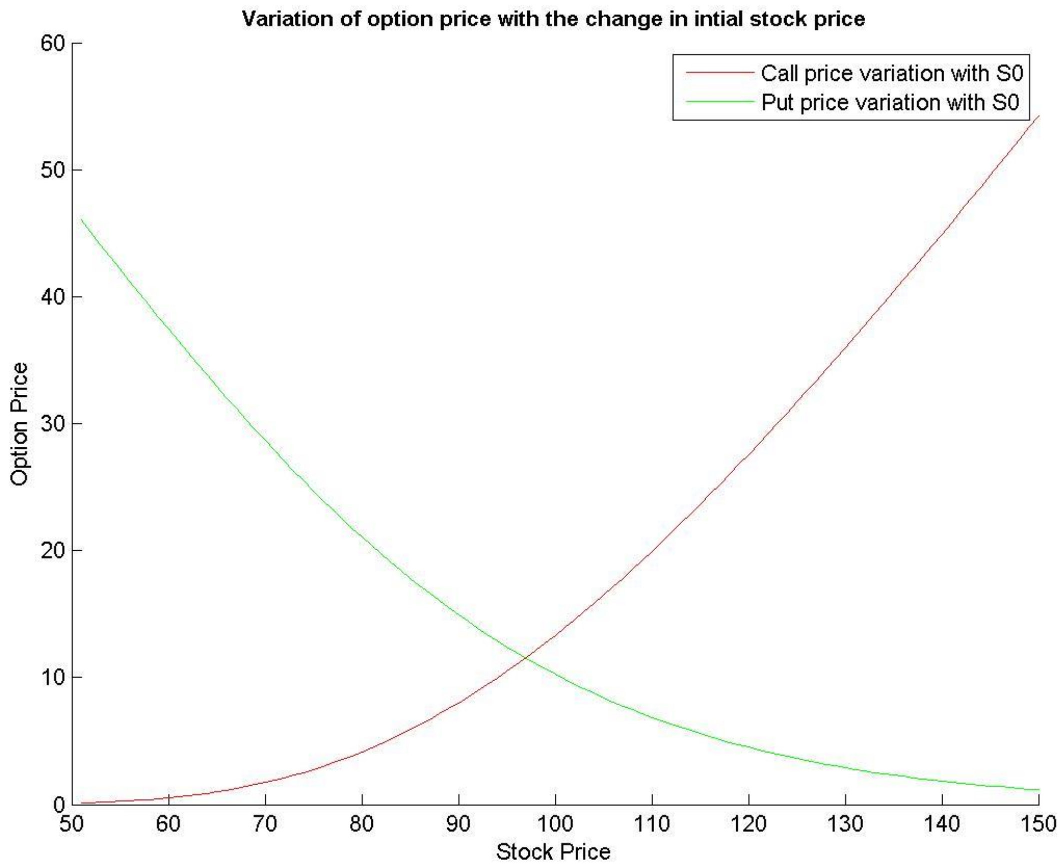
with

$$c_{1,2} = \frac{\log\frac{S_t}{K} + \left(r \ \pm \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$

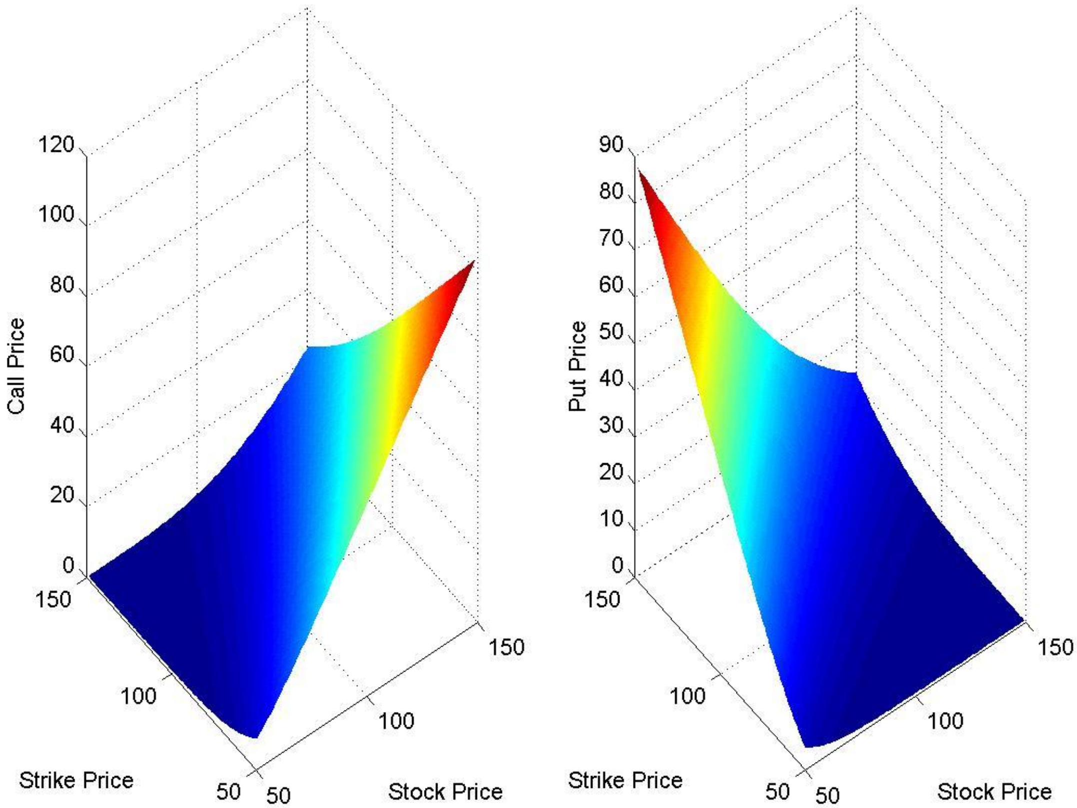
Result

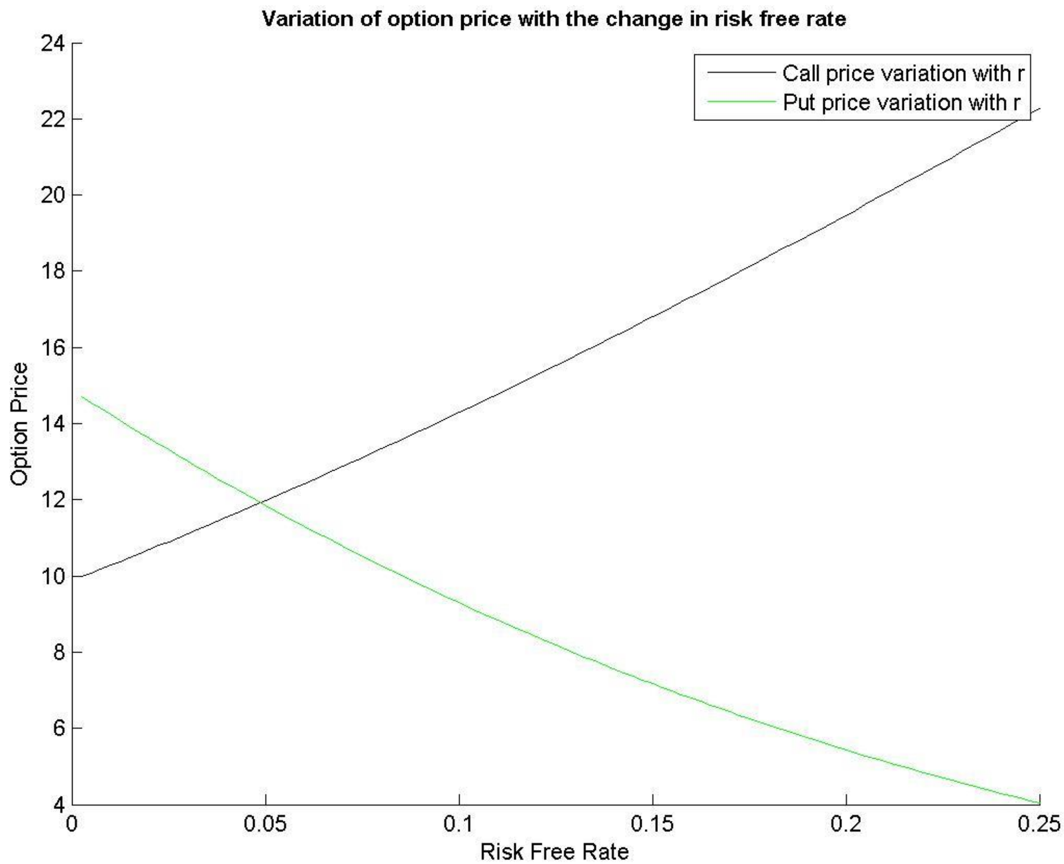
The price of European plain vanilla call option = 13.34

The price of European plain vanilla put option = 10.67

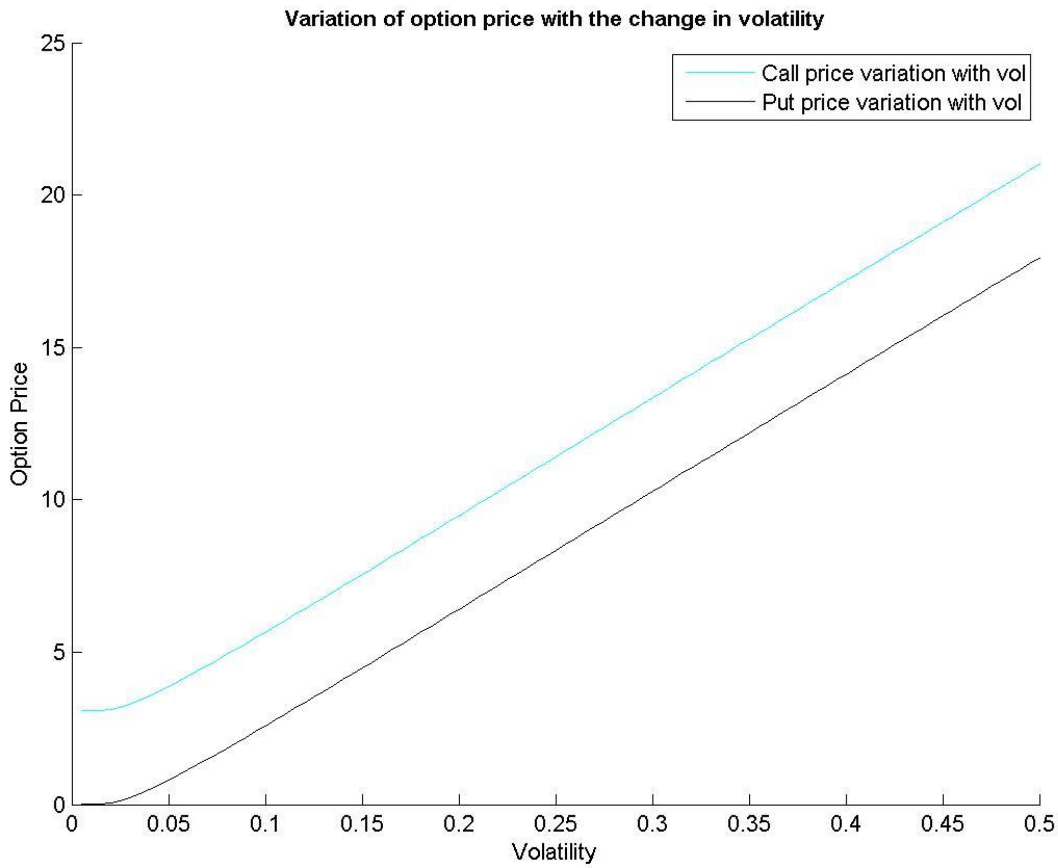

Variation of option price with the change in intial stock price
Call price variation with S0
Put price variation with S0
Option Price
Stock Price
0
10
20
30
40
50
60
50
60
70
80
90
100
110
120
130
140
150

Variation of option price with stock price & strike price

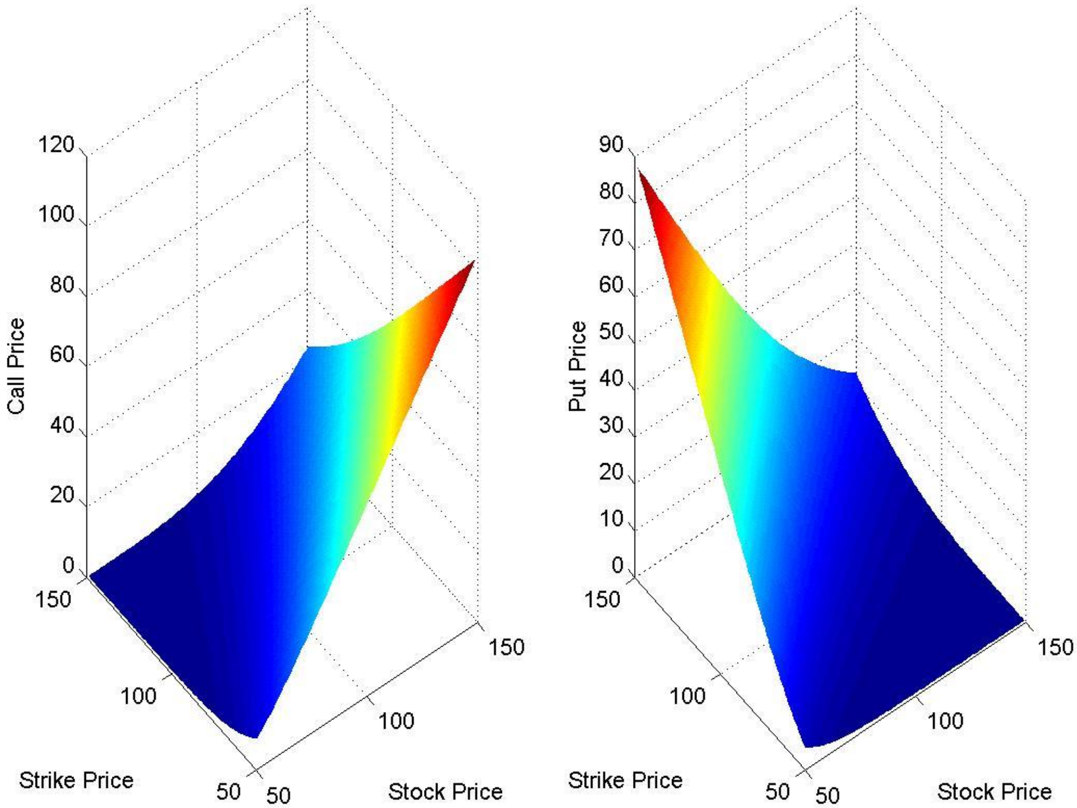




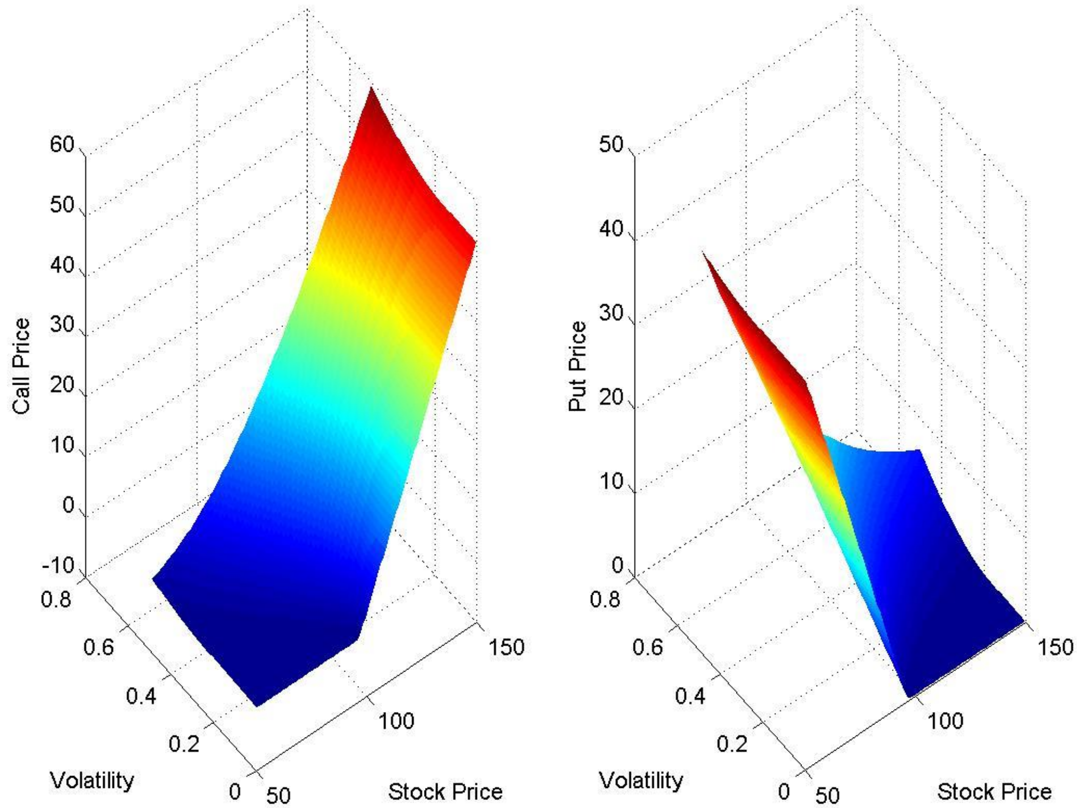
Variation of option price with the change in risk free rate
Call price variation with r
Put price variation with r
Option Price
Risk Free Rate
24
22
20
18
16
14
12
10
8
6
4
0
0.05
0.1
0.15
0.2
0.25

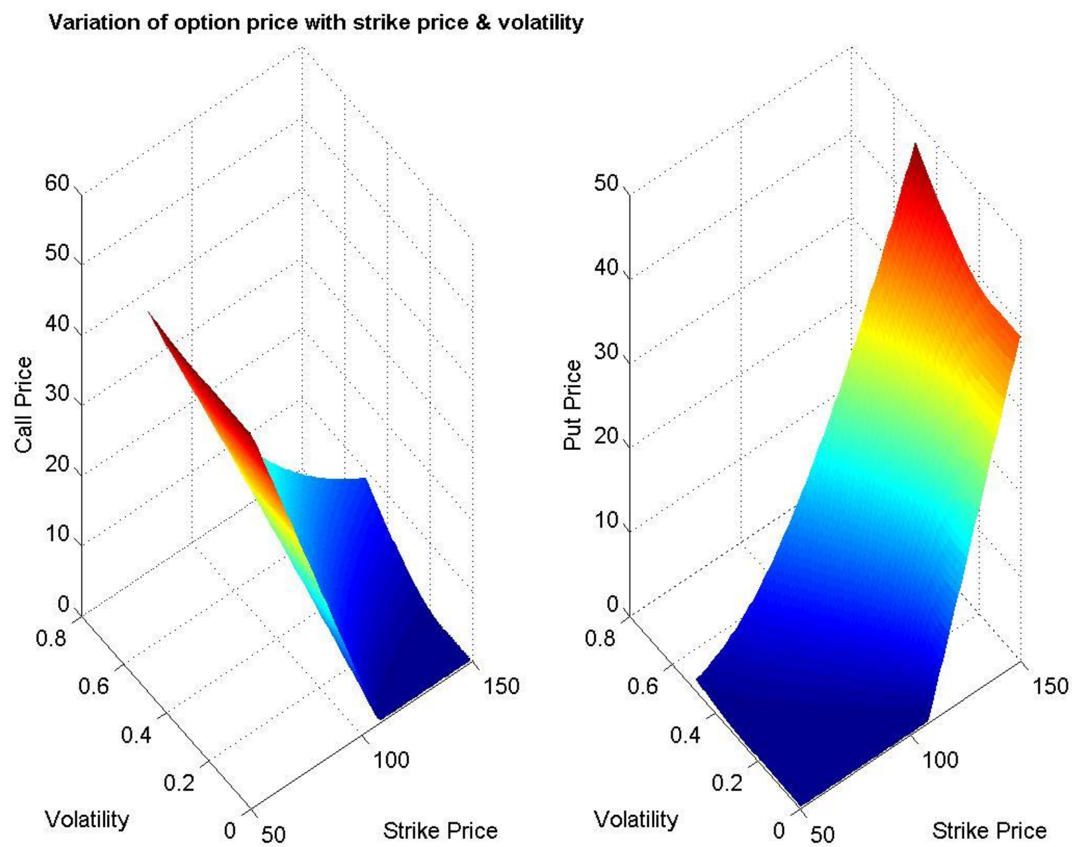

Variation of option price with the change in volatility
Call price variation with vol
Put price variation with vol
Option Price
Volatility
0
5
10
15
20
25
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5

Variation of option price with stock price & strike price



Variation of option price with stock price & volatility





Exercise 9.

(For detailed calculation see also Matlab file 'Exercise 9.9 Solution Matlab Calculation' on book webpage.)

Calculation

The BSM formula for Chooser option is given as:

$$C^h(t) = \left[S_t * \left(N(c_1) - N(d_1)\right)\right] + Ke^{-r(T-t)} * (N(-d_2) - N(c_2))$$

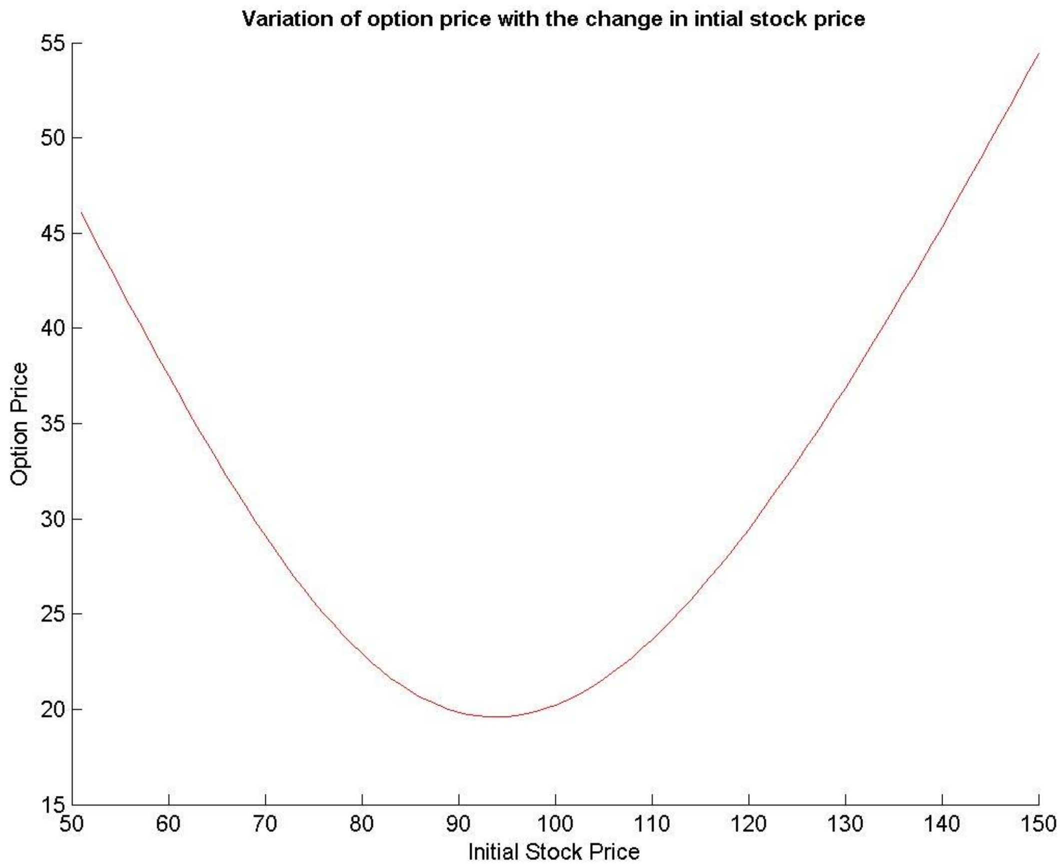
with

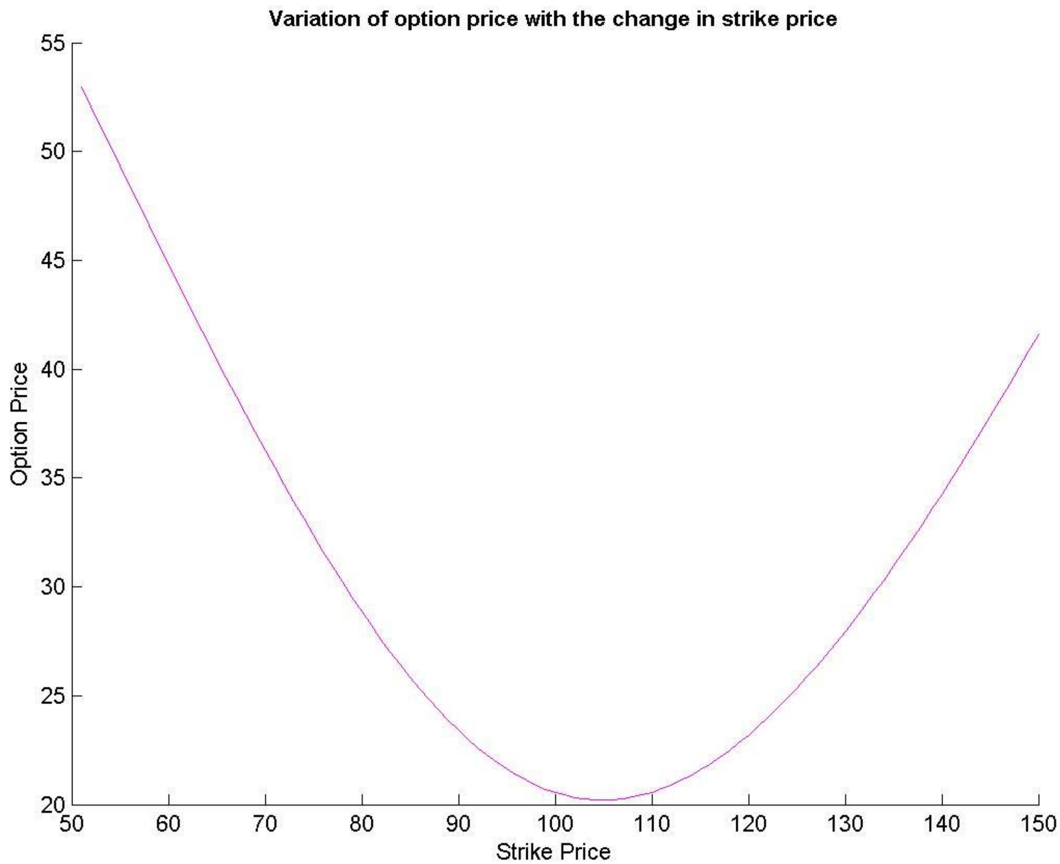
$$c_{1,2} = \frac{\log\frac{S_t}{K} + \left(r \ \pm \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$

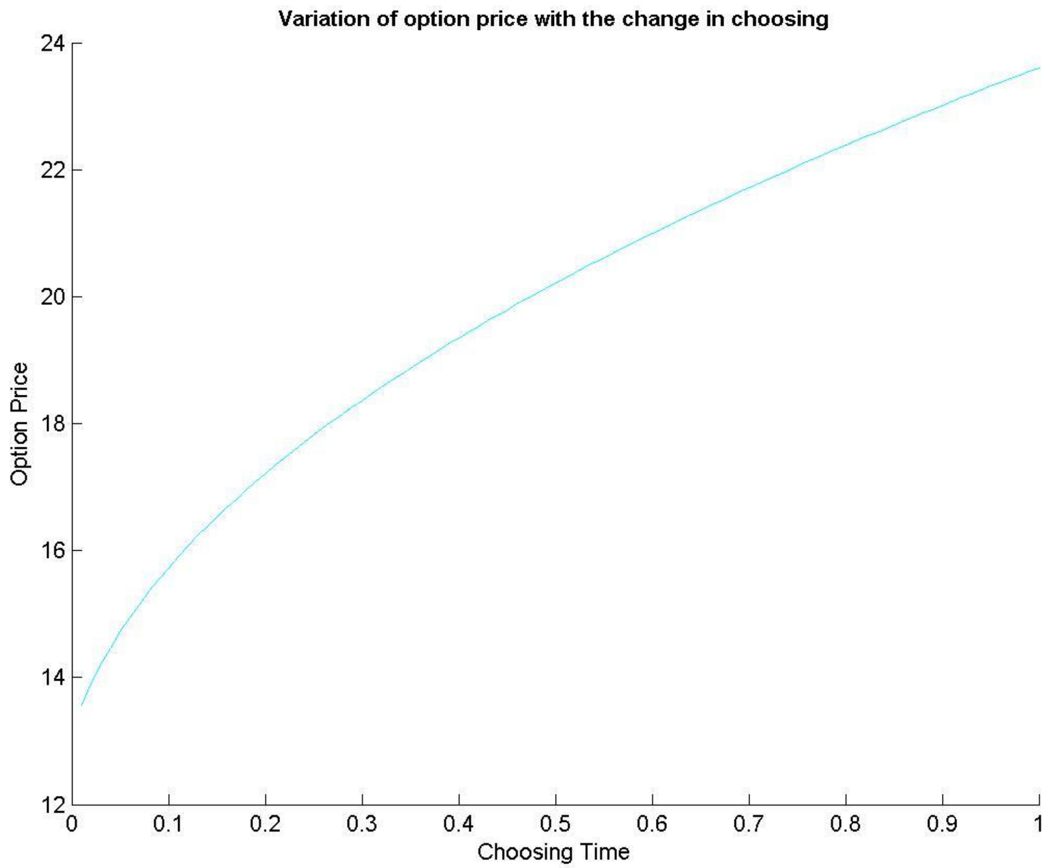
$$d_{1,2} = \frac{\log \frac{S_t}{K} + \left(r(T-t) \pm \frac{1}{2}\sigma^2 \right)(T_0 - t)}{\sigma\sqrt{T_0 - t}}$$

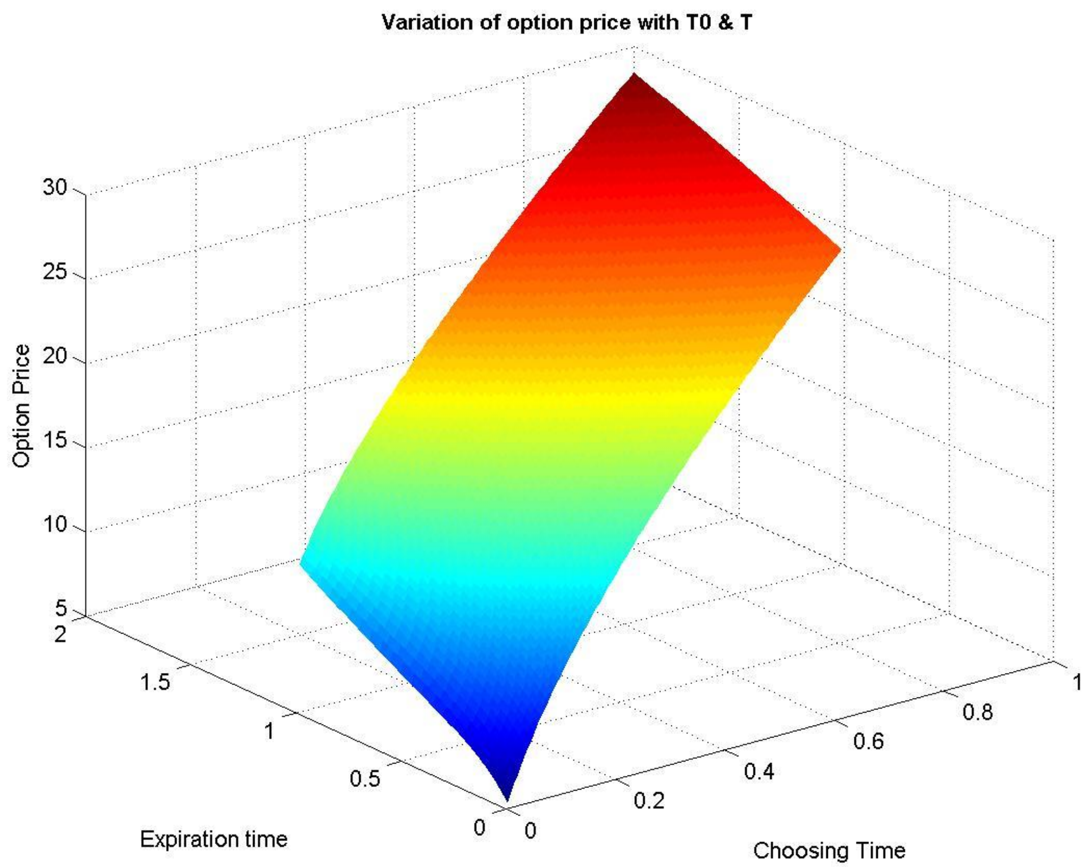
Result

The price of chooser option for the given data is **20.21**


Variation of option price with the change in intial stock price
Option Price
Initial Stock Price
15
20
25
30
35
40
45
50
55
50
60
70
80
90
100
110
120
130
140
150


Variation of option price with the change in strike price
Option Price
Strike Price
55
50
45
40
35
30
25
20
50
60
70
80
90
100
110
120
130
140
150


Variation of option price with the change in choosing
Option Price
Choosing Time
24
22
20
18
16
14
12
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



Exercise 10.

(For detailed calculation see also Matlab file ‘Exercise 9.10 Solution Matlab Calculation’ on book webpage.)

The BSM formula for Barrier down-and-out call option is given as for $H \leq S$:

$$C^b(t) = C(t) - J(t)$$

where $C(t)$ is the price of European vanilla call option and for $J(t)$,

$$J(t) = S_t \left(\frac{H}{S_t}\right)^{\frac{2\left(r-\frac{1}{2}\sigma^2\right)}{\sigma^2}+2} N(c_1) - Ke^{-r(t-T)} \left(\frac{H}{S_t}\right)^{\frac{2\left(r-\frac{1}{2}\sigma^2\right)}{\sigma^2}} N(c_2)$$

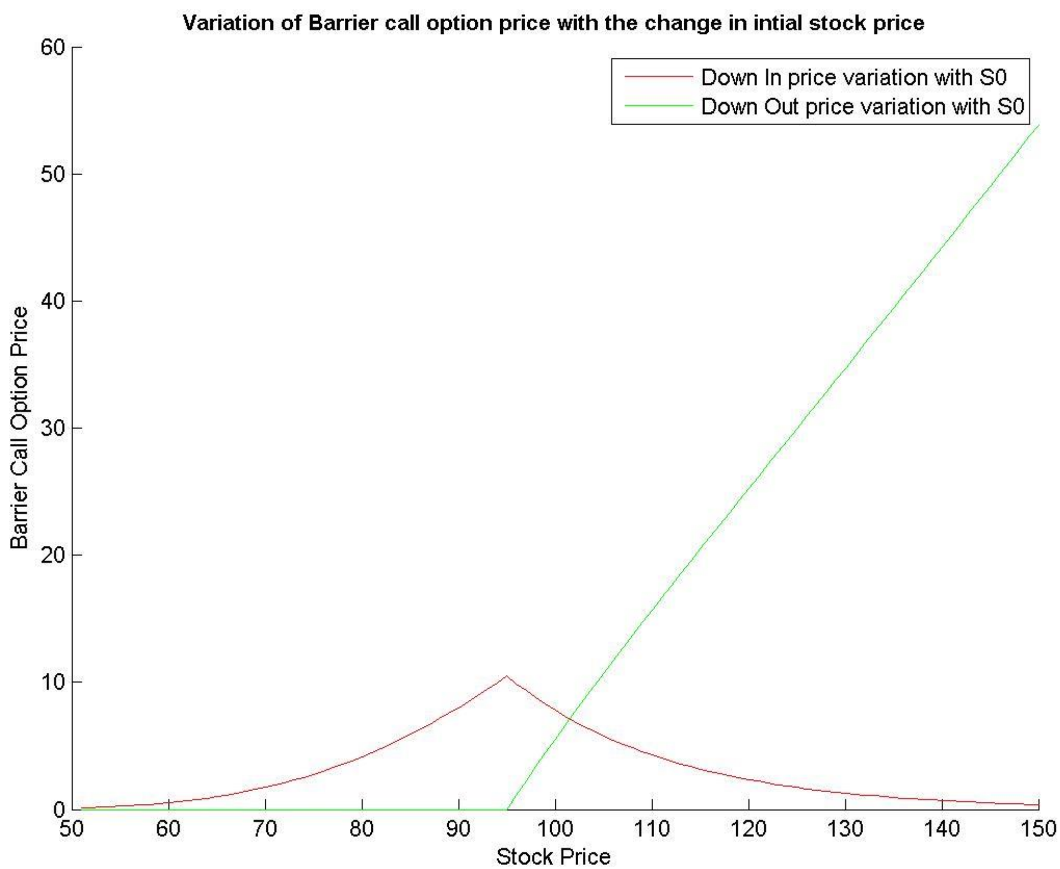
$$c_{1,2} = \frac{\log\frac{H^2}{S_t K} + \left(r \pm \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$

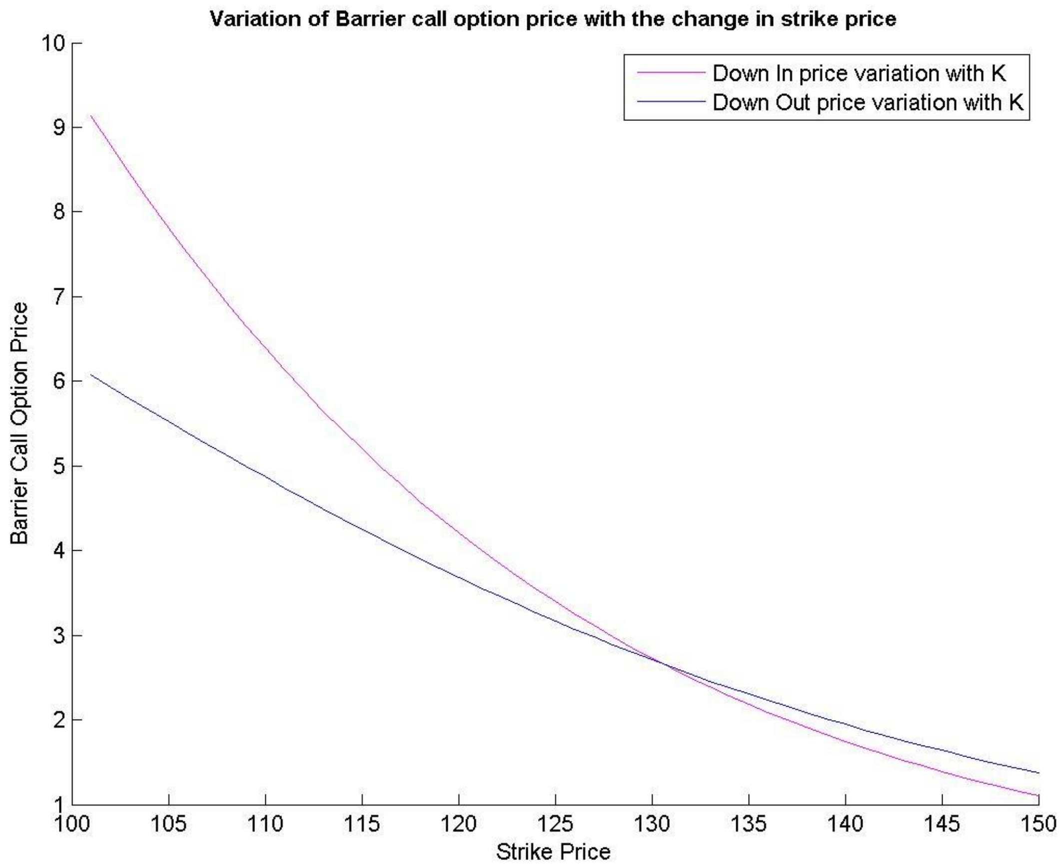
For $H \leq S \leq K$ the price of Barrier down-and-out option is given by $J(t)$

Result

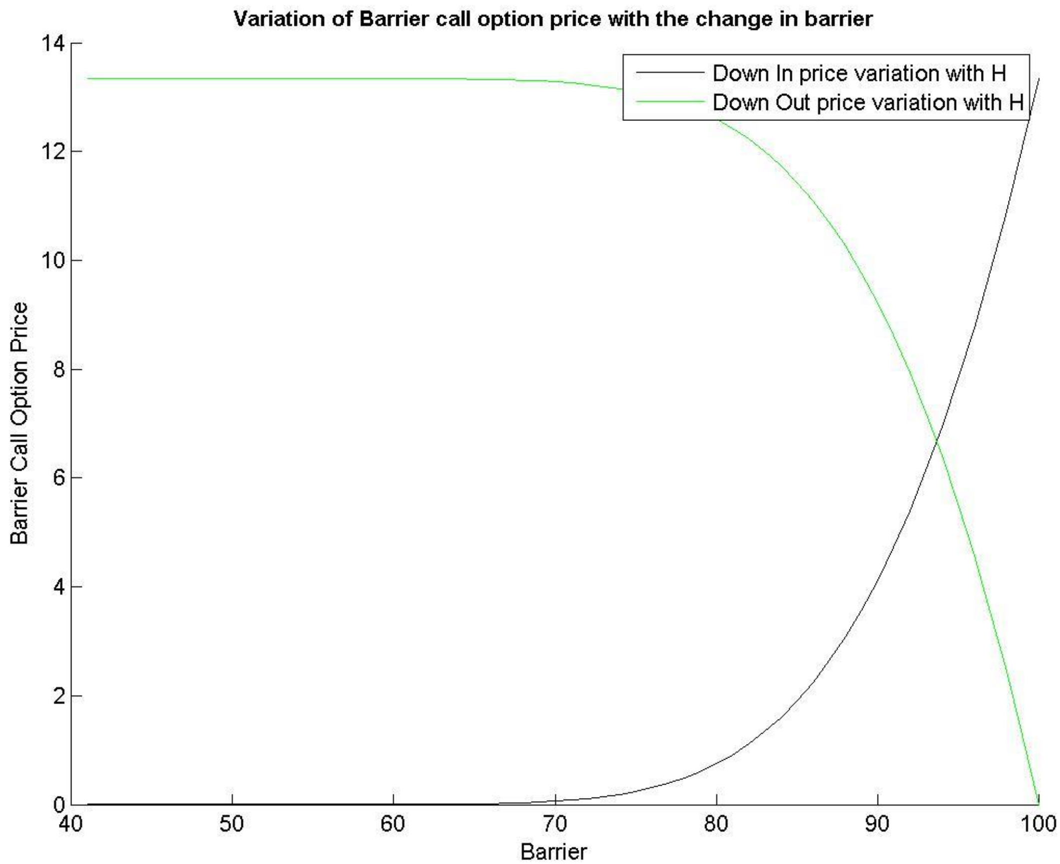
The price of Barrier down-and-out call option price = 5.5255

The price of Barrier down-and-in call option price = 7.8143





Variation of Barrier call option price with the change in strike price
Down In price variation with K
Down Out price variation with K
Barrier Call Option Price
Strike Price
10
9
8
7
6
5
4
3
2
1
100
105
110
115
120
125
130
135
140
145
150



Exercise 11.

(For detailed calculation see also Matlab file 'Exercise 9.11 Solution Matlab Calculation' on book webpage.)

<u>Calculation</u> M = 6

At time $t_0 = 0$

4. The dealer buys a call with maturity $T = 1\ yrs.$
5. He borrows the amount $C(S_0, t_0)$ amount of money at the risk free rate r for 1 year.
6. Short the Δ_0 amount of shares and deposit the amount $\Delta_0 S_0$ for $\delta = 1/M$ time period.

At time $t_1 = \delta$

4. Change in the delta value is observed i.e. $\Delta_1 - \Delta_0$
    c) If change is positive means that $\Delta_1 - \Delta_0$ no. of more shares needs to be shorted.
    d) If change is negative implies that $\Delta_1 - \Delta_0$ to be bought back from the market.
5. Now add the resultant cash flow $(\Delta_1 - \Delta_0) * S_1$ due of the above portfolio adjustment to the existing cash position.

Repeat this portfolio adjustment until expiration.

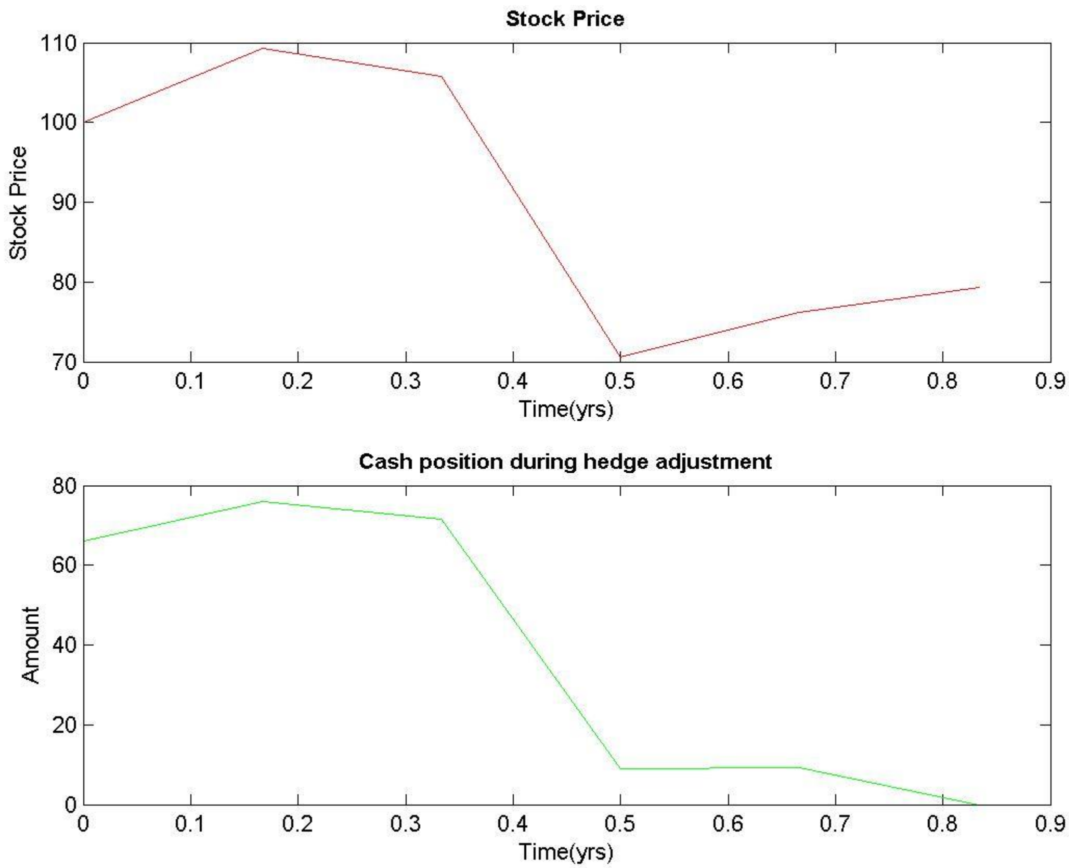
At the time of expiration T

6. Change the delta value to $\Delta = 1$ if $S_T > K$ or $\Delta = 0$ if $S_T < K$.
7. Add the resultant cash flow $(\Delta_T - \Delta_{T-1}) * S_T$ to the existing cash position.
8. Close the short stock position by buying the stock after the payment of $\Delta_T * S_T$.
9. Close the loan position by the payment of $C(S_0, t_0) * (1 + r)$.
10. Obtain the net position after adding the payoff from the call option $MAX[(S_T - K), 0]$.

To obtain the performance measure repeat the above calculation say 1000 time and observe the standard deviation of the net cash position value.

Carry out the above calculation for M = 12, 50, 100 & 300.

For delta hedge frequency = 6


Stock Price
Stock Price
110
100
90
80
70
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Time(yrs)
Cash position during hedge adjustment
Amount
80
60
40
20
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Time(yrs)