Answers, Hints, and Solutions to Selected Exercises for *Introductory Differential Equations*, Fourth Edition, Abell & Braselton

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Exercises 1.1

1. Second order linear ordinary differential equation. The forcing function is $f(x) = x^3$ so the equation is nonhomogeneous.
3. Second order linear partial differential equation.
5. This is a first order ordinary differential equation. It is nonlinear because the deriviatve dy/dx is squared.
7. Second order linear partial differential equation.
9. Nonlinear second order ordinary differential equation.
11. This is a second order partial differential equation. It is nonlinear because of the product, $uu_x = u\frac{\partial u}{\partial x}$, of functions involving the dependent variable, $u = u(x,t)$.
13. This is a first order ordinary differential equation. If we write it as $(2t - y)dt/dy - 1 = 0$, y is independent, $t = t(y)$ is dependent, and the equation is nonlinear. If we write the equation as $dy/dt + y = 2t$, t is independent, $y = y(t)$ is dependent and the equation is linear.
15. This is a first order ordinary differential equation. It is nonlinear in both x

(because of the $2x\,dx$ term) and y (because of the $-y\,dy$ term.
31. Differentiate and collect dy and dx terms:

$$\begin{aligned} 3x^2\,dx + 2xy\,dx + x^2\,dy &= 0 \\ 3x\,dx + 2y\,dx + x\,dy &= 0 \\ x\,dy &= (-2y - 3x)\,dx \\ dy/dx &= -2y/x - 3. \end{aligned}$$

If $x = 1$, $y = 99$.
35. $-\frac{1}{2}\cos\left(x^2\right) + C$
37. Use a u-substitution with $u = \ln x \Rightarrow du = 1/x\,dx$. Then,

$$y = \int \frac{1}{x\ln x}\,dx = \int \frac{1}{u}\,du = \ln|u| + C = \ln(\ln x) + C.$$

39. Use integration by parts with $u = x \Rightarrow du = dx$ and $dv = e^{-x}\,dx \Rightarrow v = -e^{-x}$.

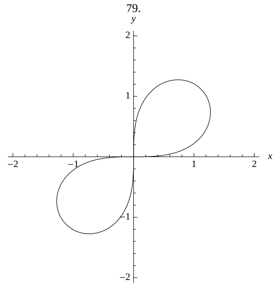
$$y = \int xe^{-x}\,dx = -xe^{-x} + \int e^{-x}\,dx = -xe^{-x} - e^{-x} + C.$$

41. $\tan^{-1}(x) - \ln(x+1) + C$
43. $6\sin^{-1}\left(\frac{x}{2}\right) - \frac{1}{4}x\sqrt{4-x^2}\left(x^2 - 10\right)$
45. Start by rewriting $\cot x = \dfrac{\cos x}{\sin x}$. Then,

$$\begin{aligned} y &= \int \frac{\cos^2 x}{\sin x}\,dx \\ &= \int \frac{1 - \sin^2 x}{\sin x}\,dx \\ &= \int (\csc x - \sin x)\,dx \end{aligned}$$

so $y = \ln(\csc x + \cot x) + \cos x + C$.
47. $y(x) = 2e^{-2x}$
49. $y(x) = \frac{5}{7}e^{-3x} + \frac{2}{7}e^{4x}$
51. $y(x) = -\frac{1}{4}\left(-3 + 3e^{2x} - 2x\right)$
53. $y(t) = -t^7 + t^6$
55. $y(x) = x^4 - 1/2\,x^2 + 2\,x + 1$
57. $y(x) = -\sin\left(x^{-1}\right) + 2$
67. $y(x) = c_1\,x + c_2\,x^2$
69. $y(x) = (e^x + C)\,e^{-2\,x}$
80. $y = \frac{1}{32}(12x - 8\sin 2x + \sin 4x$

79.



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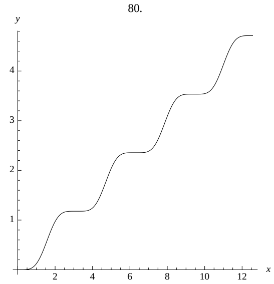

81. $y = -\frac{1}{208}e^{-x/2}(74e^{x}\cos 2x - 74\cos 3x - 111e^{x}\sin 2x - 20\sin x)$

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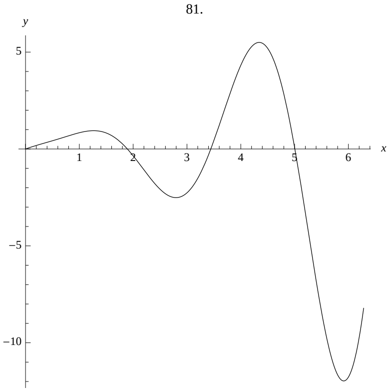

83. $x = \frac{4}{9}e^{-6t}\left(e^{9t} + 8\right)$, $y = \frac{8}{9}e^{-6t}\left(e^{9t} - 1\right)$

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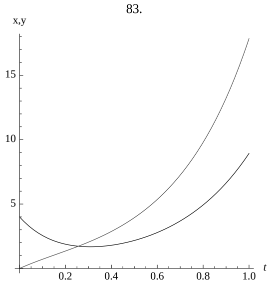

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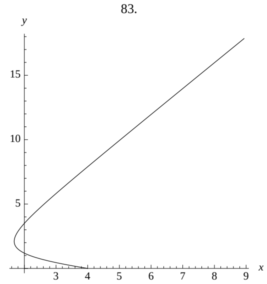

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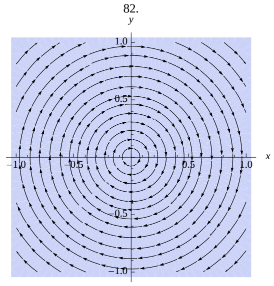

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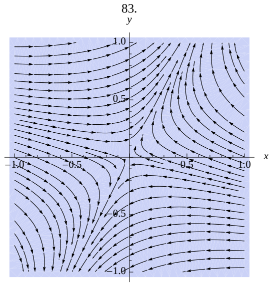
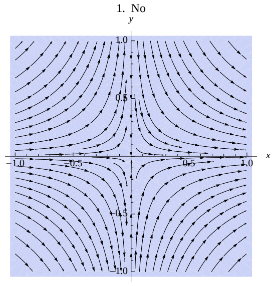
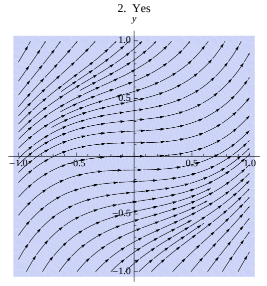


Exercises 1.2

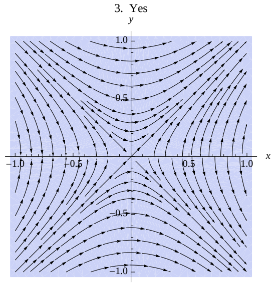
1. No



2. Yes



3. Yes



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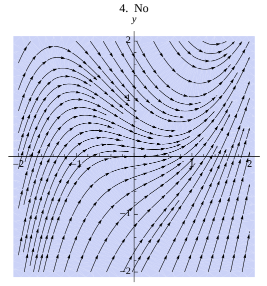

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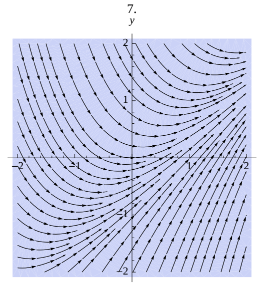

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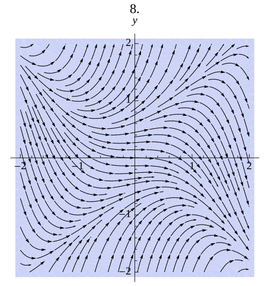

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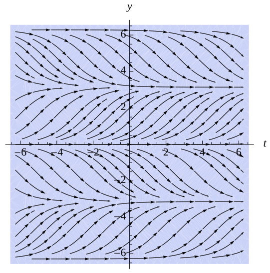

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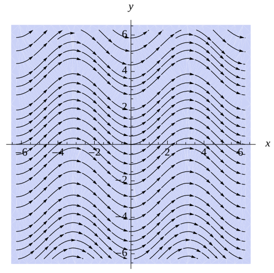

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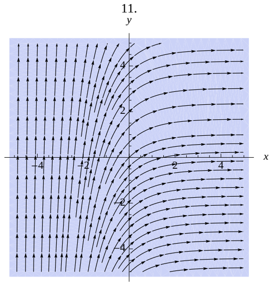

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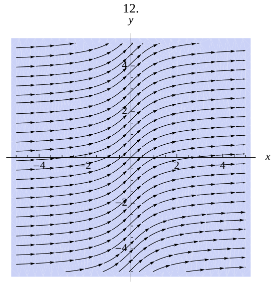

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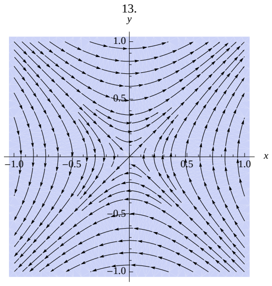

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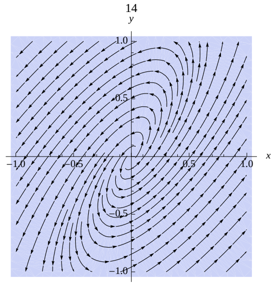


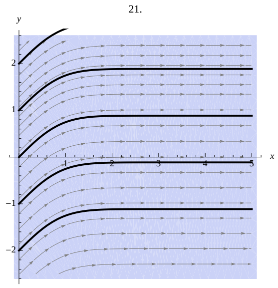
15. $x' = y$ so $y' = x'' = -4x$: $\{x' = y,\, y' = -4x\}$

17. $\{x' = y,\, y' = -13x - 4y\}$

19. $\{x' = y,\, y' = -16x + \sin t\}$

21. $y = C + \frac{\sqrt{\pi}}{2}\,\text{erf}\,(x)$, where $\text{erf}\,(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$.

21.



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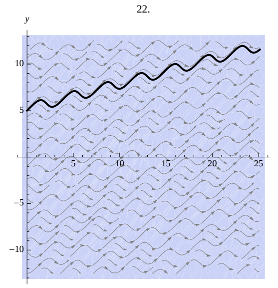
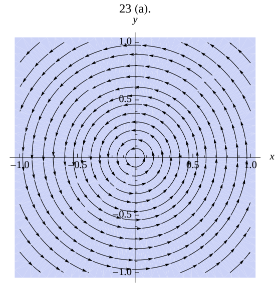
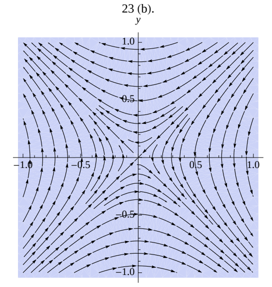


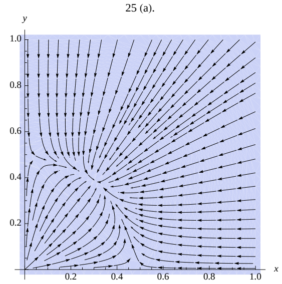
23 (a).



23 (b).



25 (a).



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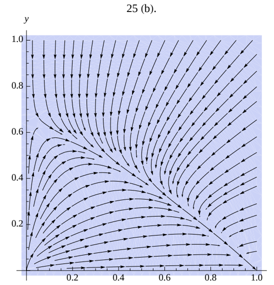

Chapter 1 Review Exercises

1. First-order ordinary linear homogeneous differential equation
3. Second-order linear homogeneous differential equation
5. Second-order non-linear partial differential equation
17. $y = (2 - x^2)\cos x + 2x \sin x$
19. $y = \frac{1}{2}\left(x\sqrt{x^2-1} + \ln\left(2x + 2\sqrt{x^2-1}\right)\right)$
21. $y = \frac{1}{4}(x + 4\cos 2x)$
23. $y = \frac{1}{3}(1 - \cos^3 x)$