

FIGURE 2.1 (a) Graphs of $y^{2}-t^{2}=C$ for several values of $C<0$. (b) Graphs of $y^{2}-t^{2}=C$ for several values of $C>0$. (c) Graphs of $y=t$ and $y=-t$.


FIGURE 2.2 (a) Plot of $y=(5 / 6)^{3 / 5} t^{6 / 5}$. (b) Graph of $t^{2} y=C\left(t^{4}+y^{2}\right)$ for various values of $C$.


FIGURE 2.3 Solution of $t y^{\prime}+2 y=t \cos t, y(2)=4$.


FIGURE 2.4 Graph of $y=1 / \sqrt{C-2 t}$ for $C=1,2,3$.


FIGURE 2.5 Graph of $t^{2}+y^{2}=k$ for various values of $k>0$.


FIGURE 2.6 Graph of the solution to IVP.


FIGURE 2.7 (a) Slope field for $\mathrm{d} y / \mathrm{d} t=\left(t^{4}+1\right)\left(y^{4}+1\right)$. (b) Solution to $\mathrm{d} y / \mathrm{d} t=\left(t^{4}+1\right)\left(y^{4}+1\right), y(0)=0$.


FIGURE 2.8 Phase line for $\mathrm{d} y / \mathrm{d} t=2 y-y^{2}$.


FIGURE 2.9 Several solutions of $\mathrm{d} y / \mathrm{d} t=2 y-y^{2}$.


FIGURE 2.10 Light wave propagating through a medium.


FIGURE 2.11 Phase line for $\mathrm{d} y / \mathrm{d} t-y=-1 / 2$.


FIGURE 2.12 (a) Solution to $\mathrm{d} y / \mathrm{d} t=y-1 / 2, y(0)=1 / 4$. (b) Solution to $\mathrm{d} y / \mathrm{d} t=y-1 / 2, y(0)=1$. (c) Solution to $\mathrm{d} y / \mathrm{d} t=y-1 / 2, y(0)=1 / 2$.


FIGURE 2.13 Graph of $y=\frac{1}{5}+\mathrm{Ce}^{-t^{5}}$ for various values of $C$.


FIGURE 2.14 Every solution of the differential equation satisfies $y(0)=0$.


FIGURE 2.15 (a) $k=0.05$, (b) $k=0.10$, (c) $k=0.15$, and (d) $k=0.20$.


FIGURE 2.16 We graph various solutions to the equation by graphing several level curves of the function $f(t, y)=t \sin y+$ $y \sin t$.


FIGURE 2.17 On the left, a graph of the equation $t^{2} \sin y-y=-1 / 2$. On the right, we see that the solution is unique only on an interval containing approximately $-1.2<t<1.2$.


FIGURE 2.18 Graph of $t \mathrm{e}^{y / t}+\tan ^{-1} t=C$ for various values of $C$.


FIGURE 2.19 (a) $C=-3 / 2$, (b) $C=-1$, (c) $C=-1 / 2$, (d) $C=0$, (e) $C=1 / 2$, (f) $C=1,(\mathrm{~g}) C=3 / 2$, and (h) $C=9 / 5$.


FIGURE 2.20 The solution is not unique near this point because when we zoom in, we see the graph of more than one function passing through the point.


FIGURE 2.21 Solution to the logistic equation ( $y_{0}=1 / 4, a=1$, and $k=3$ ).


FIGURE 2.22 Graph of $y=\sqrt{1+2 t}$.


FIGURE 2.23 First Row: (a)-(c): (a) $a_{0}=-4, a_{1}=3, a_{2}=7, a_{3}=5$; (b) $a_{0}=4, a_{1}=-5, a_{2}=-6, a_{3}=3$; and (c) $a_{0}=3$, $a_{1}=-1, a_{2}=1, a_{3}=-2$. Second Row: (d)-(f) (d) $a_{0}=3, a_{1}=-1, a_{2}=1, a_{3}=2$; (e) $a_{0}=-2, a_{1}=-7, a_{2}=-7, a_{3}=2$; and (f) $a_{0}=-5, a_{1}=1, a_{2}=2, a_{3}=-4$.


FIGURE 2.24 Graph of $\frac{3}{2} t^{2}-t y+y^{2}=C$ for various values of $C$.


FIGURE 2.25 (a) Approximating ( $x_{1}, y_{1}$ ). (b) Successive approximations using Euler's method.


FIGURE 2.26 (a) $h=0.1$ and (b) $h=0.05$.


FIGURE 2.27 Improving Euler's method.


FIGURE 2.28 Using the Improved Euler's Method to approximate the solution to an IVP.


FIGURE 2.29 (a) Graph of the solution to the IVP. (b) Several slopes together with the slope field for the ODE.

TABLE 2.1 Euler's Method with $h=0.1$

| $x_{n}$ | $y_{n}$ | $x_{n}$ | $y_{n}$ |
| :--- | :--- | :--- | :--- |
| 0.0 | 1.0 | 0.6 | 1.15873 |
| 0.1 | 1.0 | 0.7 | 1.22825 |
| 0.2 | 1.01 | 0.8 | 1.31423 |
| 0.3 | 1.0302 | 0.9 | 1.41937 |
| 0.4 | 1.06111 | 1.0 | 1.54711 |
| 0.5 | 1.10355 |  |  |

TABLE 2.2 Euler's Method with $h=0.05$

| $x_{n}$ | $y_{n}$ | $x_{n}$ | $y_{n}$ | $x_{n}$ | $y_{n}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0 | 1.0 | 0.35 | 1.05361 | 0.70 | 1.2523 |
| 0.05 | 1.0 | 0.40 | 1.07204 | 0.75 | 1.29613 |
| 0.10 | 1.0025 | 0.45 | 1.09348 | 0.80 | 1.34474 |
| 0.15 | 1.00751 | 0.50 | 1.11809 | 0.85 | 1.39853 |
| 0.20 | 1.01507 | 0.55 | 1.14604 | 0.90 | 1.45796 |
| 0.25 | 1.02522 | 0.60 | 1.17756 | 0.95 | 1.52357 |
| 0.30 | 1.03803 | 0.65 | 1.21288 | 1.00 | 1.59594 |

TABLE 2.3 Improved Euler's Method with $h=0.1$

| $x_{n}$ | $y_{n}($ IEM $)$ | $y_{n}($ EM $)$ | Actual Value |
| :--- | :--- | :--- | :--- |
| 0.0 | 1.0 | 1.0 | 1.0 |
| 0.1 | 1.005 | 1.0 | 1.00501 |
| 0.2 | 1.0201755 | 1.01 | 1.0202 |
| 0.3 | 1.0459859 | 1.0302 | 1.04603 |
| 0.4 | 1.083223 | 1.06111 | 1.08329 |
| 0.5 | 1.1330513 | 1.10355 | 1.13315 |
| 0.6 | 1.1970687 | 1.15873 | 1.19722 |
| 0.7 | 1.277392 | 1.22825 | 1.27762 |
| 0.8 | 1.3767731 | 1.31423 | 1.37713 |
| 0.9 | 1.4987552 | 1.41937 | 1.49930 |
| 0.10 | 1.6478813 | 1.54711 | 1.64872 |

TABLE 2.4 Runge-Kutta Method of Order 2 with $h=0.1$

| $x_{n}$ | $y_{n}(\mathbf{R K})$ | Actual Value |
| :--- | :--- | :--- |
| 0.0 | 1.0 | 1.0 |
| 0.1 | 1.005 | 1.00501 |
| 0.2 | 1.0201755 | 1.0202 |
| 0.3 | 1.0459859 | 1.04603 |
| 0.4 | 1.083223 | 1.08329 |
| 0.5 | 1.1330513 | 1.13315 |
| 0.6 | 1.1970687 | 1.19722 |
| 0.7 | 1.277392 | 1.27762 |
| 0.8 | 1.3767731 | 1.37713 |
| 0.9 | 1.4987552 | 1.4993 |
| 1.0 | 1.6478813 | 1.64874 |

TABLE 2.5 Fourth-Order Runge-Kutta Method with $h=0.1$

| $x_{n}$ | $y_{n}($ RK Order 4$)$ | $y_{n}$ (IEM) | Actual Value |
| :--- | :--- | :--- | :--- |
| 0.0 | 1.0 | 1.0 | 1.0 |
| 0.1 | 1.00501 | 1.005 | 1.00501 |
| 0.2 | 1.0202 | 1.0201755 | 1.0202 |
| 0.3 | 1.04603 | 1.0459859 | 1.04603 |
| 0.4 | 1.08329 | 1.083223 | 1.08329 |
| 0.5 | 1.13315 | 1.1330513 | 1.13315 |
| 0.6 | 1.19722 | 1.1970687 | 1.19722 |
| 0.7 | 1.27762 | 1.277392 | 1.27762 |
| 0.8 | 1.37713 | 1.3767731 | 1.37713 |
| 0.9 | 1.4993 | 1.4987552 | 1.4993 |
| 1.0 | 1.64872 | 1.6478813 | 1.64874 |

