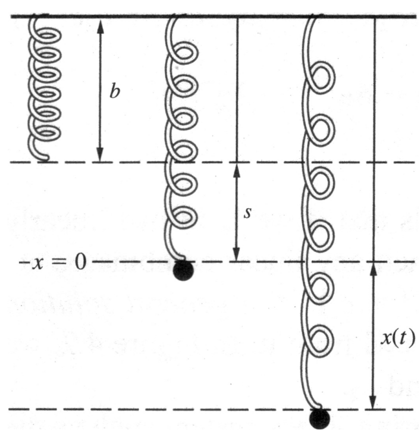

FIGURE 4.1 A spring-mass system.

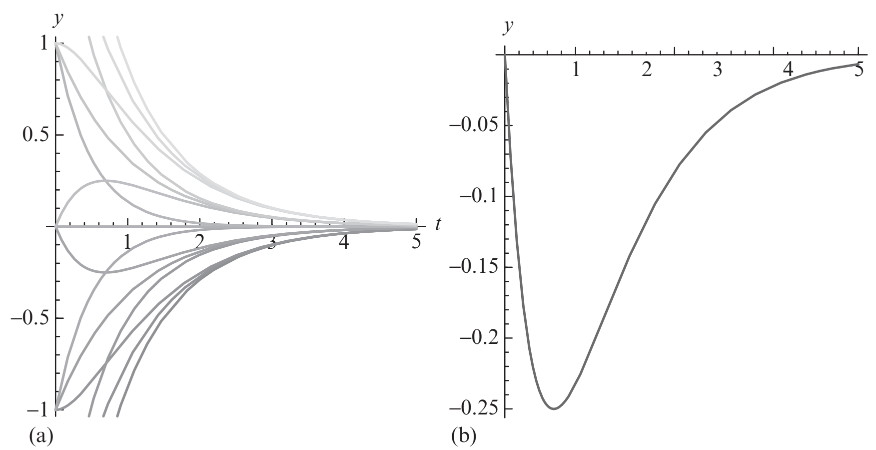

FIGURE 4.2 (a) Graph of general solution for various values of c_1 and c_2. (b) Graph of the solution $x(t) = e^{-2t} - e^{-t}$ to the IVP.

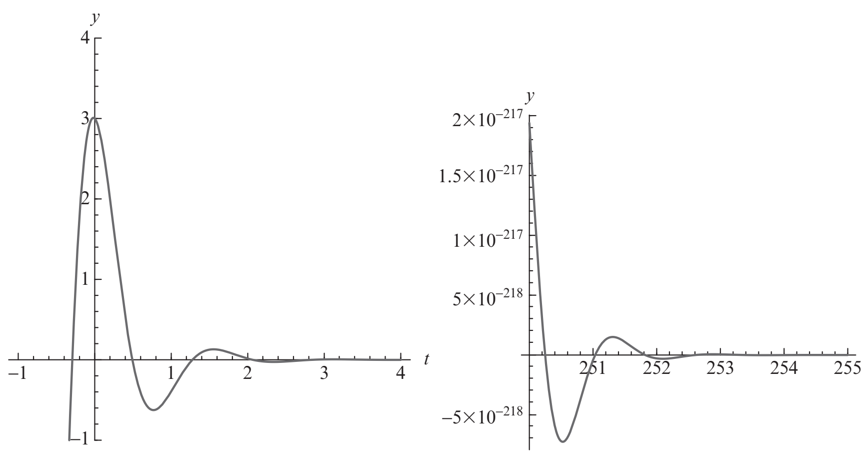

FIGURE 4.3 Graph of $y = e^{-2t}\left(3\cos 4t + \frac{5}{4}\sin 4t\right)$. Note that y is not identically the zero function but converges to the zero function quickly so it appears that y and the zero function are identical in the plot as t increases, even though the function oscillates forever.

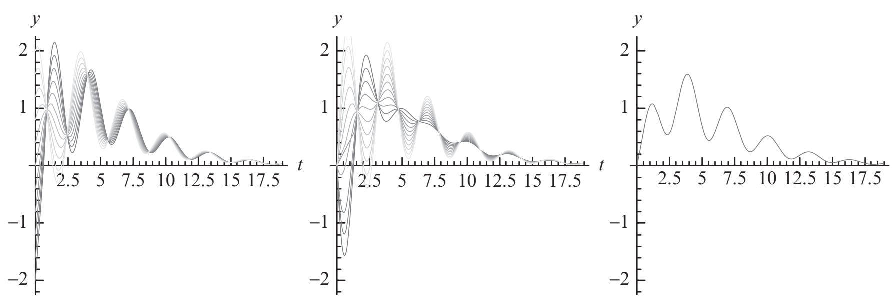

FIGURE 4.4 (a) Plots of some solutions satisfying $y'(0) = 1$. (b) Plots of some solutions satisfying $y(0) = 0$. (c) Plot of *the* solution satisfying $y(0) = 0$ and $y'(0) = 1$.

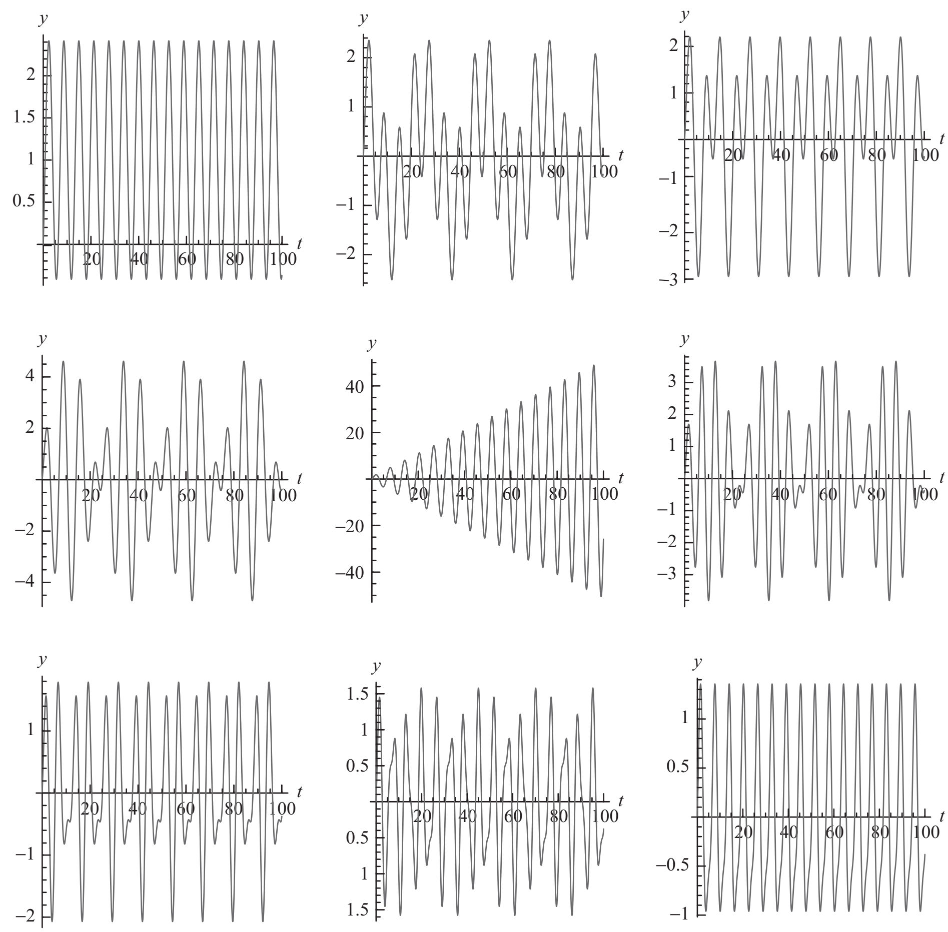

FIGURE 4.5 From left to right, $\omega = 0, 1/4, 1/2, 3/4, 1, 5/4, 3/2, 7/4$, and 2. Notice that the solution is bounded unless $\omega = 1$.

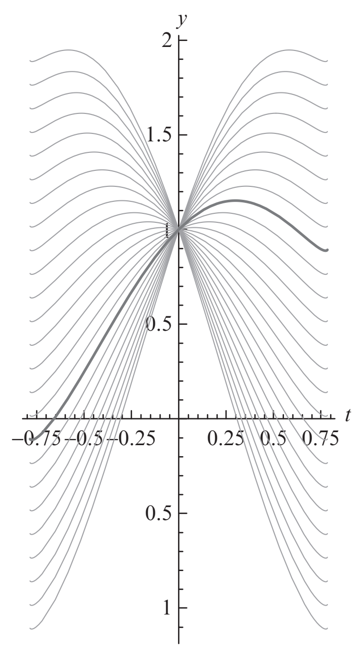

FIGURE 4.6 Plots of $y'' + 4y = \sec 2t,\ y(0) = 1,\ y'(0) = c$ for various values of c. Which is the plot of the solution satisfying $y'(0) = 1$?

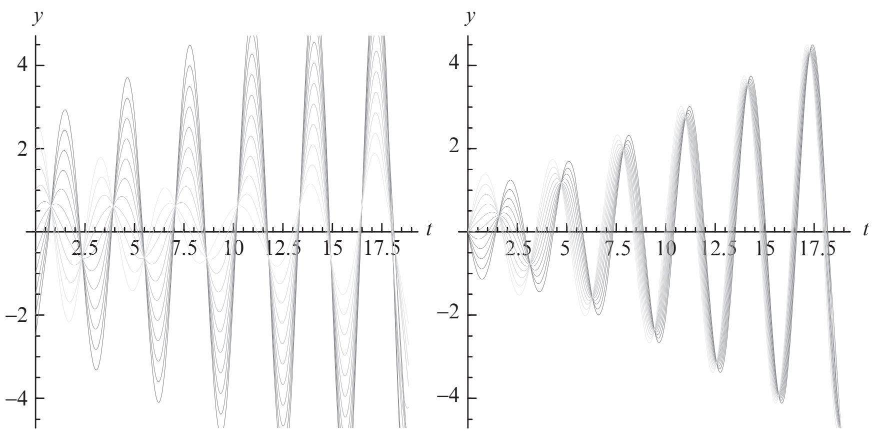

FIGURE 4.7 (a) Various solutions satisfying $y'(0) = 1$. (b) Various solutions satisfying $y(0) = 0$.

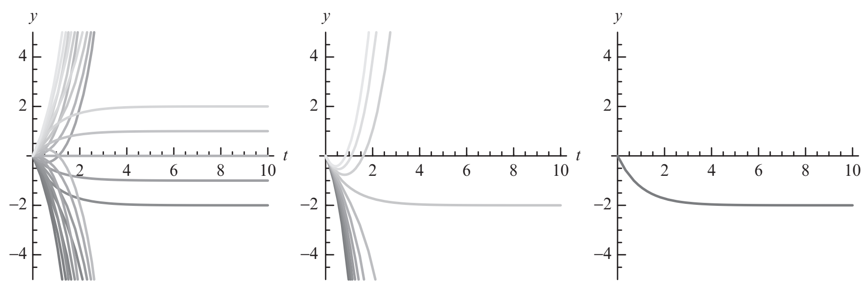

FIGURE 4.8 (a) Various solutions of the differential equation satisfying $y(0) = 0$. (b) Various solutions of the differential equation satisfying $y(0) = 0$ and $y'(0) = -2$. (c) The solution of the differential equation satisfying $y(0) = 0$, $y'(0) = -2$, and $y''(0) = 2$.

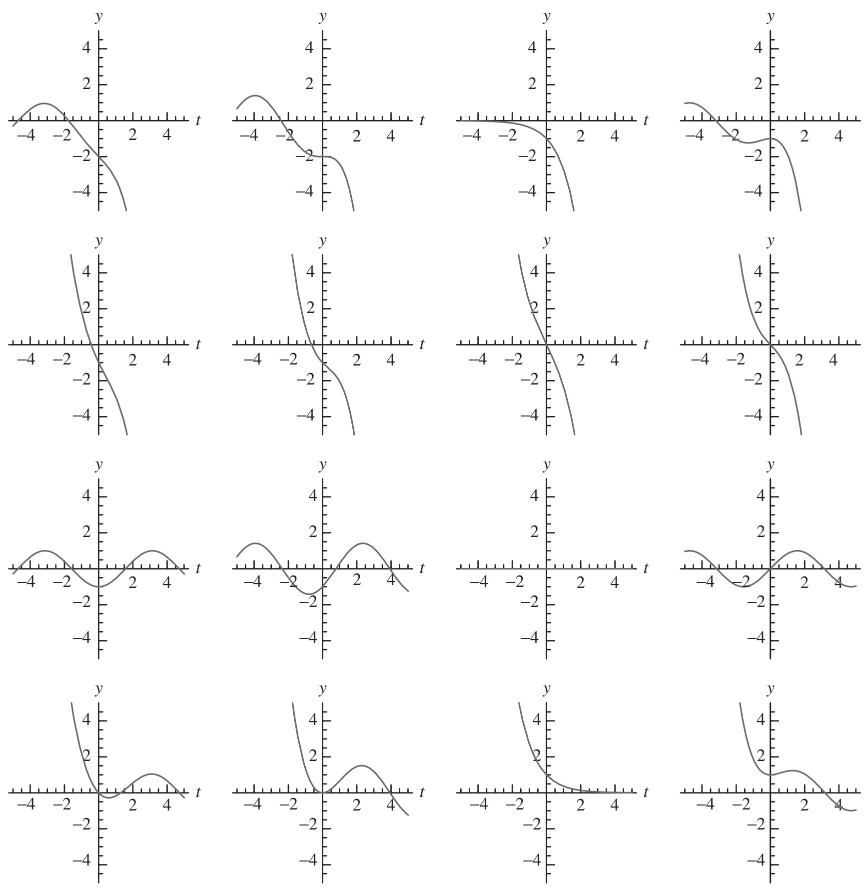

FIGURE 4.9 (a)-(d) in row 1, (e)-(h) in row 2, (i)-(l) in row 3, and (m)-(p) in row 4.

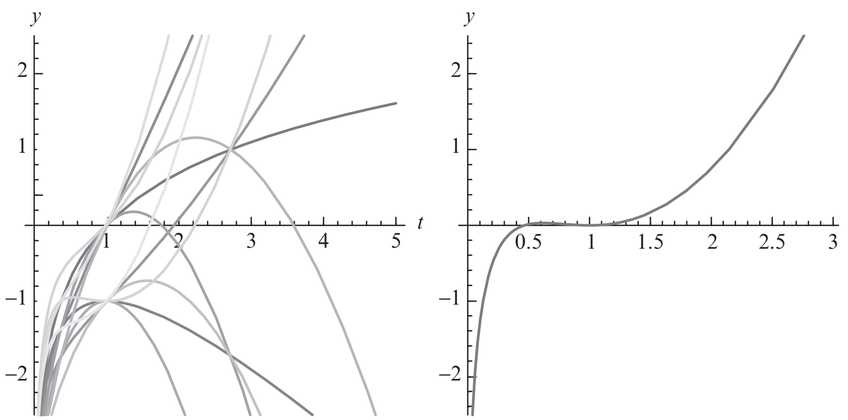

FIGURE 4.10 (a) Plots of various solutions of $t^3y''' + ty' - y = 3 - \ln t$. (b) Plot of *the* solution of $t^3y''' + ty' - y = 3 - \ln t$ that satisfies the initial conditions $y(1) = 0$, $y'(1) = 0$, and $y''(1) = 1$.

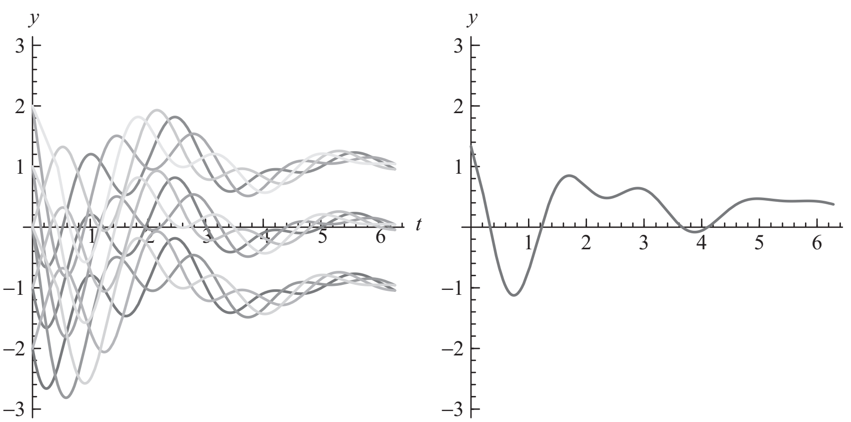

FIGURE 4.11 (a) Various solutions of the nonhomogeneous equation. (b) The solution of the nonhomogeneous equation that satisfies the initial conditions $y(0) = 4/3$, $y'(0) = -5/2$, and $y''(0) = -173/12$.

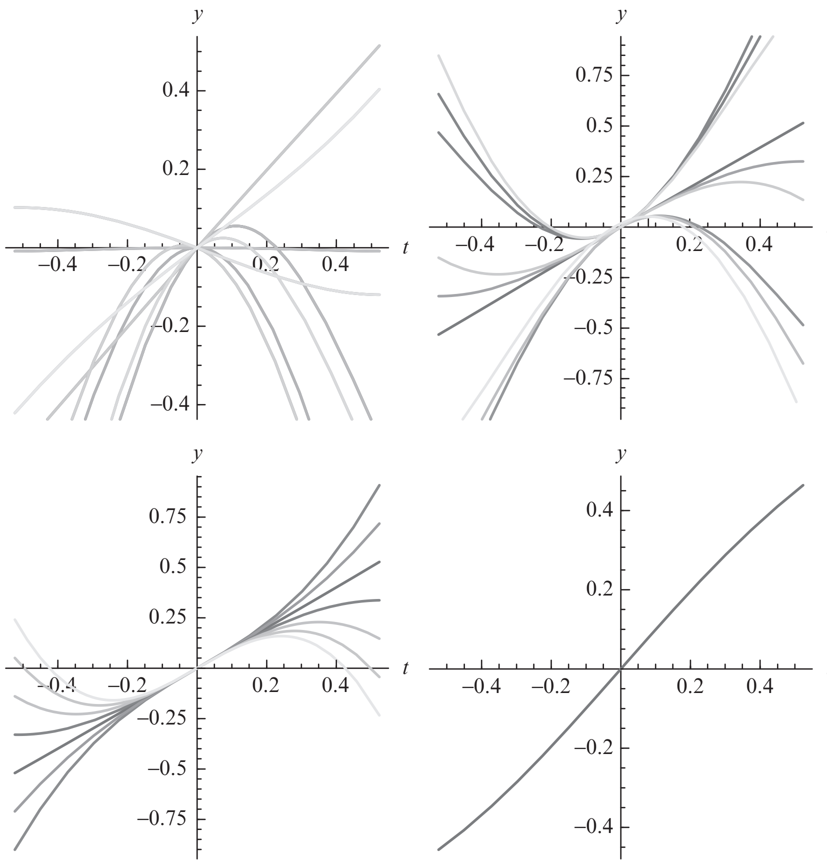

FIGURE 4.12 (From left to right) (a) Various solutions of the nonhomogeneous equation that satisfy $y(0) = 0$. (b) Various solutions that satisfy $y(0) = 0$ and $y'(0) = 1$. (c) Various solutions that satisfy $y(0) = 0$, $y'(0) = 1$, and $y''(0) = 0$. (d) *The* solution that satisfies $y(0) = 0$, $y'(0) = 1$, $y''(0) = 0$, and $y'''(0) = -3$.

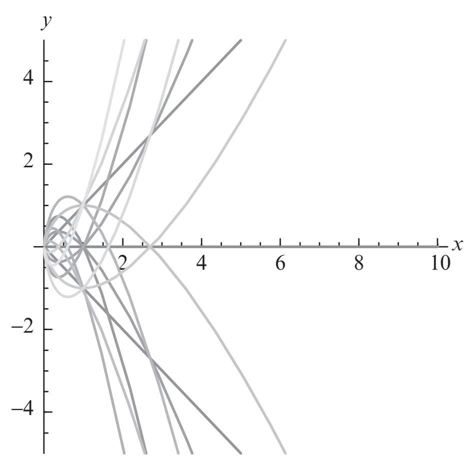

FIGURE 4.13 Plots of various solutions of $x^2y'' - xy' + y = 0$, $x > 0$.

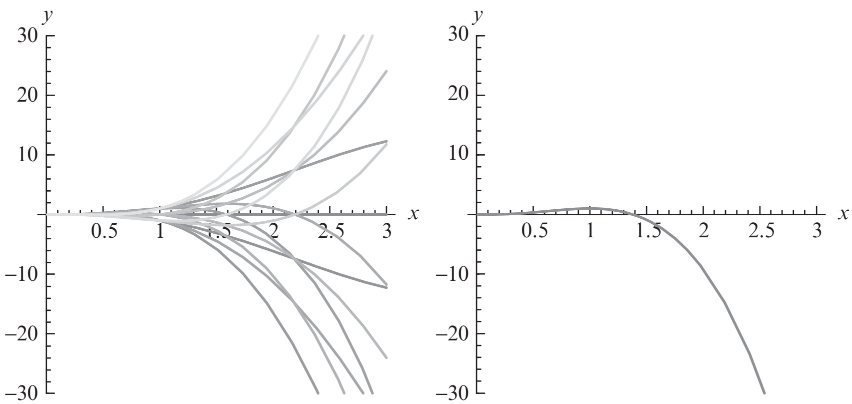

FIGURE 4.14 (a) Plots of various solutions of $x^2y'' - 5xy' + 10y = 0$, $x > 0$. (b) Plot of *the* solution of the IVP.

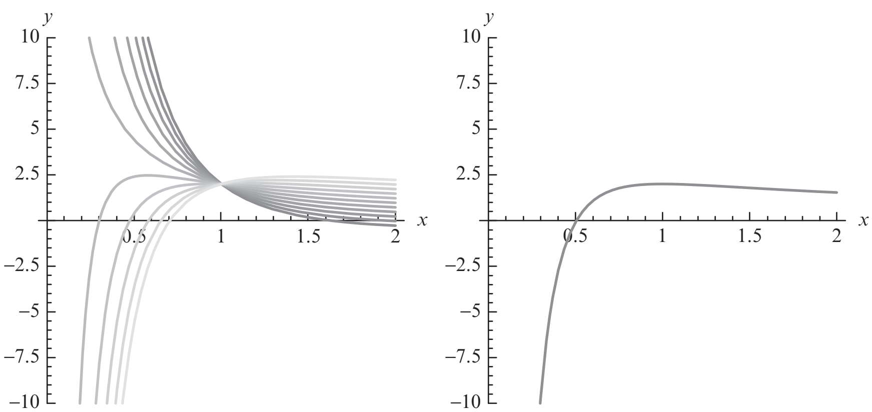

FIGURE 4.15 (a) Plots of various solutions of the nonhomogeneous equation. (b) Plot of *the* solution of the nonhomogeneous equation that satisfies the initial conditions.

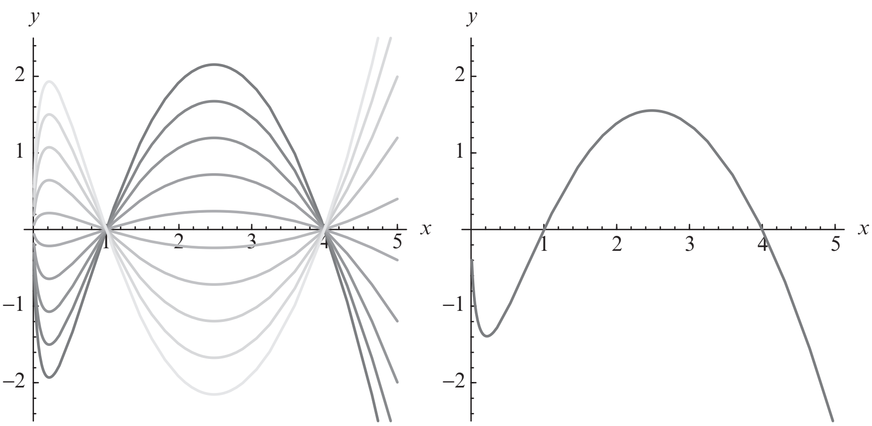

FIGURE 4.16 (a) Plots of various solutions of the differential equation that satisfy $y(1) = y(4) = 0$. (b) Plot of *the* solution of the differential equation that satisfies $y(1) = y(4) = 0$ and $y'(1) = 2$.

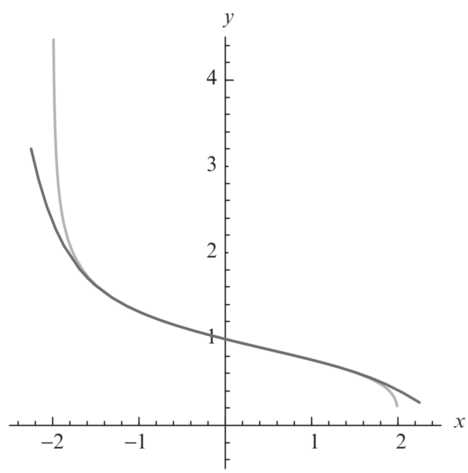

FIGURE 4.17 Comparison of exact (black) and approximate (gray) solutions to the initial-value problem $(4-x^2)y'+y=0$, $y(0)=1$.

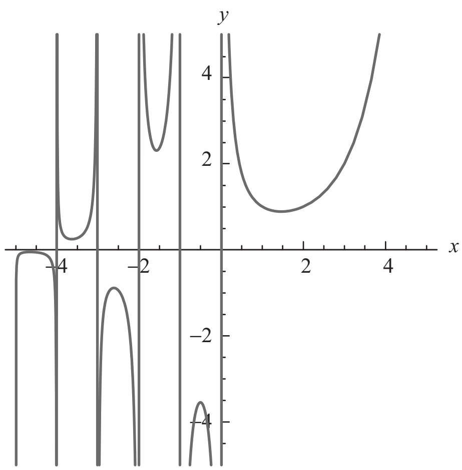

FIGURE 4.18 Although we have only defined $\Gamma(x)$ for $x > 0$, $\Gamma(x)$ can be defined for all real numbers *except* $x = 0$, $x = -1$, $x = -2, \ldots$. (This topic is discussed in most complex analysis texts such as *Functions of One Complex Variable*, Second Edition, by John B. Conway, Springer-Verlag (1978), pp. 176-185.)

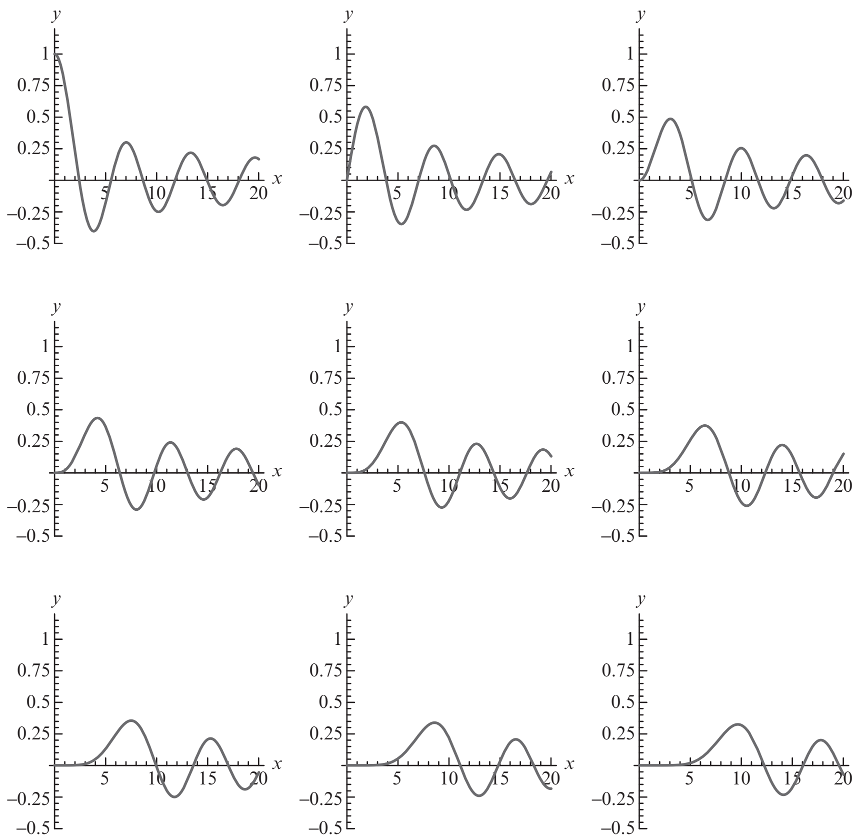

FIGURE 4.19 In the top row, from left to right, plots of $J_0(x)$, $J_1(x)$, and $J_2(x)$; in the second row, from left to right, plots of $J_3(x)$, $J_4(x)$, and $J_5(x)$; and in the third row, from left to right, plots of $J_6(x)$, $J_7(x)$, and $J_8(x)$.

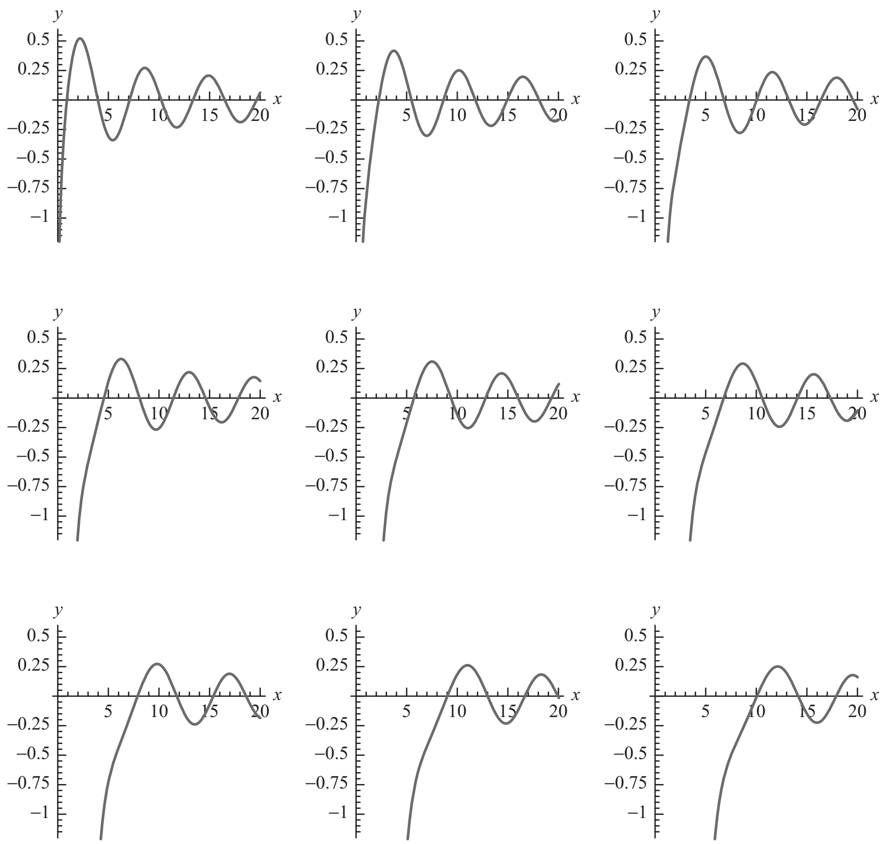

FIGURE 4.20 In the top row, from left to right, plots of $Y_0(x)$, $Y_1(x)$, and $Y_2(x)$; in the second row, from left to right, plots of $Y_3(x)$, $Y_4(x)$, and $Y_5(x)$; and in the third row, from left to right, plots of $Y_6(x)$, $Y_7(x)$, and $Y_8(x)$.

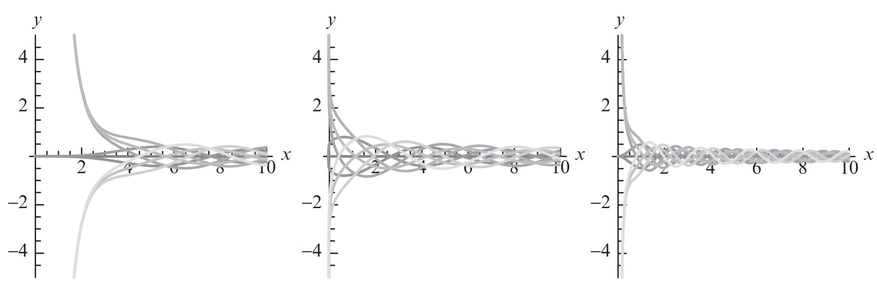

FIGURE 4.21 (a) $y = c_1 J_4(x) + c_2 Y_4(x)$, (b) $y = c_1 J_{1/5}(x) + c_2 Y_{1/5}(x)$, and (c) $y = c_1 J_2(3x) + c_2 Y_2(3x)$.

Differential Equation	Characteristic Equation	Roots of Characteristic Equation	General Solution
$y^{(n)} = 0$			
	$(r - k)^n$		
		$r_{1,2} = \alpha \pm i\beta$,	
		$\beta \neq 0$	
			$y = e^{\alpha t}[(c_{1,1} + c_{1,2}t + \cdots c_{1,n-1}t^{n-1}) \cos \beta t + (c_{2,1} + c_{2,2}t + \cdots c_{2,n-1}t^{n-1}) \sin \beta t]$

TABLE 4.1 Linearly Independent Solutions of $2y^{(6)} - 7y^{(5)} - 4y^{(4)} = 0$

Root	Multiplicity	Corresponding Solution(s)
$r = 0$	$k = 4$	$y_1 = 1, y_2 = t, y_3 = t^2, y_4 = t^3$
$r = -1/2$	$k = 1$	$y_5 = e^{-t/2}$
$r = 4$	$k = 1$	$y_6 = e^{4t}$

TABLE 4.2 Linearly Independent Solutions of $y^{(4)} - y = 0$

Root	**Multiplicity**	**Corresponding Solution(s)**
$r_1 = 1$	$k = 1$	$y_1 = \mathrm{e}^t$
$r_2 = -1$	$k = 1$	$y_2 = \mathrm{e}^{-t}$
$r_{3,4} = \pm\mathrm{i}$	$k = 1, k = 1$	$y_3 = \cos t, y_4 = \sin t$

TABLE 4.3 a_n Values for $n = 0, 1, 2, 3, \ldots, 10$

n	a_n	n	a_n	n	a_n	n	a_n
0	a_0	3	$-\frac{3}{128}a_0$	6	$\frac{69}{65536}a_0$	9	$-\frac{4859}{33554432}a_0$
1	$-\frac{1}{4}a_0$	4	$\frac{11}{2048}a_0$	7	$-\frac{187}{262144}a_0$	10	$\frac{12767}{268435456}a_0$
2	$\frac{1}{32}a_0$	5	$-\frac{31}{8192}a_0$	8	$\frac{1843}{8388608}a_0$		

TABLE 4.4 Coefficients for $n = 0, \ldots, 7$

n	a_n	n	a_n	n	a_n
0	a_0	3	$\frac{1}{6}(a_0 - a_1)$	6	$\frac{1}{720}(18a_1 - 7a_0)$
1	a_1	4	$\frac{1}{24}(2a_1 - a_0)$	7	$\frac{1}{5040}(33a_0 - 85a_1)$
2	$-\frac{1}{2}a_0$	5	$\frac{1}{120}(2a_0 - 5a_1)$		

TABLE 4.5 $P_n(x)$ for $n = 0, 1, \ldots, 5$

n	$P_n(x)$
0	$P_0(x) = 1$
1	$P_1(x) = x$
2	$P_2(x) = \frac{1}{2}(3x^2 - 1)$
3	$P_3(x) = \frac{1}{2}(5x^3 - 3x)$
4	$P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3)$
5	$P_5(x) = \frac{1}{8}(63x^5 - 70x^3 + 15x)$