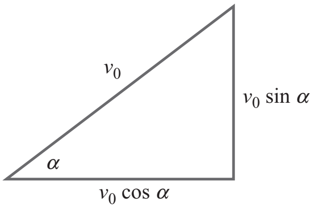

FIGURE 6.1 Sketch to determine components of $\mathbf{v}(0)$.

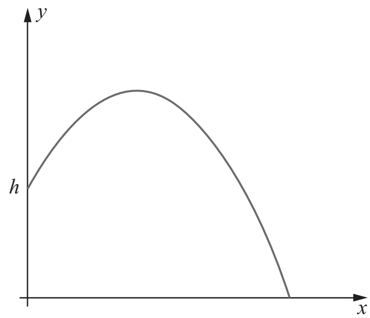

FIGURE 6.2 Sketch to determine $\mathbf{r}(0)$.

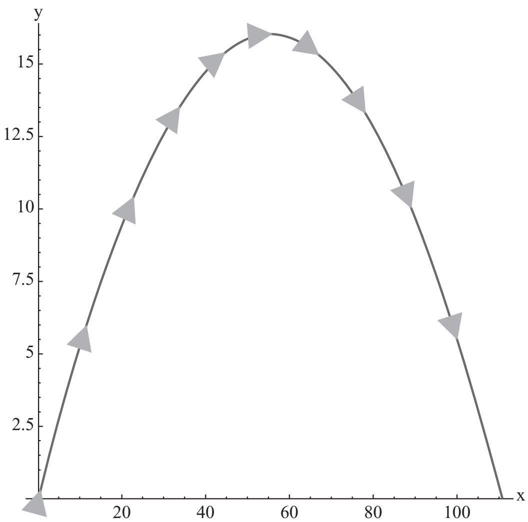

FIGURE 6.3 Path of projectile with orientation.

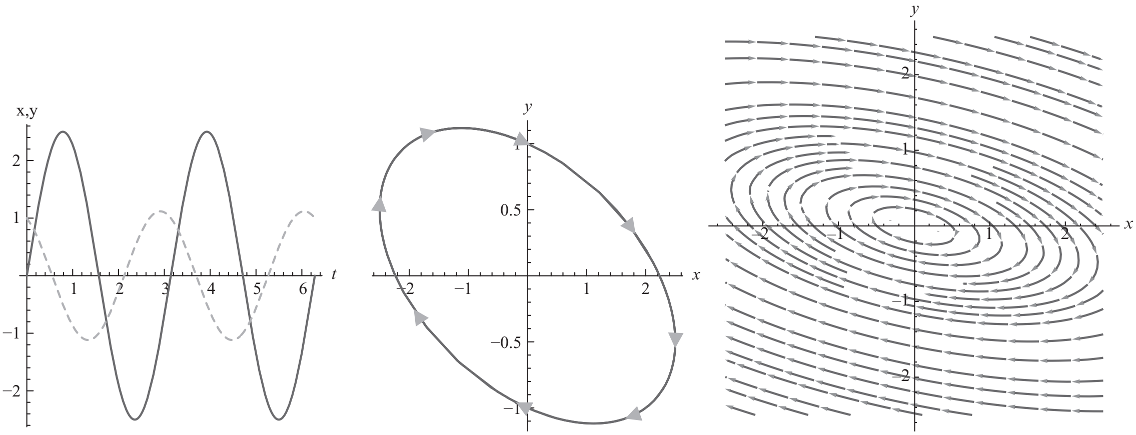

FIGURE 6.4 (a) Graphs of $x(t) = \frac{5}{2}\sin 2t$ and $y(t) = \cos 2t - \frac{1}{2}\sin 2t$. (b) Parametric plot of $\{x(t) = \frac{5}{2}\sin 2t, y(t) = \cos 2t - \frac{1}{2}\sin 2t\}$. (c) All solutions are periodic around the origin. We say that $(0, 0)$ is a *stable center*.

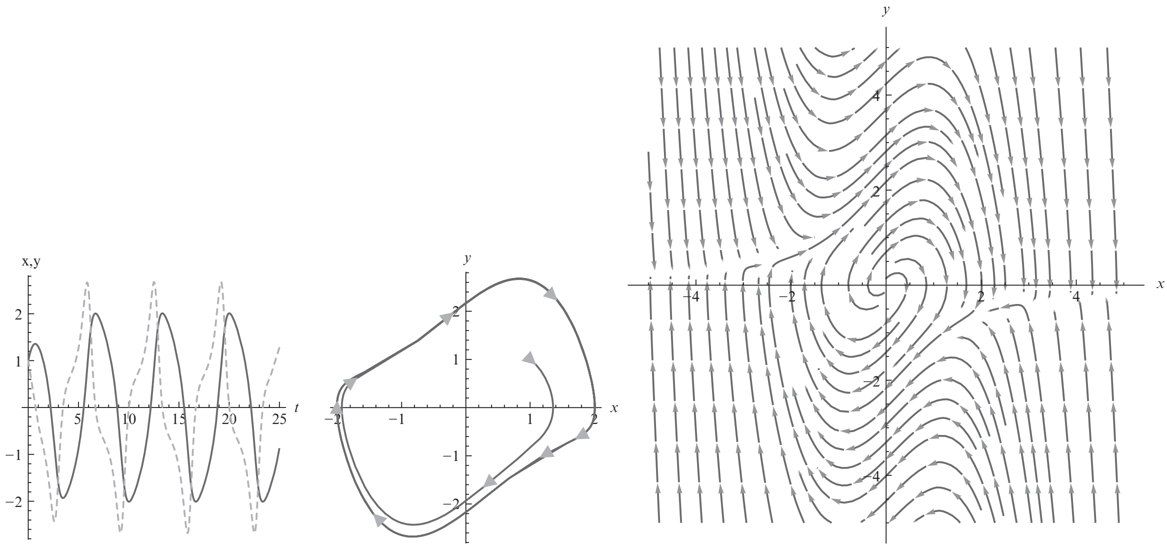

FIGURE 6.5 (a) Graph of $x(t)$ and $y(t)$ (dashed). (b) The graph of $\{x(t), y(t)\}$ indicates that the solution approaches an isolated periodic solution, which is called a *limit cycle*. (c) The phase portrait gives us a better understanding of the behavior of the solutions of the system.

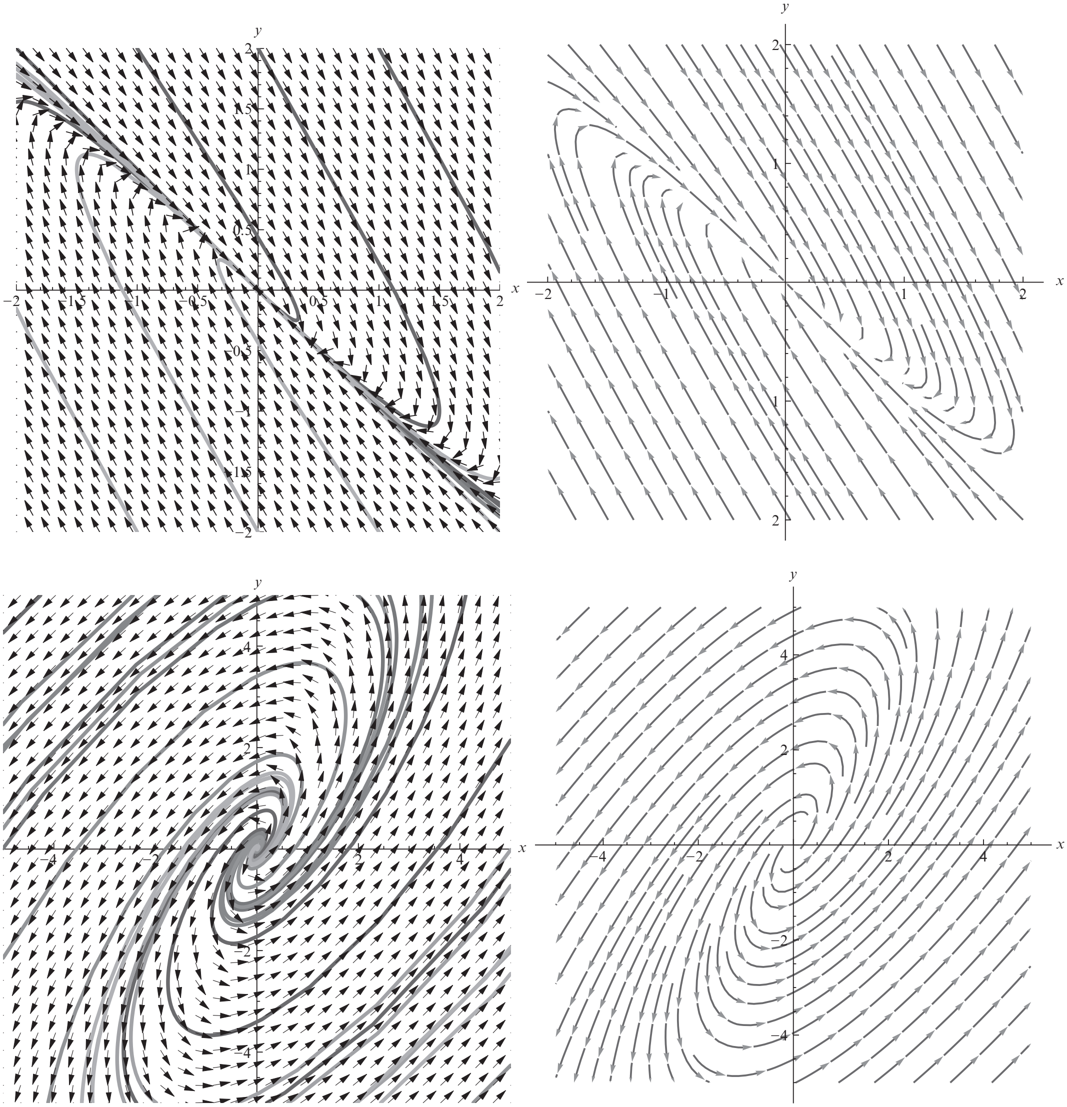

FIGURE 6.6 (a) All nontrivial solutions of this linear homogeneous system approach the origin as t increases. (b) A phase portrait confirms that all solutions tend to the origin as $t \to \infty$. (c) We see that all nontrivial solutions spiral away from the origin. (d) A phase portrait confirms that all solutions spiral away from the origin.

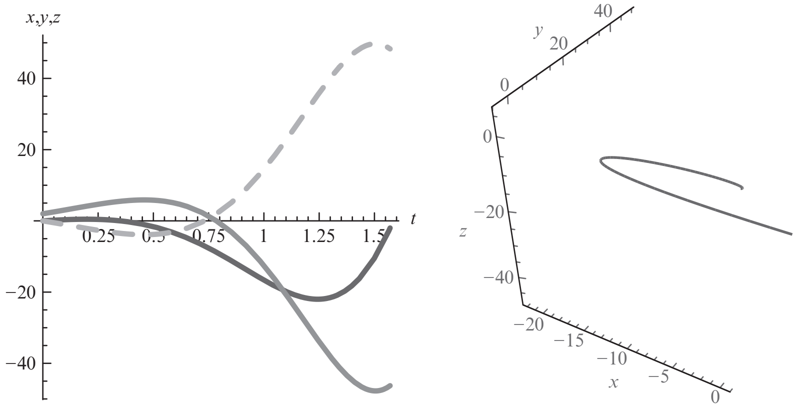

FIGURE 6.7 (a) Graphs of $x(t)$, $y(t)$ (dashed) and $z(t)$ (light pink; light gray in print versions). (b) Parametric plot of $\{x(t), y(t), z(t)\}$ in three dimensions for $0 \leq t \leq \pi/2$.

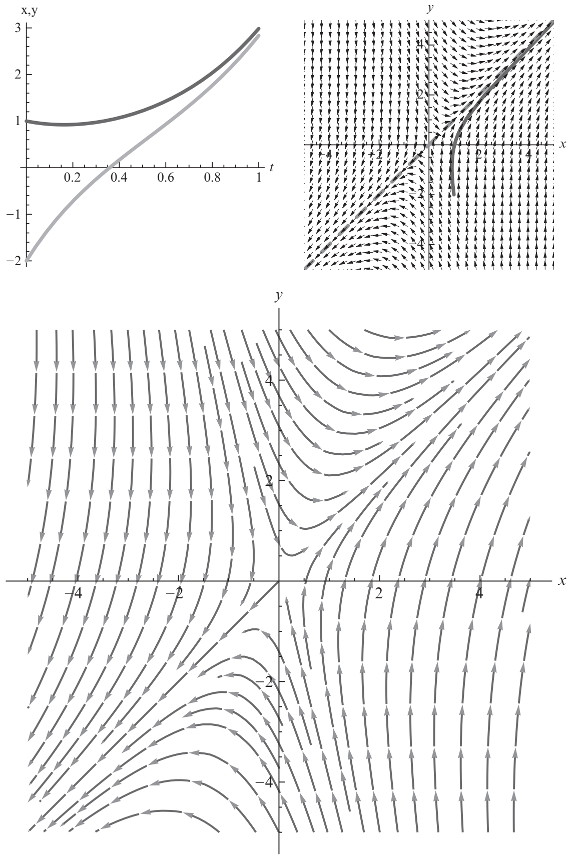

FIGURE 6.8 (a) Graph of $x(t)$ (dark red; dark gray in print versions) and $y(t)$. (b) Graph of $\{x(t), y(t)\}$ along with the direction field associated with the system of equations. (c) Phase portrait.

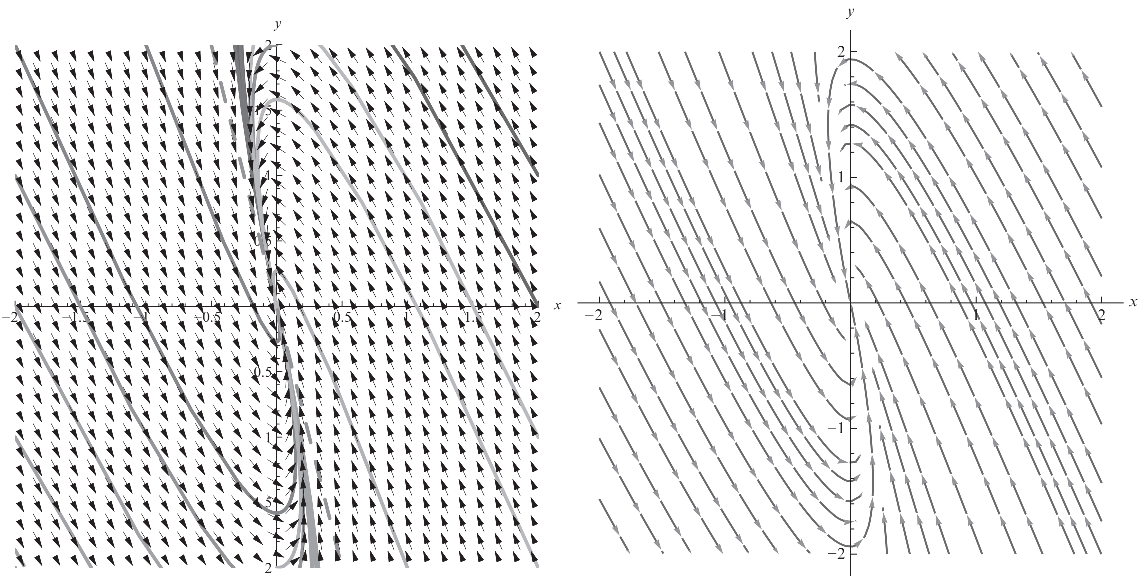

FIGURE 6.9 (a) All nontrivial solutions approach the origin. (b) A different view of the phase portrait emphasizes the behavior of the solutions.

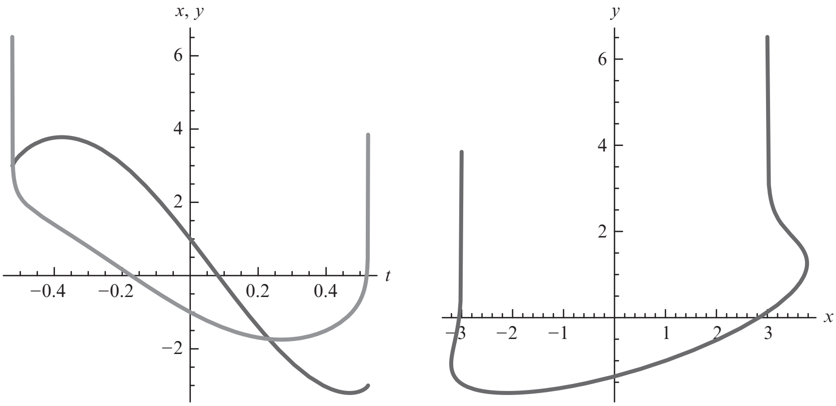

FIGURE 6.10 (a) Plots of x and y (light red; light gray in print versions) as functions of t. (b) Parametric plot of x versus y.

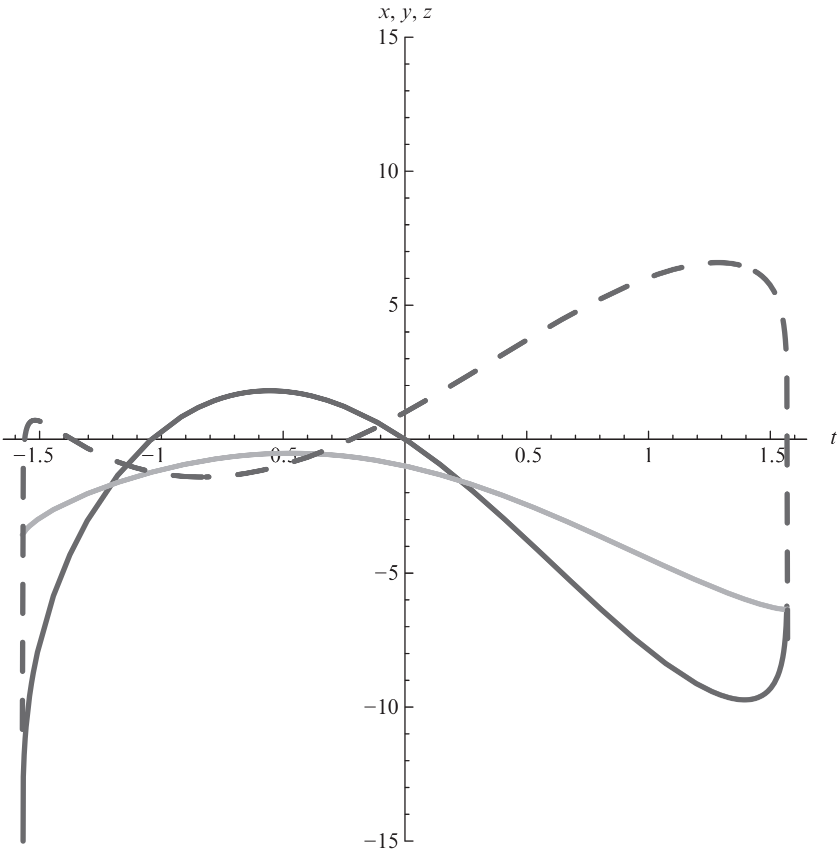

FIGURE 6.11 Plots of $x = x(t)$, $y = y(t)$ (in pink; light gray in print versions), and $z = z(t)$ (dashed).

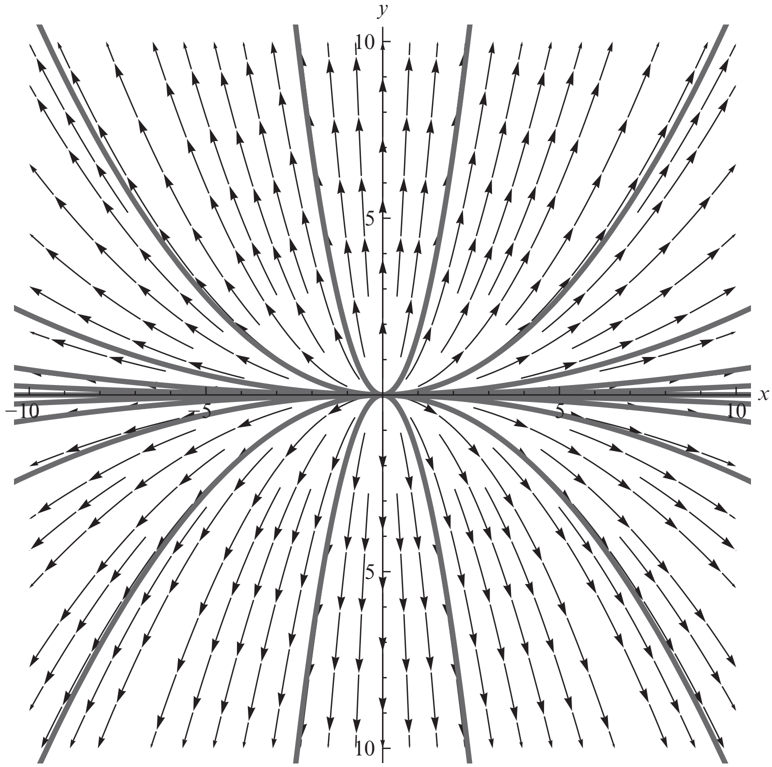

FIGURE 6.12 (a) Phase portrait of $\{x' = x, y' = y\}$. (b) Solutions with direction field of $\{x' = x, y' = y\}$.

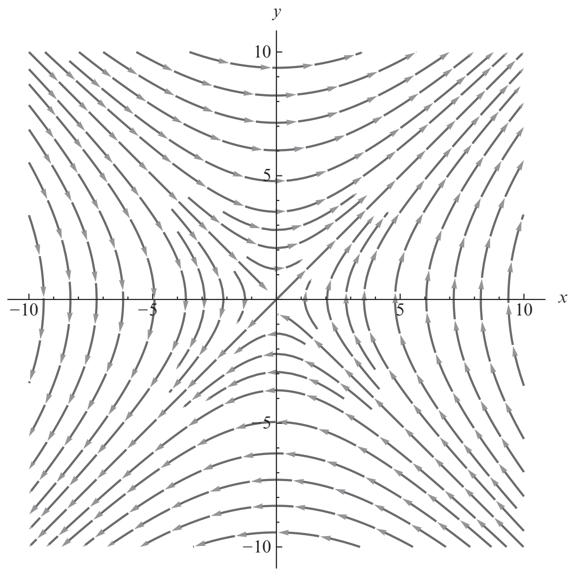

FIGURE 6.13 Phase portrait of $\{x' = y, y' = x\}$.

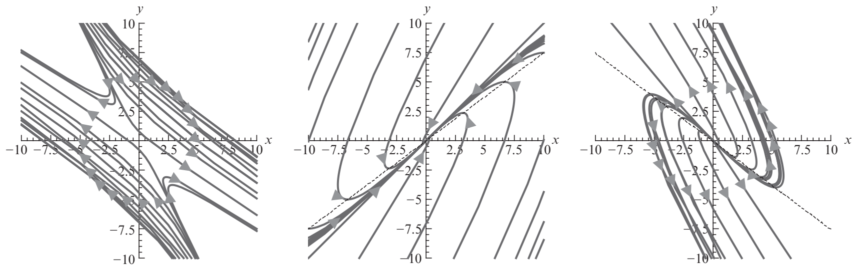

FIGURE 6.14 (a) Phase portrait for Example 6.6.2, solution (a). (b) Phase portrait for Example 6.6.2, solution (b). (c) Phase portrait for Example 6.6.2, solution (c).

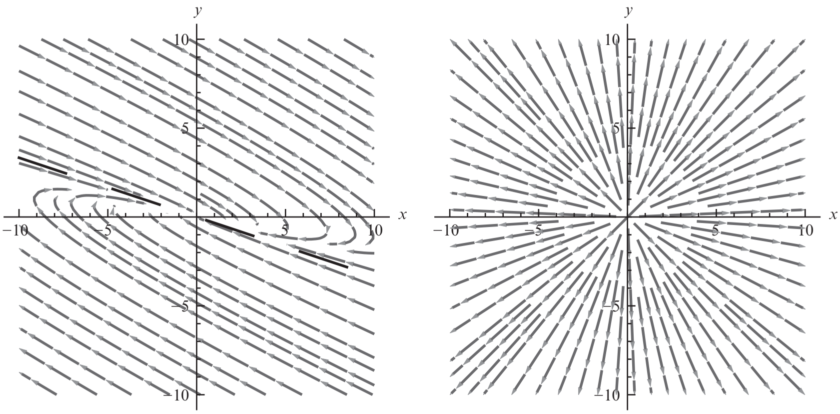

FIGURE 6.15 (a) Phase portrait for Example 6.6.3, solution (a). (b) Phase portrait for Example 6.6.3, solution (b).

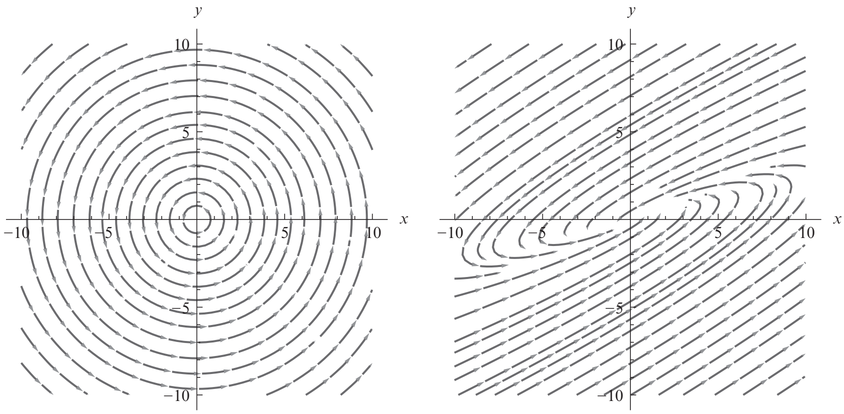

FIGURE 6.16 (a) Phase portrait for Example 6.6.4, solution (a). (b) Phase portrait for Example 6.6.4, solution (b).

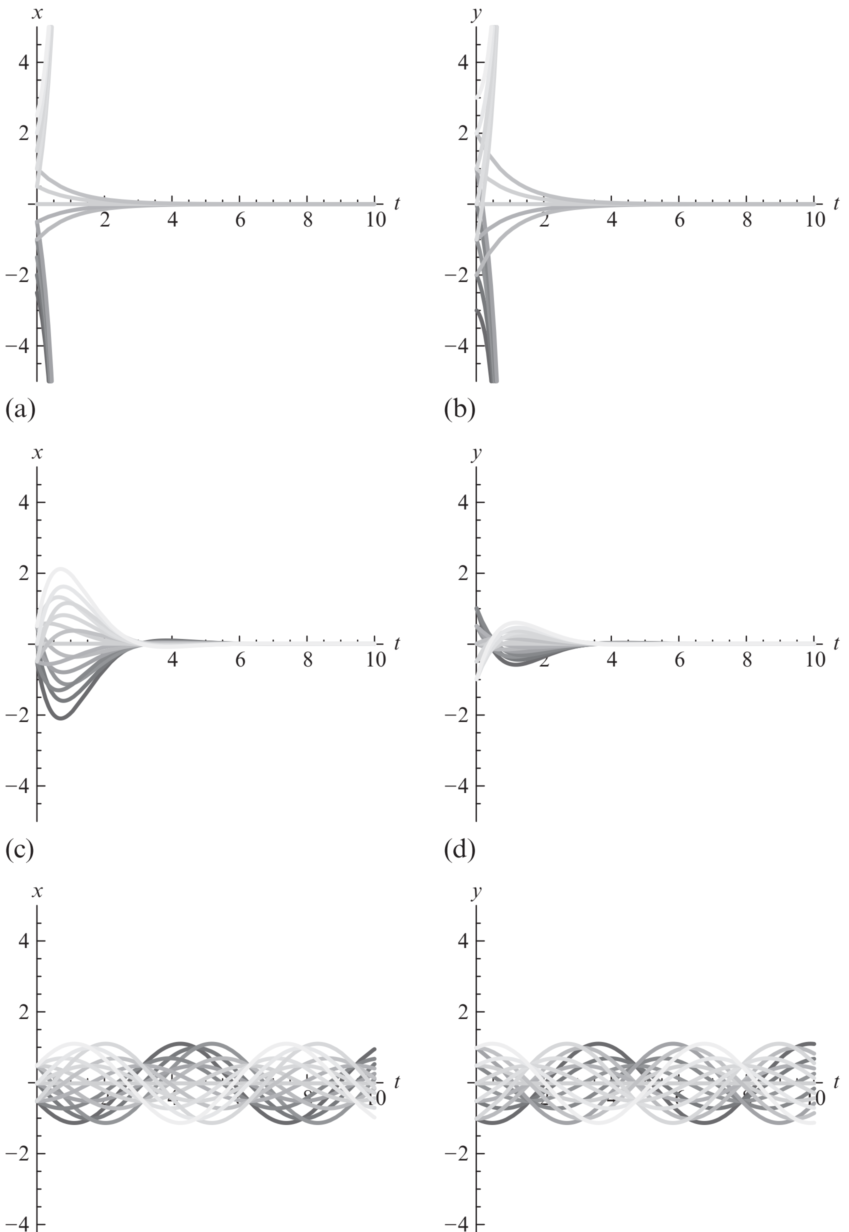

FIGURE 6.17 Graphically illustrating unstable, asymptotically stable, and stable.

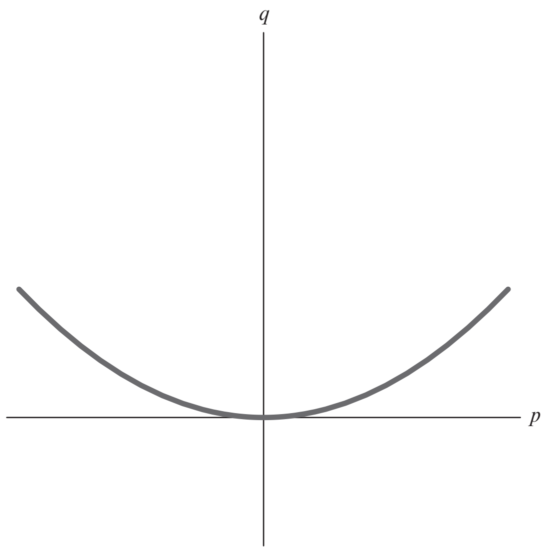

FIGURE 6.18 Graph of $\Delta = p^2 - 4q = 0$.

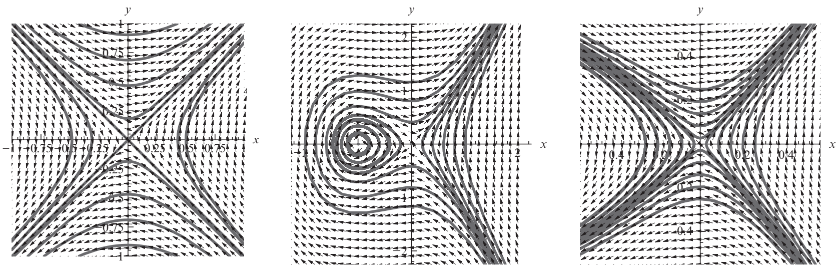

FIGURE 6.19 (a) Trajectories of corresponding linear system with direction field. (b) Trajectories of nonlinear system with direction field. (c) Trajectories of nonlinear system with direction field near the origin.

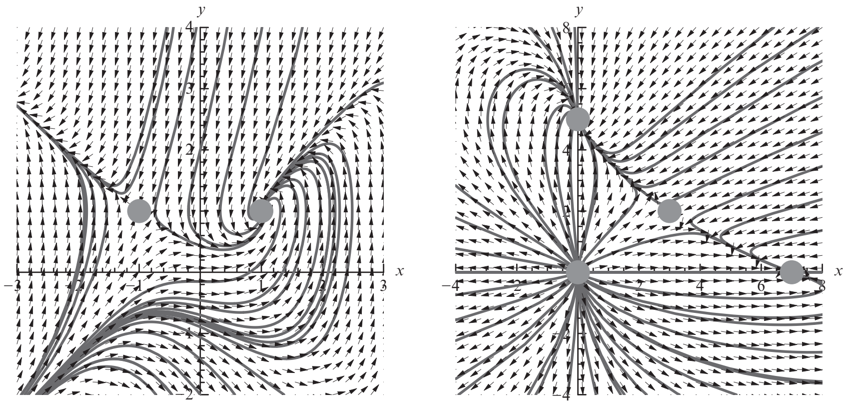

FIGURE 6.20 (a) $(1, 1)$ is a stable spiral and $(-1, 1)$ is a saddle. (b) $(0, 0)$ is an unstable node, $(0, 5)$ is an asymptotically stable improper node, $(7, 0)$ is an asymptotically stable improper node, and $(3, 2)$ is a saddle point.

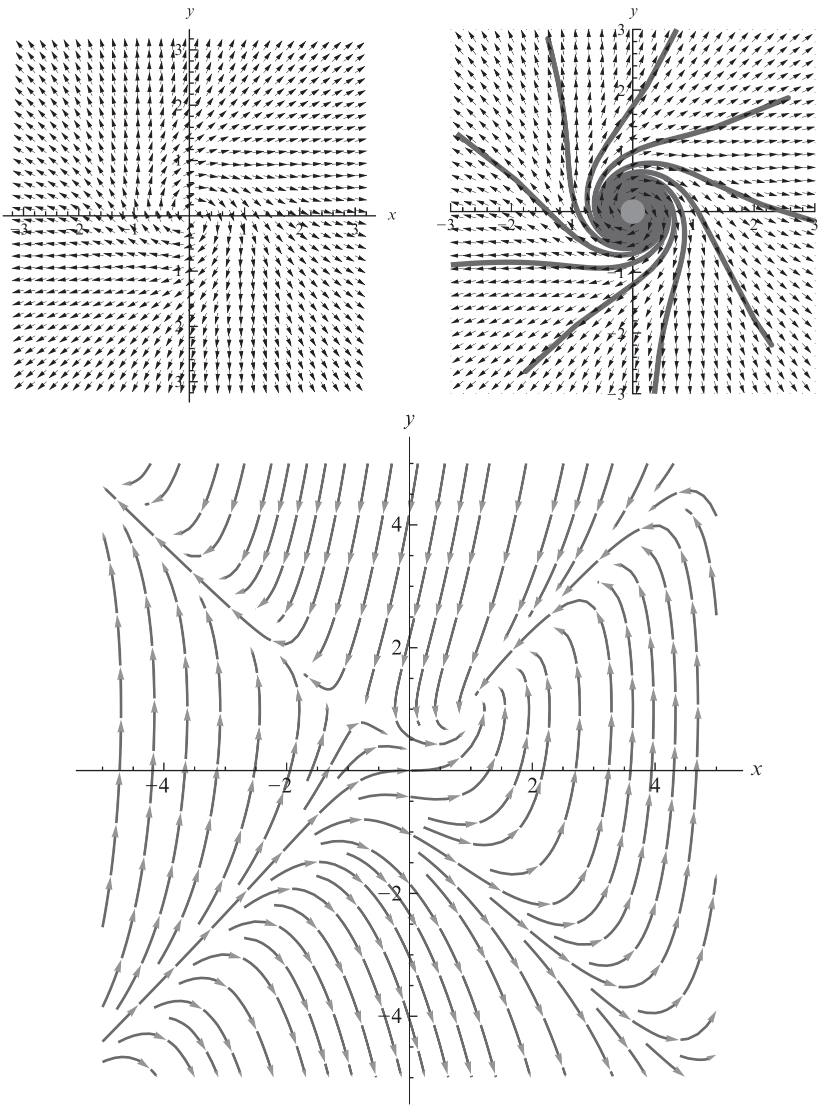

FIGURE 6.21 (a) The direction field indicates that $(0, 0)$ is unstable. (b) All (nontrivial) trajectories spiral away from the origin. (c) Phase portrait.

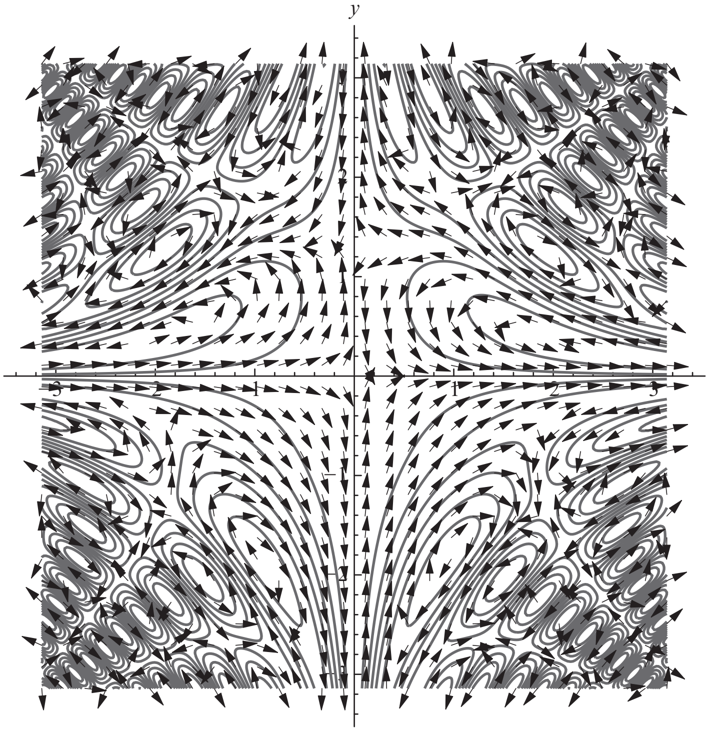

FIGURE 6.22 Direction field for $\mathrm{d}x/\mathrm{d}t = -2xy\cos(x^2y) + y^2\sin(xy^2)$, $\mathrm{d}y/\mathrm{d}t = x^2\cos(x^2y) - 2xy\sin(xy^2)$ along with plots of $H(x, y) = C$ for various values of C.

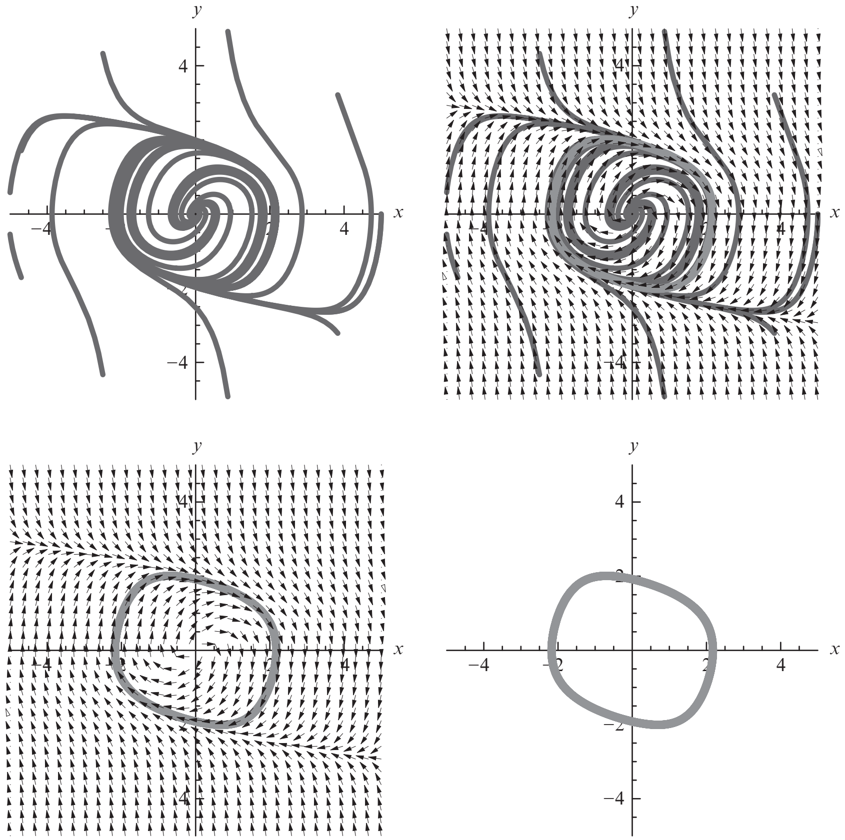

FIGURE 6.23 (a) If $x(0)$ and $y(0)$ are both close to 0, the solutions spiral outward while if $x(0)$ and $y(0)$ are both sufficiently large, the solutions spiral inward. (b) All nontrivial solutions tend to a closed curve, L. (c) An isolated period solution like this is called a *limit cycle*. This limit cycle is stable because all nontrivial solutions spiral into it. (d) Finding L.

TABLE 6.1 Classification of Equilibrium Point in Linear System

Eigenvalues	Geometry	Stability
λ_1, λ_2 real; $\lambda_1 > \lambda_2 > 0$	Improper node	Unstable
λ_1, λ_2 real; $\lambda_1 = \lambda_2 > 0$; 1 eigenvector	Deficient node	Unstable
λ_1, λ_2 real; $\lambda_1 = \lambda_2 > 0$; 2 eigenvectors	Star node	Unstable
λ_1, λ_2 real; $\lambda_2 < \lambda_1 < 0$;	Improper node	Asymptotically stable
λ_1, λ_2 real; $\lambda_1 = \lambda_2 < 0$; 1 eigenvector	Deficient note	Asymptotically stable
λ_1, λ_2 real; $\lambda_1 = \lambda_2 < 0$; 2 eigenvectors	Star node	Asymptotically stable
λ_1, λ_2 real; $\lambda_2 < 0 < \lambda_1$	Saddle point	Unstable
$\lambda_1 = \alpha + \beta i, \lambda_2 = \alpha - \beta i, \beta \neq 0, \alpha > 0$	Spiral point	Unstable
$\lambda_1 = \alpha + \beta i, \lambda_2 = \alpha - \beta i, \beta \neq 0, \alpha < 0$	Spiral point	Asymptotically stable
$\lambda_1 = \beta i, \lambda_2 = -\beta i, \beta \neq 0$	Center	Stable

TABLE 6.2 Classification of Equilibrium Point in Nonlinear System

Eigenvalues of $J(x_0, y_0)$	**Geometry**	**Stability**
λ_1, λ_1 real; $\lambda_1 > \lambda_2 > 0$	Improper node	Unstable
λ_1, λ_2 real; $\lambda_1 = \lambda_2 > 0$	Node or spiral point	Unstable
λ_1, λ_2 real; $\lambda_2 < \lambda_1 < 0$	Improper node	Asymptotically stable
λ_1, λ_2 real; $\lambda_1 = \lambda_2 < 0$	Node or spiral point	Asymptotically stable
λ_1, λ_2 real; $\lambda_2 < 0 < \lambda_1$	Saddle point	Unstable
$\lambda_1 = \alpha + \beta i, \lambda_2 = \alpha - \beta i, \beta \neq 0, \alpha > 0$	Spiral point	Unstable
$\lambda_1 = \alpha + \beta i, \lambda_2 = \alpha - \beta i, \beta \neq 0, \alpha < 0$	Spiral point	Asymptotically stable
$\lambda_1 = \beta i, \lambda_2 = -\beta i, \beta \neq 0$	Center or spiral point	Inconclusive

TABLE 6.3

t_n	x_n (approx)	x_n (exact)	y_n (approx)	y_n (exact)
0.0	0. 0	0. 0	1. 0	1.0
0.1	0. 0	−0.02270	1. 4	1.46032
0.2	−0.04	−0.10335	1. 91048	2.06545
0.3	−0.13505	−0.26543	−2. 5615	2.85904
0.4	−0.30470	−0.54011	13. 39053	3.89682
0.5	−0.57423	−0.96841	4. 44423	5.24975
0.6	−0.97607	−1.60412	5. 78076	7.00806
0.7	−1.55176	−2.51737	7. 47226	9.28638
0.8	−2.35416	−3.79926	9. 60842	12.23
0.9	−3.45042	−5.56767	12. 3005	16.0232
1.0	−4.9255	−7.97468	15. 6862	20.8987

TABLE 6.4

t_n	x_n (approx)	x_n (exact)	y_n (approx)	y_n (exact)
0.0	0.0	0.0	1.0	1.0
0.05	0.0	−0.00532	1.2	1.21439
0.10	−0.01	−0.02270	1.42756	1.46032
0.15	−0.03188	−0.05447	1.68644	1.74321
0.20	−0.06779	−0.10335	1.98084	2.06545
0.25	−0.12023	−0.17247	2.31552	2.43552
0.30	−0.192013	−0.26543	2.69577	2.85904
0.35	−0.28640	−0.38639	3.12758	3.34338
0.40	−0.40710	−0.54011	3.61763	3.89682
0.45	−0.55834	−0.73203	4.17344	4.52876
0.50	−0.74493	−0.96841	4.80342	5.24975
0.55	−0.97234	−1.25639	5.51701	6.07171
0.60	−1.24681	−1.60412	6.32479	7.00806
0.65	−1.57529	−2.02091	7.23861	8.07394
0.70	−1.96609	−2.51737	8.27174	9.28638
0.75	−2.42798	−3.10558	9.43902	10.6645
0.80	−2.97133	−3.79926	10.7571	12.23
0.85	−3.60776	−4.61405	12.2446	14.0071
0.90	−4.35037	−5.56767	13.9222	16.0232
0.95	−5.214	−6.68027	15.8134	18.3088
1.00	−6.21537	−7.97468	17.944	20.8987

TABLE 6.5

t_n	x_n (approx)	x_n (exact)	y_n (approx)	y_n (exact)
0.0	0. 0	0. 0	1. 0	1. 0
0.1	−0.02269	−0.02270	1.46031	1.46032
0.2	−0.10332	−0.10335	2.06541	2.06545
0.3	−0.26538	−0.26543	2.85897	2.85904
0.4	−0.54002	−0.54011	3.8967	3.89682
0.5	−0.96827	−0.96841	5.24956	5.24975
0.6	−1.60391	−1.60412	7.00778	7.00806
0.7	−2.51707	−2.51737	9.28596	9.28638
0.8	−3.79882	−3.79926	12.2294	12.23
0.9	−5.56704	−5.56767	16.0223	16.0232
1.0	−7.97379	−7.97468	20.8975	20.8987

TABLE 6.6

t_n	x_n **(R-K)**	x_n **(linear)**	y_n **(R-K)**	y_n **(linear)**
0.0	0.0	0.0	1.0	1.0
0.1	0.09983	0.09983	0.99500	0.99500
0.2	0.19867	0.198669	0.98013	0.98007
0.3	0.29553	0.29552	0.95566	0.95534
0.4	0.38950	0.389418	0.922061	0.92106
0.5	0.47966	0.47943	0.87994	0.87758
0.6	0.56523	0.56464	0.83002	0.82534
0.7	0.64544	0.64422	0.77309	0.764842
0.8	0.71964	0.71736	0.70999	0.69671
0.9	0.78726	0.78333	0.641545	0.62161
1.0	0.84780	0.84147	0.568569	0.54030