

FIGURE 8.1 Graph of $\mathcal{U}(t-a)$.

(a)
(b)

FIGURE 8.2 (a) Graph of $\mathcal{U}(t-5)$. (b) Graph of $\mathcal{U}(t)$.


FIGURE 8.3 (a) Graph of a function $h(t)$. (b) $h(t) \mathcal{U}(t-a)$. (c) $h(t) \mathcal{U}(t-b)$. (d) $g(t)=h(t) \mathcal{U}(t-a)-h(t) \mathcal{U}(t-b)=h(t) \mathcal{U}(t-$ a) $-\mathcal{U}(t-b)$. The graph of the function obtained by subtracting $h(t) \mathcal{U}(t-b)$ from $h(t) \mathcal{U}(t-a)$ is the graph of the function $h(t)$ between $a$ and $b$.

(a)

(b)

(c)

FIGURE 8.4 (a) Graph of $g(t)$ in black and $h(t)$ in gray. (b) Graph of $g(t)[\mathcal{U}(t-a)-\mathcal{U}(t-b)]$ in black and $h(t)[\mathcal{U}(t-a)-\mathcal{U}(t-b)]$ in gray. (c) Graph of $g(t)[\mathcal{U}(t-a)-\mathcal{U}(t-b)]+h(t)[\mathcal{U}(t-a)-\mathcal{U}(t-b)]$.


FIGURE 8.5 Graphs of three solutions of $y^{\prime \prime}+9 y=\mathcal{U}(t)-\mathcal{U}(t-\pi)$ that satisfy $y(0)=0$. Which one is the graph of the solution that satisfies $y^{\prime}(0)=0$ ?


FIGURE 8.6 Although $f$ is discontinuous, its Laplace transform is continuous for $s>0$.


FIGURE 8.7 The half-wave rectification of $2 \sin t$.


FIGURE 8.8 Even though the forcing function is discontinuous, the solution to the initial-value problem is continuous.


FIGURE 8.9 The area under the graph of $y=\delta_{\alpha}\left(t-t_{0}\right)$ is independent of $\alpha$ and $t_{0}$ : the area is 1 .


FIGURE 8.10 (a) Shows the graph of the solution in the example. In (b), (c), and (d), the forcing functions are $f(t)=$ $-3 \sin t \delta(t-\pi / 2), f(t)=-3 \sin t(\delta(t-\pi / 2)+\delta(t-3 \pi / 2))$, and $f(t)=3 \sin t(\delta(t-\pi / 2)+\delta(t-3 \pi / 2))$. Explain how to match each solution graph with its forcing function.


FIGURE 8.11 Observe that $R=\{(t, v): v \leq t<\infty, 0 \leq v<\infty\}=\{(t, v): 0 \leq v \leq t, 0 \leq t<\infty\}$.


FIGURE 8.12 (a) Graph of $x(t)$ and $y(t)$ for $0 \leq t \leq 8 \pi$. (b) Graph of $(x(t), y(t))$ for $0 \leq t \leq 8 \pi$.


FIGURE 8.13 (a) Graph of $x(t)$ and $y(t)$ for $0 \leq t \leq 10$. (b) Graph of $(x(t), y(t))$ for $0 \leq t \leq 10$.


FIGURE 8.14 (a) Graph of $x(t)$ and $y(t)$ for $0 \leq t \leq 5 \pi / 2$. (b) Graph of $(x(t), y(t))$ for $0 \leq t \leq 5 \pi / 2$.

(a)

FIGURE 8.15 (a) $L-R-C$ circuit, (b) $L-R$ circuit, and (c) an $R-C$ circuit with the property that $R=C=1$.


FIGURE 8.16 (a) $a=1$, (b) $a=10$, (c) $a=20$, and (d) $a=50$.


FIGURE 8.17 (a) The graph of $y(t)$. (b) The graph of $y(t)$ together with the graph of the solution to the problem that does not include a delay, $z(t)$.


FIGURE 8.18 (a) A coupled spring-mass system. (b) Force diagram for a coupled spring-mass system.


FIGURE 8.19 (a) simultaneous plot and (b) parametric plot.


FIGURE 8.20 (a) $t=0$, (b) $t=1 / 2$, (c) $t=1$, (d) $t=3 / 2$, (e) $t=2$, (f) $t=5 / 2$, (g) $t=3$, (h) $t=7 / 2$, and (i) $t=4$.


FIGURE 8.21 Applying external forces to a spring-mass system.


FIGURE 8.22 A double pendulum.


FIGURE 8.23 (a) simultaneous plot; and (b) parametric plot.


FIGURE 8.24 (a) $t=0$, (b) $t=5 / 4$, (c) $t=5 / 2$, (d) $t=15 / 4$, (e) $t=5$, (f) $t=25 / 4$, (g) $t=15 / 2$, (h) $t=35 / 4$, and (i) $t=10$.


FIGURE 8.25 See Exercise 17.


FIGURE 8.26 Two pendulums coupled with a spring.


FIGURE 8.27 Two objects connected by three springs.


FIGURE 8.28 A mass connected to a bar that is connected to its support by two springs.


FIGURE 8.29 Two pairs of three masses connected by three springs.


FIGURE 8.30 The tautochrone.


FIGURE 8.31 (a) The principal system and the vibration absorber attached to the principal system. (b) A simple model of an airplane.


FIGURE 8.32 Diagram used to model the sway of a three-story building.

TABLE 8.1 Laplace Transforms of Frequently Encountered Functions

| $f(t)$ | $F(s)=\mathcal{L}(f(t))$ | $f(t)$ | $F(s)=\mathcal{L}(f(t))$ |
| :--- | :--- | :--- | :--- |
| 1 | $\frac{1}{s}, s>0$ | $t^{n}, n=1,2, \ldots$ | $\frac{n!}{s^{n+1}, s>0}$ |
| $\mathrm{e}^{a t}$ | $\frac{1}{s-a}, s>a$ | $t^{n} \mathrm{e}^{a t}, n=1,2, \ldots$ | $\frac{n!}{(s-a)^{n+1}}$ |
| $\sin k t$ | $\frac{k}{s^{2}+k^{2}}$ | $\mathrm{e}^{a t} \sin k t$ | $\frac{k}{(s-a)^{2}+k^{2}}$ |
| $\cos k t$ | $\frac{s}{s^{2}+k^{2}}$ | $\mathrm{e}^{a t} \cos k t$ | $\frac{s-a}{(s-a)^{2}+k^{2}}$ |
| $\sinh k t$ | $\frac{k}{s^{2}-k^{2}}$ | $\mathrm{e}^{a t} \sinh k t$ | $\frac{k}{(s-a)^{2}-k^{2}}$ |
| $\cosh k t$ | $\frac{s}{s^{2}-k^{2}}$ | $\mathrm{e}^{a t} \cosh k t$ | $\frac{s-a}{(s-a)^{2}-k^{2}}$ |
| $t^{n} f(t), n=1,2, \ldots$ | $(-1)^{n} \frac{\mathrm{~d}^{n}}{\mathrm{~d} s^{n}} \mathcal{L}\{f(t)\}=(-1)^{n} \frac{\mathrm{~d}^{n} F}{\mathrm{~d} s^{n}}(s)$ |  |  |

