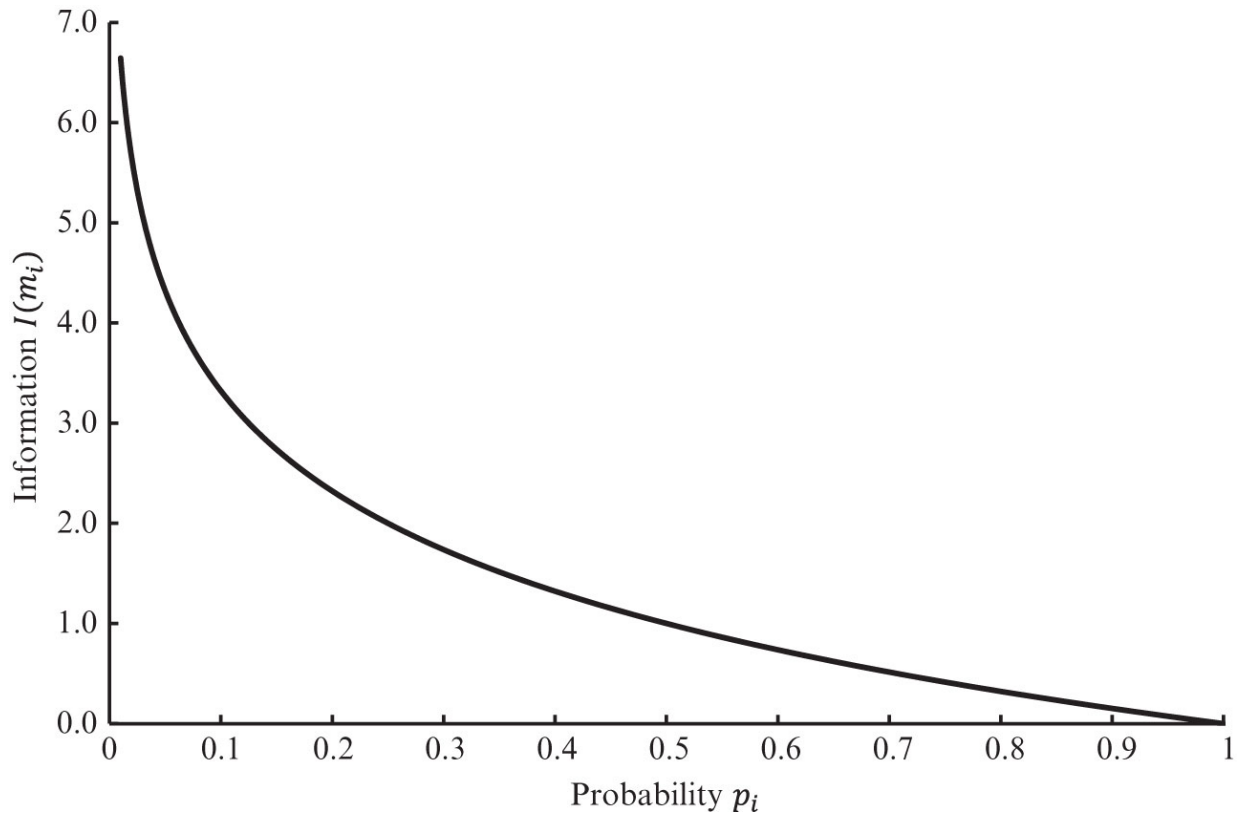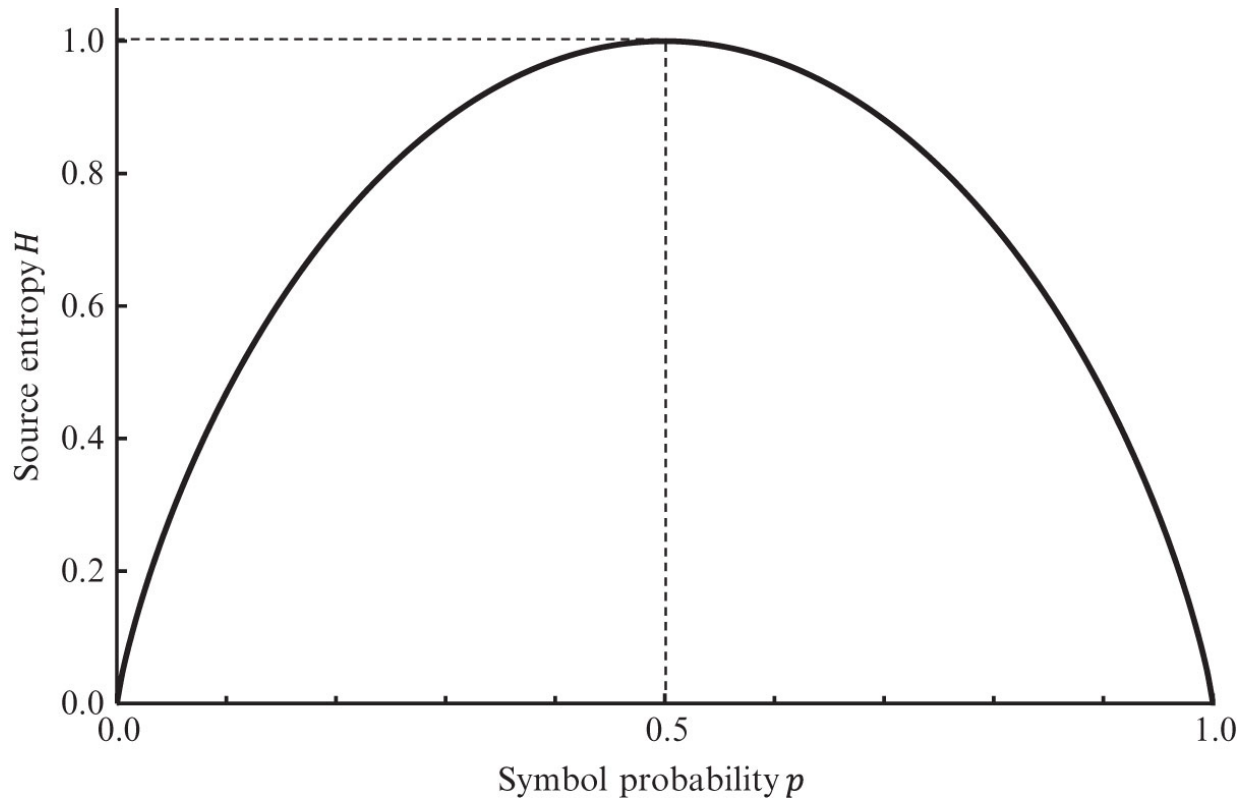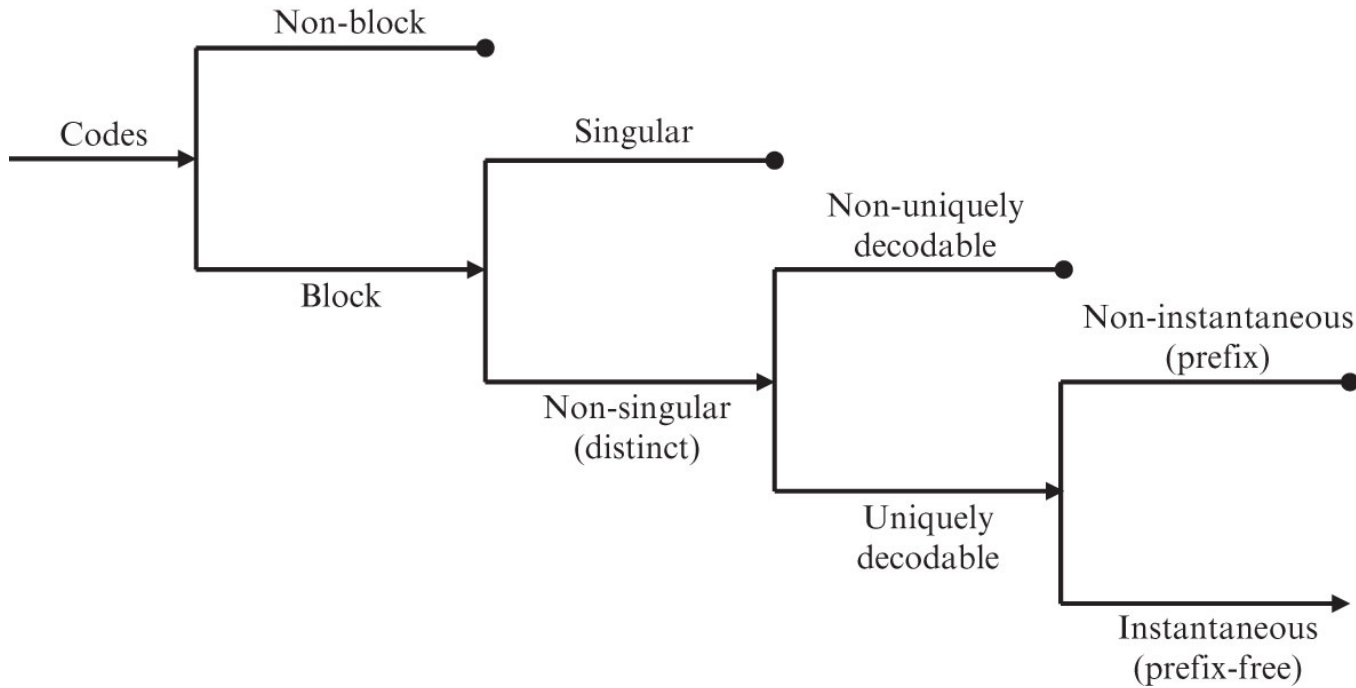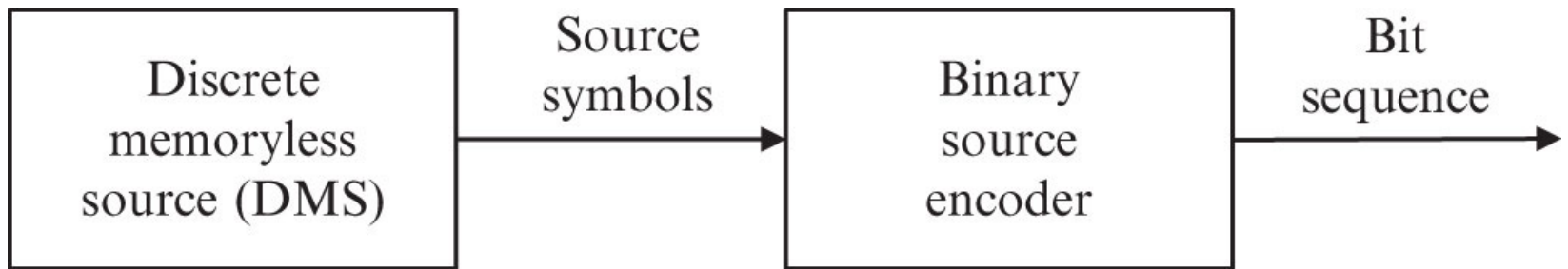# Chapter 09

Information Theory

Figure 9.1 Information content of an event in terms of the probability of the event.

2

Figure 9.2 Entropy of a binary memoryless source in terms of symbol probability.
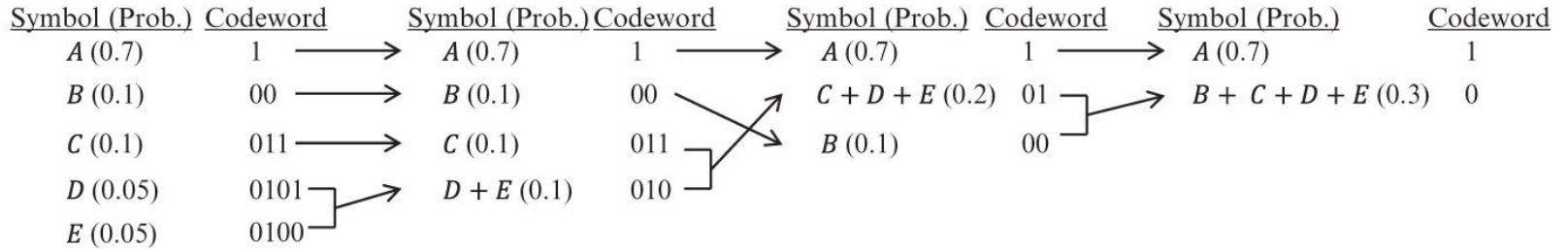
Figure 9.3 Classification of codes.

Figure 9.4 Source encoding.

Symbol	Probability	Codeword
x_1	0.2	000
x_2	0.2	001
x_3	0.2	010
x_4	0.2	011
x_5	0.2	100

(a)

Symbol	Probability	Codeword
$x_1 x_1$	0.04	00000
$x_1 x_2$	0.04	00001
$x_1 x_3$	0.04	00010
$x_1 x_4$	0.04	00011
$x_1 x_5$	0.04	00100
$x_2 x_1$	0.04	00101
$x_2 x_2$	0.04	00110
$x_2 x_3$	0.04	00111
$x_2 x_4$	0.04	01000
$x_2 x_5$	0.04	01001
$x_3 x_1$	0.04	01010
$x_3 x_2$	0.04	01011
$x_3 x_3$	0.04	01100
$x_3 x_4$	0.04	01101
$x_3 x_5$	0.04	01110
$x_4 x_1$	0.04	01111
$x_4 x_2$	0.04	10000
$x_4 x_3$	0.04	10001
$x_4 x_4$	0.04	10010
$x_4 x_5$	0.04	10011
$x_5 x_1$	0.04	10100
$x_5 x_2$	0.04	10101
$x_5 x_3$	0.04	10110
$x_5 x_4$	0.04	10111
$x_5 x_5$	0.04	11000

(b)

Symbol	Probability	Codeword
$x_1 x_1 x_1$	0.008	0000000
$x_1 x_1 x_2$	0.008	0000001
$x_1 x_1 x_3$	0.008	0000010
$x_1 x_1 x_4$	0.008	0000011
$x_1 x_1 x_5$	0.008	0000100
$x_1 x_2 x_1$	0.008	0000101
$x_1 x_2 x_2$	0.008	0000110
.	.	.
.	.	.
.	.	.
$x_5 x_4 x_4$	0.008	1110110
$x_5 x_4 x_5$	0.008	1110111
$x_5 x_5 x_1$	0.008	1111000
$x_5 x_5 x_2$	0.008	1111001
$x_5 x_5 x_3$	0.008	1111010
$x_5 x_5 x_4$	0.008	1111011
$x_5 x_5 x_5$	0.008	1111100

(c)

Figure 9.5 Extension codes for Example 9.7: (a) n=1 and 5 codewords, (b) n=2 and 25 codewords, and (c) n=3 and 125 codewords.
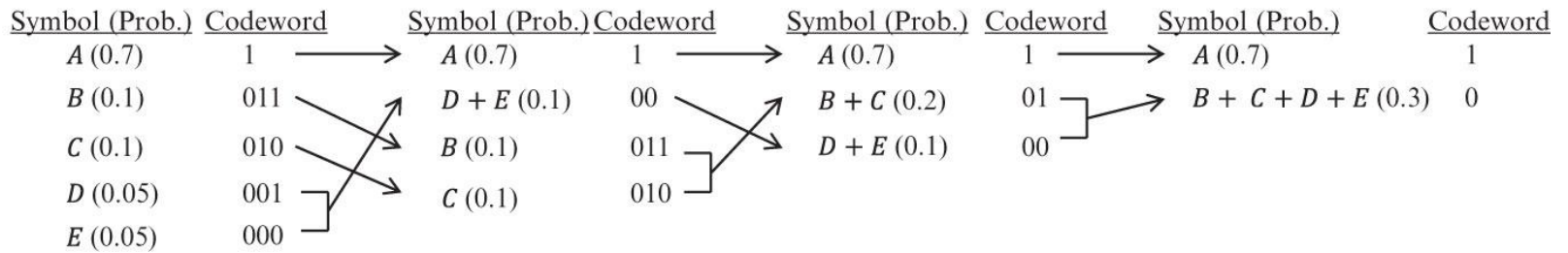
Code_1

Symbol (Prob.)	Codeword		Symbol (Prob.)	Codeword		Symbol (Prob.)	Codeword		Symbol (Prob.)	Codeword
A (0.7)	1	→	A (0.7)	1	→	A (0.7)	1	→	A (0.7)	1
B (0.1)	00	→	B (0.1)	00		$C + D + E$ (0.2)	01	→	$B + C + D + E$ (0.3)	0
C (0.1)	011	→	C (0.1)	011		B (0.1)	00			
D (0.05)	0101	→	$D + E$ (0.1)	010						
E (0.05)	0100									

Code_2

Symbol (Prob.)	Codeword		Symbol (Prob.)	Codeword		Symbol (Prob.)	Codeword		Symbol (Prob.)	Codeword
A (0.7)	1	→	A (0.7)	1	→	A (0.7)	1	→	A (0.7)	1
B (0.1)	011		$D + E$ (0.1)	00		$B + C$ (0.2)	01	→	$B + C + D + E$ (0.3)	0
C (0.1)	010		B (0.1)	011		$D + E$ (0.1)	00			
D (0.05)	001		C (0.1)	010						
E (0.05)	000									

Figure 9.6 Huffman codes for Example 9.8.

Symbol	Probability	Codeword
A	0.9	1
B	0.1	0

(a)

Symbol	Probability	Codeword
AA	0.81	1
AB	0.09	01
BA	0.09	001
BB	0.01	000

(b)

Symbol	Probability	Codeword
AAA	0.729	1
AAB	0.081	001
BAA	0.081	000
ABA	0.081	011
BBA	0.009	01011
BAB	0.009	01010
ABB	0.009	01001
BBB	0.001	01000

(c)

| Symbol	Probability	Codeword
AAAA	0.6561	1
AAAB	0.0729	011
AABA	0.0729	010
ABAA	0.0729	001
BAAA	0.0729	0000
AABB	0.0081	000111
ABAB	0.0081	0001101
BAAB	0.0081	0001100
ABBA	0.0081	0001011
BABA	0.0081	0001010
BBAA	0.0081	0001001
ABBB	0.0009	000100011
BABB	0.0009	000100010
BBAB	0.0009	000100001
BBBA	0.0009	0001000001
BBBB	0.0001	0001000000

(d)

Figure 9.7 Huffman coding using extension codes for Example 9.9.

| Dictionary Locations		Dictionary Contents	Codewords
1	0001	0	0000 0
2	0010	1	0000 1
3	0011	00	0001 0
4	0100	000	0011 0
5	0101	11	0010 1
6	0110	110	0101 0
7	0111	0001	0100 1
8	1000	1100	0110 0
9	1001	01	0001 1
10	1010	10	0010 0
11	1011	010	1001 1
12	1100	101	1010 1
13	1101	010	1001 0
14	1110	0000	0100 0
15	1111	00011	0111 1

Figure 9.8 Lempel-Ziv coding algorithm for Example 9.10.
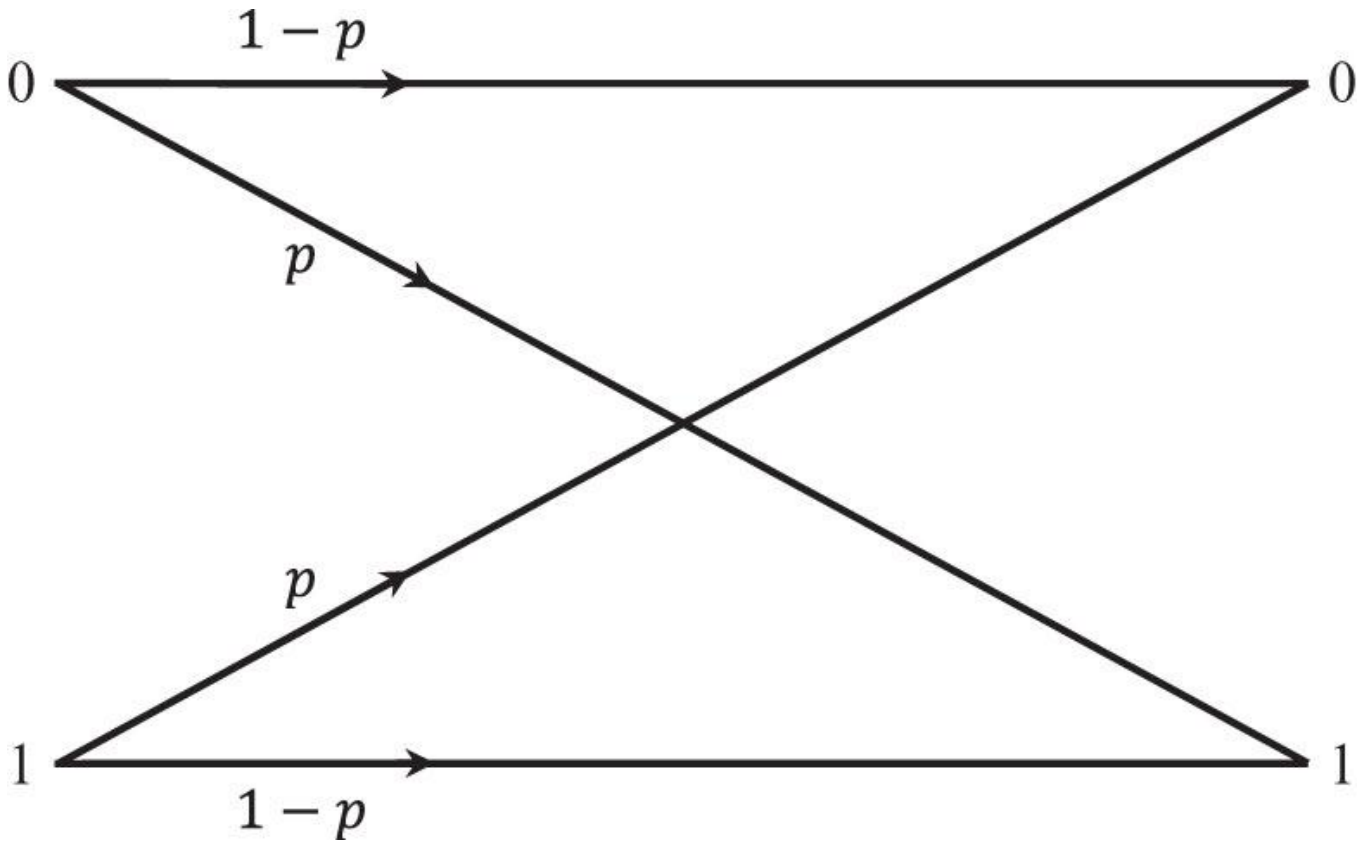
Figure 9.9 Binary symmetric channel (BSC).
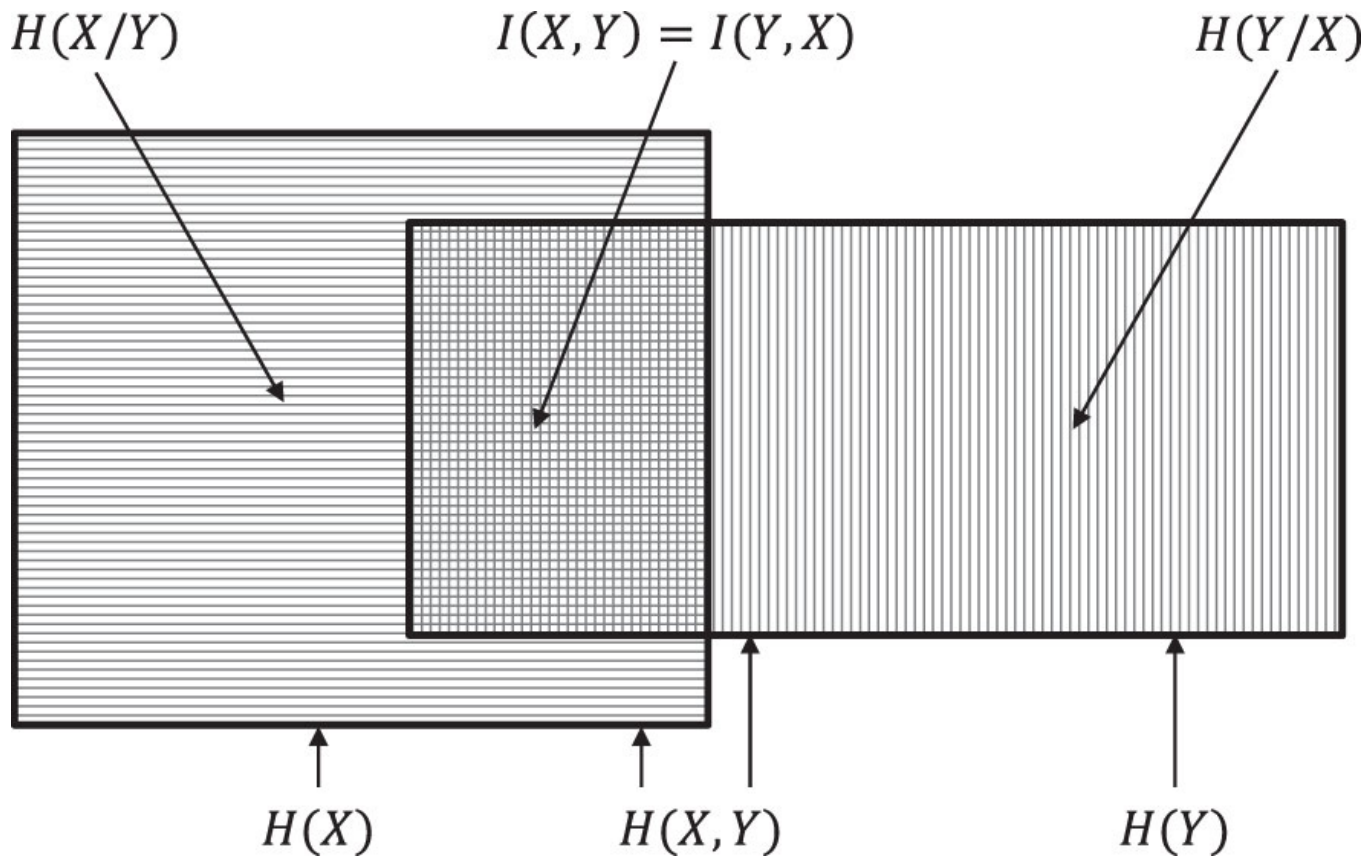
Figure 9.10 Entropy, conditional entropy, and mutual information.
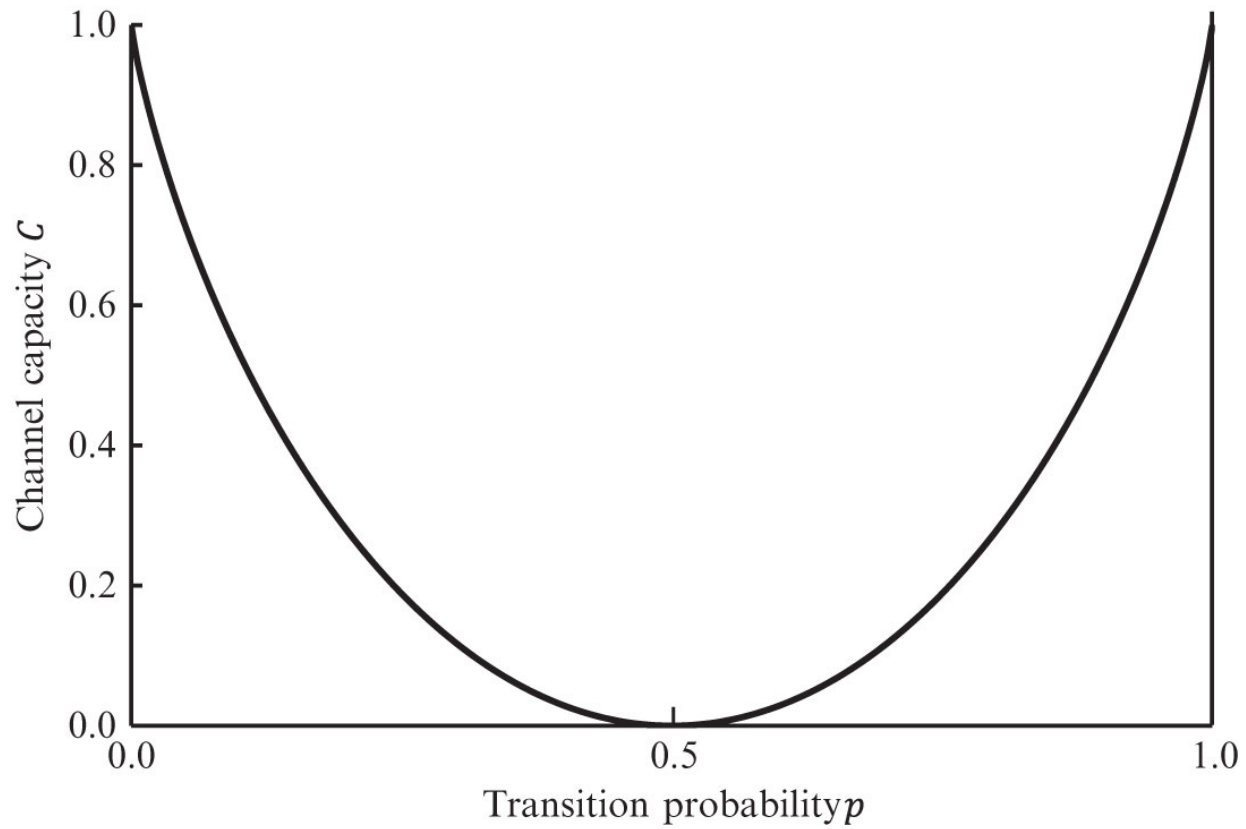
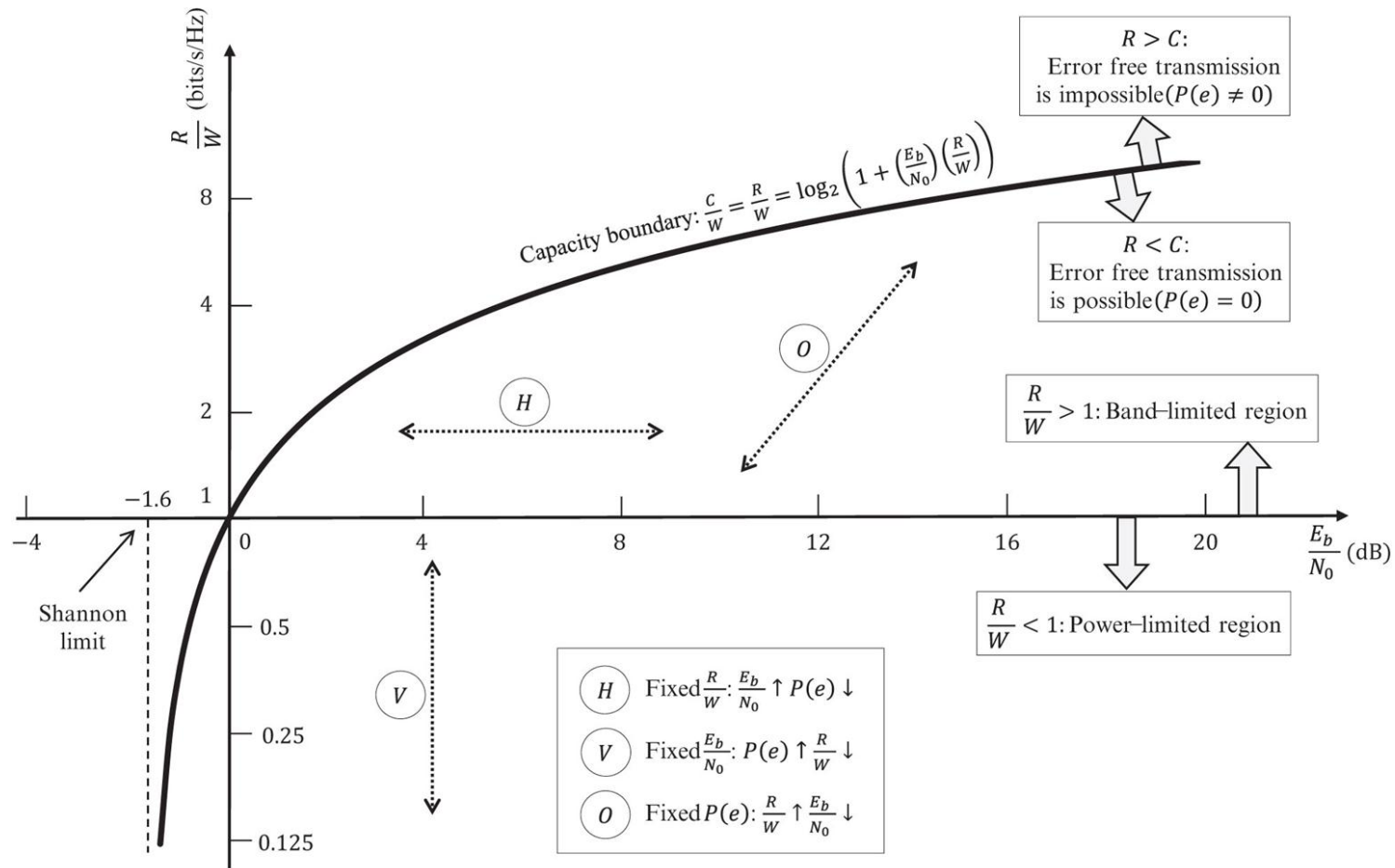**Figure 9.11** Channel capacity of a BSC.

Figure 9.12 Bandwidth efficiency diagram.