Diagnostic Ultrasound Imaging: Inside Out by T. L. Szabo
Prerequisites: Read Chapter 2 and Appendix A
List of Problems

1. Derive and graph the minus i Fourier transforms of the following:
   (a) $\sin(\pi bt)$ (b) $\sin\left[2\pi b\left(t-1/(4b)\right)\right]$ and (c) $\sin\left[2\pi b\left(t-d\right)\right]$

Assume you know that $\Im_{-i}\left[\exp\left(i2\pi at\right)\right]=\delta(f-a)$.

2. Find $\Im_{-i}\left[\exp(-at^2)\cos(2\pi f_c t)\right]$
3. (a) If the running mean is $rm(t)=1/a\prod\left(t/a\right)*g(t)=\frac{1}{a}\int_{t-a/2}^{t+a/2}g(t)dt$, find its minus –i Fourier transform.
   (b) Find the transform of $g(t)=\exp(-a|t|)$ Note this is an even function , you can use limits 0 to ∞
   (c) What is RM(f) for this function?
4. Find $\Im_{-i}\left[g(t)\right]$, where $g(t)=\delta(t-1)+\delta(t-1/2)+\delta(t+1/2)+\delta(t+1)$ in terms of trigonometric functions. Plot G(f).
5. Given the triangle function defined as $\begin{aligned}\Lambda\left(\text{t/L}\right) &= 0 & |\text{t}|>\text{L}\\ &=1-|\text{t}|/\text{L} & |\text{t}|<\text{L}\end{aligned}$, use a MATLAB program to find and calculate the minus i Fourier transform of $g(t)=\Lambda(t/L)\cos\left(2\pi f_c t\right)$ and its numerical DFT equivalent . Compare and plot the numerical transform to that obtained by a direct continuous (exact) transform. Explain how you selected the sampling interval Δt and number of points N. Is there a way to determine Δt directly from the time waveform in this case? Use units of f_c =10 MHz and L=1.5 microseconds.
6. Use G(f) from problem 5 as a filter function block. Determine the output of the filter as R(f)=G(f)V(f) for the following two inputs of the form $v(t)=\exp(-at^2)\cos(2\pi f_c t)$: (a) a=5.0, f_c =10 MHz (b) a=50.0, f_c =10 MHz . Find the corresponding r(t) for these inputs using an numerical DFT method equivalent to an inverse minus i Fourier transform. Plot all results. Hint: don't use an fftshift command before doing an inverse fft.