**Chapter 14**

**The Second-Best Theory of Taxation with General Production Technologies and Many Consumers**

1. Deadweight loss in one-consumer-equivalent economies with general technology—modeling differences relative to linear technology

a. The possibility of pure (lump-sum) profits or losses from production that have to be accounted for—

{Insert Figures 14.1 and 14.2}

1). The production function relating input X2 to output X1 exhibits decreasing returns to scale

2). In the first figure, at the no-tax equilibrium A, with marginal cost pricing, the firm earns a profit oa, which is received by the consumer—the consumer's budget line is oa

3). In the second figure, a tax on X1 rotates the budget line with slope equal to the slope of line cd

4). Compensating the consumer for the tax leads to a with-tax equilibrium at D on I0.

5). The tax collected at D is cg, the difference between the with-tax and no-tax budget lines projected back to the X-axis.

6). Production at the compensated equilibrium occurs at point E on the production frontier, and generates pure profits of 0e for the consumer

7). The dead-weight loss from the tax is (eg), the difference between the income (in terms of X1) needed to compensate the consumer for the tax (0c), less the two sources of income received by the consumer: the tax revenue cg returned lump-sum and the pure profit with production at E (0e)

8). The economy cannot produce enough X1 to fully compensate the consumer—it would need to receive a gift of DE (= to (eg)) units of X1 from outside the country for the consumer to reach the original no-tax level of utility on indifference curve I0.

b. The first requirement is to develop a valid production relationship, specified in terms of producer prices, that measures the pure economic profits in the economy for any given vector of production prices.

1). Assuming perfectly competitive goods and factor markets, the profit function, , is derived by assuming that a planner maximizes aggregate profits at fixed producer prices subject to the aggregate production-possibilities frontier .

2). The resulting general equilibrium aggregate goods supply and input demand functions  are then substituted back into the profit function .

3). , the supply of (demand for) the kth good (factor), from Shepard’s lemma.

c. The valid general equilibrium expression for deadweight loss under general technology: 

1). The lump-sum income at any vector of consumer prices required to keep the consumer at utility level U less the tax revenue and profit returned lump-sum

d. Market clearance

1). In general, when any tax changes, all prices are affected since goods supply curves (factor demand curves) are upward (downward) sloping

2). Need market clearance equations, along with the pricing identities qi = ti + pi , to solve for the consumer and producer prices given the tax rates



--the market clearance equations solve for p given t

-- qi = ti + pi then solves for the q

3). A difficulty: the consumer's goods demands and factor supplies are compensated demands and supplies in the calculation of loss—market clearance cannot hold for all of them simultaneously since there are not enough resources for that—if all could be satisfied there would be no deadweight loss

4). Therefore, assume markets clear for all goods except good 1, the untaxed numeraire, with q1 = p1 = 1—the choice of which market to select as uncleared is arbitrary

5). The general equilibrium prices used to measure loss at the compensated equilibrium will in general differ from the actual general equilibrium prices with general technology (but not with linear technology)

6). It can be shown that loss can also be expressed as M1, the amount of excess demand at the compensated equilibrium for the untaxed numeraire—the resource transfer needed from elsewhere to fully compensate the consumer for the vector of taxes—Distance ED = eg in the figure above

2. Loss and marginal loss with general technology

a. The full model describing loss

Loss: 

Market clearance: 

Pricing identities: qi = pi + ti i = 2, …, N

q1 = pl = 1, t1 = 0

b. Marginal loss

1). Given , and p1 = 1





2). From market clearance



and



Therefore,



3). With qi = pi + ti, for i = 2, …, N,



--as with linear technology, marginal loss depends upon the pattern of existing taxes and the change in compensated demands (factors supplies) in response to the tax.

-- The major qualitative difference between marginal loss in linear vs. general technologies is that the derivative ∂Xicomp/∂tk depends on both consumption *and* production responses, since ∂qj/∂tk depends upon all the consumption and production elasticities through the (N – 1) market clearance equations

4). In vector notation





where:

(t) = the (N – 1) × 1 column vector 

(Mij) = the (N – 1) × (N – 1) matrix:



 = the (N – 1) × (N – 1) matrix of differentials:



(dt) = the (N – 1) ×1 column vector of differentials:

Mi = the (N – 1) ×1 column vector of compensated demands (factor supplies):



πi = the (N – 1) × 1 column vector of supplies (input demands):



q, p = the (N – 1) × 1 column vectors of prices.

5). Totally differentiate the market clearance equations

Mijdq = Yij(dq – dt)

6). Solving for dq/dt and substituting the notation X for Mi and Y for πi yields:



7). Substituting the consumer price derivatives into the loss equation



--marginal loss depends upon both consumption and production derivatives

8). For illustrative purposes: assume dtk affects only prices qk and pk, and (inappropriately) all cross-price Slutsky substitution effects Mkj = 0.

{Insert Figure 14.5}

9). Under these assumptions



-- The shaded area can be thought of as representing the combined (marginal) decrease in consumer’s and producer’s surplus from consuming and producing good k, where the former is measured with reference to the compensated demand for good k and the latter with reference to the general equilibrium aggregate supply function 

3. Optimal commodity taxation

a. With general technology, the problem is





b. The FOC











c. Without CRS

1).



equivalent to



where ∂Xicomp/∂t refer to the general equilibrium changes in Xicomp in response to the tax, which in turn depend upon the changes in the full set of producer and consumer prices as tk changes

d. With CRS—no pure profits in the economy

1). , with p1 = 1

2). Therefore,

3). Subtract  from the FOC to obtain



4). In matrix notation



where: I is the (N – 1) × (N – 1) identity matrix

 = the (N – 1) × (N – 1) matrix of price derivatives:



5). Since [I + (∂p/∂t)] is nonsingular,



or



same as with linear technology

6). Optimal pattern of commodity taxes depends only on compensated demand (factor supply) derivatives, not the production derivatives

4. Many person economies—fixed producer prices

a. No simple loss measure applied across individuals is equivalent to maximizing social welfare—no obvious pattern exists for compensating each of the individuals to measure each person's loss

1). Need one-consumer-equivalence to make loss minimization correspond to social welfare maximization—

2). Three sufficient conditions for one-consumer-equivalence

-- Lump-sum income is continuously and optimally redistributed in accordance with the interpersonal equity conditions of first-best social welfare maximization. That is, the social marginal utility of income is always equal for all consumers.

-- Consumers have identical and homothetic tastes so that for any given consumer price vector, , and all lump-sum income distributions I1, …, IH, the aggregate Engel’s (income-consumption) curves are straight parallel lines.

--The covariance of person h’s social marginal utility of income and his proportion of aggregate consumption of any one good (Xhk/Xk) is identical for all goods (and factors) k = 1, …, N (Green’s condition).

b. In social welfare maximization, efficiency and equity considerations are commingled

c. To express social welfare in terms of prices, use the indirect utility function

, obtained from by solving for the consumer’s demand (input supply) functions  from utility maximization and substituting them for the arguments of the direct utility function Uh(Xhk )

1). Note: , where ah is the private marginal utility of income for person h (Roy's identity)

d. Social welfare:



e. The government's problem:





and 

f. FOC

, 



g. Let βh = (∂W\*/∂Vh)αh represent the social marginal utility of income for person h, the product of the marginal social welfare weight and the private marginal utility of income,

 k = 2, …, N

h. No simple equal percentage change rule as in one-consumer case—all one can say in general

is



--,the marginal change in social welfare resulting from a change in any given tax rate must be proportional to the change in tax revenues resulting from changing the tax rate

i. To get a percentage change rule, use the Slutsky equation for each person

 k = 1,…,N

1). Plug it into the FOC to obtain





2). Rearranging terms, dividing through by and noting that yields, after some manipulation



3). Feldstein's distributional coefficient for persona h is



--the social marginal utility of person h times the proportion of his/her consumption of good k in the overall consumption of k

4). In terms of Feldstein's distributional coefficient



5). LHS is the percentage change in the aggregate compensated demand for good k (approximately)

6). RHS not independent of k; the percentage changes depend in a complicated manner on Feldstein’s distributional coefficients and the change in tax revenue in response to changes in the pattern of lump-sum incomes

7). Use the Slutsky equations again to solve for percentage changes in actual demands



8). Substituting for the above and rearranging terms:







9). The actual percentage changes in demand (factor supply) resulting from the optimal pattern of commodity taxes should be greater

-- The lower its distributional coefficient λk or the more it is demanded by people with low social marginal utilities of income

-- The more it is demanded by people whose total taxes change least as lump-sum income changes.

-- The more it is demanded by people for whom, other things equal, the product of the fraction of income paid as taxes and the income elasticity of demand for the good is highest.

10). Not clear who the latter two people might be

j. Diamond's covariance interpretation of the optimal tax rule

1). Allow the government to offer a lump-sum head subsidy, I, equal for everyone

2). The government's problem





3). FOC wrt tk are unchanged



4). FOC wrt I



5). Define  as the full social marginal utility of income for person h, consisting of the conventional direct increase in social utility when Ih increases, the βh term, plus the social marginal utility of the increased tax revenues when Ih increases, equal to .

6). Using h, the FOC for tk are



and the FOC for I are

 or 

--λ is the *average* full social marginal utility of income given that the government employs an optimal head subsidy

7). Divide the FOC for tk by λXk = 



8). But, . Hence,  where 

9). Therefore

 (14.54)

-- the aggregate percentage change in the compensated demand (supply) of good (factor) k should be proportional to the covariance between the full marginal social utility of income and the consumption (supply) of good (factor) k.

---the simplest interpretation of the many-person optimal tax rule to date. (Although it requires the simultaneous imposition of a uniform head subsidy/tax.)

k. A two-class tax rule

1). Recall 

2). Multiply each equation by tk and sum over k = 1, …, N to obtain:



3). Because  is negative semidefinite,



4). Therefore,



5). Divide all the people into two classes rich (R) and poor (P)

6). With an optimal head subsidy



7). Therefore



8). But, , or



9). 

or



10). Assuming  > 0 , 

--the optimal pattern of commodity taxes should, in general, collect more taxes from the rich than the poor.

l. U.S Taxes: How far from Optimal

1). Balcer *et al.* applied a many-person, fixed-producer-price model to U.S. data to get a sense of how optimal commodity tax rates would vary with the government’s revenue needs and society’s aversion to inequality

2). Assumptions: Atkinson social welfare function with the aversion to inequality ranging from 0 to 2; Stone-Geary utility functions; ten income classes; fixed labor supply; no saving; government budget equal to 30% of disposable income; nine commodity groups

3). Found that only the tax on gasoline varied much from its optimal value (39.8% versus the optimal value of 8.8%); therefore moving to optimal taxes would have only a small effect on social welfare

4). Welfare cost of moving to a uniform tax system is zero under no aversion to inequality since that is equivalent to a one-consumer-economy and the Stone-Geary utility function satisfies the sufficient condition for uniform taxation

5. Many person economy with general technology

a. The same social welfare function as above



b. Production

1). Cannot use the generalized profit function because social welfare is not measured in terms of lump-sum income.

2). Use the aggregate production frontier F() = 0, as in first-best analysis

3). Replace the quantities with the general equilibrium aggregate market supply (input demand) functions Yi = Yi(), i = 1, …, N

-- the functions that would result from a social planner maximizing aggregate profits at given competitive prices

4). F[()] = 0, the production-price frontier, specifies all relevant production parameters assuming competitive market behavior

c. Market clearance



--solve for the vector of producer prices given a vector of tax rates

d. The pricing identities 

-- solve for the vector of consumer prices

e. The government's problem:



f. Simplify by incorporating market clearance directly into the production frontier and thinking of the government as solving directly for the vector of consumer prices,  rather than the vector of taxes, 



g. Walras' Law in general equilibrium—can assume either

1). All markets clear and all but one agent are on their budget constraints: implies remaining agent is on its budget constraint

or

2). All agents are on their budget constraints and all but one market clears: implies that the remaining market also clears.

h. Assume the first interpretation of Walras' Law and ignore the government's budget constraint

i. The FOC of the government's problem above with respect to qk and an equal head subsidy I





Note: The FOC implicitly assume CRS because the distribution of lump-sum incomes Ih is assumed to be constant

1). From Roy’s Identity on indirect utility functions, the definition of marginal social utility βh, and the assumption of profit maximization with p1 ≡ 1



2). But pi = qi – ti, for i = 1, …, N. Hence,



3). IF consumers are on their budget constraints,



4). Therefore:



--identical to the FOC above with fixed producer prices

5). The head subsidy-- making use of profit maximization, the definition of marginal social utility, and the definitional relationships among prices and taxes, the FOC wrt I is



6). If consumers are on their budget constraints,



7). Hence,



8). But



the full social marginal utility of income.

9). Therefore:



also as with fixed producer prices. assuming CRS

6. The social welfare implications of any given change in taxes

a. To consider the effects on social welfare of any given change in taxes, totally differentiate W wrt prices and income (and using Roy's identity and the social marginal utility of income





b. Totally differentiate the production-price frontier F() = 0, in which the market clearance equations have been used to substitute consumers’ demands and factor supplies for the production aggregates Yi:



1). Assuming perfect competition and p1 = 1



2). But, qi = pi + ti, for i = 1, …, N. Multiplying each price by dXhi, and summing over all goods and people yields:





3). Totally differentiate each consumer’s budget constraint and sum over all consumers to obtain:



4). Therefore,



5). Thus



6). The dXhi in the last term can be eliminated by noting that:



--Totally differentiating:



7). Therefore, 

c. CRS production

1). The first term in dW can be ignored since lump-sum incomes are unchanged

2). Even so, production derivatives matter for tax reforms

3). Totally differentiating the market clearance equations,



yields:



4). Solving for  and expressing the N equations in vector notation yields



where E =  in vector notation

5). Therefore,



where:

β = , an (H × 1) column vector of marginal social utilities of income

--the fundamental equation for evaluating tax changes in a many-consumer economy with CRS general production technology

6). The change in loss in a one-consumer-equivalent economy with general technology is 

7). Comparisons with the many-person rule

--the demand derivatives in the loss expression are the compensated derivatives

--as a rough interpretation, can think of the first term in the many person rule as incorporating the equity considerations and the second term as incorporating the efficiency considerations

--in some applications, the equity and efficiency implications are tightly intertwined, as in the optimal commodity tax rule