CHAPTER 4 PROBLEMS AND EXERCISES

Problem 1: Find the first, second, and third natural frequencies of in-plane axial vibration of a circular aluminum plate of thickness 0.8 mm, diameter 100 mm, modulus E = 70 GPa, Poisson ratio $\nu = 0.33$, and density $\rho = 2700$ kg/m^3

Solution

Frequency expression is given by Eq. (4.52) in conjunction with Eqs. (4.29), (4.51):

$$f_j = \frac{1}{2\pi}\frac{c_L}{a} z_j \quad , \qquad j = 1, 2, 3, \ldots \tag{4.52}$$

$$c_L^2 = \frac{E}{\rho(1-\nu^2)} \qquad \text{(axial wave speed in a plate)} \tag{4.29}$$

$$z = 2.067364; \quad 5.395106; \quad 8.575401; \quad 11.734360 \ldots \tag{4.51}$$

Upon calculations, we get $c_L = 5394$ m/s and $f_1 = 35.5$ kHz, $f_2 = 92.6$ kHz, $f_3 = 147.2$ kHz

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Problem 2: Find all the natural frequencies in the interval 10 kHz to 40 kHz of in-plane axial vibration of a circular aluminum plate of thickness 0.8 mm, diameter 100 mm, modulus E = 70 GPa, Poisson ratio $\nu = 0.33$, and density $\rho = 2700$ kg/m^3

Solution

Frequency expression is given by Eq. (4.52) in conjunction with Eqs. (4.29), (4.51):

$$f_j = \frac{1}{2\pi}\frac{c_L}{a} z_j \qquad j = 1, 2, 3, \ldots \tag{4.52}$$

$$c_L^2 = \frac{E}{\rho(1-\nu^2)} \qquad \text{(axial wave speed in a plate)} \tag{4.29}$$

$$z = 2.067364; \quad 5.395106; \quad 8.575401; \quad 11.734360 \ldots \tag{4.51}$$

Upon calculations, we get $c_L = 5394$ m/s and $f_1 = 35.5$ kHz, $f_2 = 92.6$ kHz, $f_3 = 147.2$ kHz

Of these frequencies, only one resides in the interval 10 kHz to 40 kHz, i.e., $f_1 = 35.5$ kHz.

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SOLUTION Problems 4.1 and 4.2

$ORIGIN := 1$

$h := 1 \cdot 10^{-3}$ $\quad a := \frac{1}{2} \cdot 100 \cdot 10^{-3}$ $\quad E := 70 \cdot 10^{9}$ $\quad \rho := 2700$ $\quad \nu := 0.33$

$cL := \sqrt{\frac{E}{\rho \cdot \left(1 - \nu^2\right)}}$ $\quad cL = 5394$

$n := 1 .. 4$

$\gamma a_1 := 2.067364$ $\quad \gamma a_2 := 5.395106$ $\quad \gamma a_3 := 8.575401$ $\quad \gamma a_4 := 11.734360$

$c := cL$ $\quad f_n := \frac{1}{2 \cdot \pi} \cdot \frac{c}{a} \cdot \gamma a_n$ $\quad f_n =$

$$\begin{array}{|r|} \hline 35.5 \\ \hline 92.6 \\ \hline 147.2 \\ \hline 201.5 \\ \hline \end{array} \quad 10^3$$

Problem 3: Find the first, second, and third natural frequencies of out-of-plane flexural vibration of a circular plate aluminum of thickness 0.8 mm, diameter 100 mm, modulus E = 70 GPa, Poisson ratio $\nu = 0.33$, and density $\rho = 2700$ kg/m^3

<u>Solution</u>

Frequency expression is given by Eq. (4.209) in conjunction with Eqs. (4.108) and (4.208):

$$f_j = \frac{\lambda_j^2}{2\pi}\sqrt{\frac{D}{\rho h a^4}} \qquad j = 1, 2, 3, \ldots \tag{4.209}$$

$$D = \frac{Eh^3}{12(1-\nu^2)} \tag{4.108}$$

$$\lambda = 3.011461, \quad 6.205398, \quad 9.370844, \quad 12.525181, \; \ldots \tag{4.208}$$

where a is the radius of the circular plate. In addition, we recall that $f = \omega / 2\pi$, and hence Eq. (4.209) becomes

$$f_j = \frac{\lambda_j^2}{2\pi}\sqrt{\frac{D}{\rho h a^4}} \qquad j = 1, 2, 3, \ldots$$

Upon calculations, we get $D = 3.352 \text{ Nm}$ and $f_1 = 0.719 \text{ kHz}$, $f_2 = 3.054 \text{ kHz}$, $f_3 = 6.964 \text{ kHz}$.

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SOLUTION Problems 4.3 and 4.4

$\mathrm{ORIGIN} := 1$

$h := 0.8 \cdot 10^{-3}$ $\quad a := \frac{1}{2} \cdot 100 \cdot 10^{-3}$ $\quad E := 70 \cdot 10^{9}$ $\quad \rho := 2700$ $\quad \nu := 0.33$

$D := \frac{E \cdot h^3}{12 \cdot \left(1 - \nu^2\right)}$ $\quad D = 3.352$

$j := 1 .. 8$ $\quad \lambda := \begin{pmatrix} 3.011461 \\ 6.205398 \\ 9.370844 \\ 12.525181 \\ 15.674661 \\ 18.821611 \\ 21.967077 \\ 25.111601 \end{pmatrix}$

$$f_j := \frac{1}{2 \cdot \pi} \cdot \left(\frac{D}{\rho \cdot h \cdot a^4}\right)^{\frac{1}{2}} \cdot \left(\lambda_j\right)^2$$

$f_j =$

0.719	10^3
3.054	
6.964	
12.441	
19.484	
28.093	
38.267	
50.007	

$ORIGIN := 1$

$TOL := 10^{-12}$

$h := 1 \cdot 10^{-3}$ $\quad a := \frac{1}{2} \cdot 7 \cdot 10^{-3}$ $\quad E := 65.4 \cdot 10^{9}$ $\quad \rho := 7700$ $\quad \nu := 0.35$

$cL := \sqrt{\frac{E}{\rho \cdot (1 - \nu^2)}}$ $\quad cL = 3111$

$F(x) := 1 - \frac{(1 - \nu)}{x} \cdot \frac{J1(x)}{J0(x)}$

$j := 1 .. 8$

find roots of the eigenvalue equation

$F(x) := x \cdot J0(x) - (1 - \nu) \cdot J1(x)$

$x := 2 \quad z_1 := root(F(x), x) \quad z_1 = 2.07950762929693 \quad F(z_1) = 0.000000000000000$

$x := 5 \quad z_2 := root(F(x), x) \quad z_2 = 5.39892763320741 \quad F(z_2) = -6.10622663543836 \times 10^{-15}$

$x := 8 \quad z_3 := root(F(x), x) \quad z_3 = 8.57776133292638 \quad F(z_3) = 0.000000000000102$

$x := 12 \quad z_4 := root(F(x), x) \quad z_4 = 11.7360756926028 \quad F(z_4) = -5.10702591327572 \times 10^{-15}$

$x := 15 \quad z_5 := root(F(x), x) \quad z_5 = 14.8872198440198 \quad F(z_5) = -0.000000000000428$

$x := 18 \quad z_6 := root(F(x), x) \quad z_6 = 18.0350025454700 \quad F(z_6) = -0.000000000000268$

$x := 21 \quad z_7 := root(F(x), x) \quad z_7 = 21.1809360637258 \quad F(z_7) = 0.000000000000000$

$x := 24 \quad z_8 := root(F(x), x) \quad z_8 = 24.3257425526341 \quad F(z_8) = -2.65065747129256 \times 10^{-15}$

$x := 27 \quad z_9 := root(F(x), x) \quad z_9 = 27.4698110185590 \quad F(z_9) = -2.33146835171283 \times 10^{-15}$

$c := cL$ $\qquad f_j := \frac{1}{2\cdot\pi}\cdot\frac{c}{a}\cdot z_j$ $\qquad f_j =$

294.2	10^3
763.8	
1213.5	
1660.3	
2106.1	
2551.5	
2996.5	
3441.4	

Problem 4: Find all the natural frequencies in the interval 10 kHz to 40 kHz of out-of-plane flexural vibration of a circular aluminum plate of thickness 0.8 mm, diameter 100 mm, modulus E = 70 GPa, Poisson ratio $\nu = 0.33$, and density $\rho = 2700$ kg/m^3

<u>Solution</u>

Frequency expression is given by Eq. (4.209) in conjunction with Eqs. (4.108) and Table 3.3:

$$f_j = \frac{\lambda_j^2}{2\pi}\sqrt{\frac{D}{\rho h a^4}} \qquad j = 1, 2, 3, \ldots \tag{4.209}$$

$$D = \frac{Eh^3}{12(1-\nu^2)} \tag{4.108}$$

where a is the radius of the circular plate. In addition, we recall that $f = \omega / 2\pi$, and hence Eq. (4.209) becomes

$$f_j = \frac{\lambda_j^2}{2\pi}\sqrt{\frac{D}{\rho h a^4}} \qquad j = 1, 2, 3, \ldots$$

The values of λ_j are taken from Table 3.3. Upon calculations, we get $D = 3.352\ \text{Nm}$; the frequencies are $f_1 = 0.719\ \text{kHz}$, $f_2 = 3.054\ \text{kHz}$, $f_3 = 6.964\ \text{kHz}$, $f_4 = 12.441\ \text{kHz}$, $f_5 = 19.484\ \text{kHz}$, $f_6 = 28.093\ \text{kHz}$, $f_7 = 38.267\ \text{kHz}$, $f_8 = 50.007\ \text{kHz}$. We note that only f_4, f_5, f_6, f_7 frequencies fall within the interval 10—40 kHz.

Hence, the answer to the problem is as follows: The natural frequencies in the interval 10 kHz to 40 kHz of out-of-plane flexural vibration are $f_4 = 12.441\ \text{kHz}$, $f_5 = 19.484\ \text{kHz}$, $f_6 = 28.093\ \text{kHz}$, $f_7 = 38.267\ \text{kHz}$.

Problem 5: Find the first, second, and third natural frequencies of in-plane axial vibration of a circular plate made of piezoceramic material with thickness 0.2 mm, diameter 7 mm, modulus $E = 65.4$ GPa, Poisson ratio $\nu = 0.35$, and density $\rho = 7700$ kg/m^3

<u>Solution</u>

Frequency expression is given by Eq. (4.52) in conjunction with Eqs. (4.29), (4.48):

$$f_j = \frac{1}{2\pi}\frac{c_L}{a} z_j \quad , \qquad j = 1, 2, 3, \ldots \tag{4.52}$$

$$c_L^2 = \frac{E}{\rho(1-\nu^2)} \qquad \text{(axial wave speed in a plate)} \tag{4.29}$$

$$zJ_0(z) - (1-\nu)J_1(z) = 0 \tag{4.48}$$

One has to first find the roots of Eq. (4.48) for $\nu = 0.35$ (these roots are different from the roots listed in Eq. (4.51), which correspond to $\nu = 0.33$). Upon calculations, we get

$$z = 2.079508; \quad 5.398928; \quad 8.577761; \quad 11.736076 \ldots$$

Then, one uses Eq. (4.29) to get $c_L = 3111$ m/s; upon substitution into Eq. (4.52), one gets $f_1 = 294.2$ kHz, $f_2 = 763.8$ kHz, $f_3 = 1213.5$ kHz

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Problem 6: Find all the natural frequencies in the interval 100 kHz to 2 MHz of in-plane axial vibration of a circular plate made of piezoceramic material with thickness 0.2 mm, diameter 7 mm, modulus $E = 65.4\text{ GPa}$, Poisson ratio $\nu = 0.35$, and density $\rho = 7700\text{ kg/m}^3$

Solution

Frequency expression is given by Eq. (4.52) in conjunction with Eqs. (4.29), (4.48):

$$f_j = \frac{1}{2\pi}\frac{c_L}{a} z_j \quad , \qquad j = 1, 2, 3, \ldots \tag{4.52}$$

$$c_L^2 = \frac{E}{\rho(1-\nu^2)} \qquad \text{(axial wave speed in a plate)} \tag{4.29}$$

$$zJ_0(z) - (1-\nu)J_1(z) = 0 \tag{4.48}$$

One has to first find the roots of Eq. (4.48) for $\nu = 0.35$ (these roots are different from the roots listed in Eq. (4.51), which correspond to $\nu = 0.33$). We find more roots than in previous problem because we need to get beyond 2,000 kHz. Upon calculations, we get

$z =$ 2.079508; 5.398928; 8.577761; 11.736076, 14.887220, 18.035003…

Then, one uses Eq. (4.29) to get $c_L = 3111\text{ m/s}$; upon substitution into Eq. (4.52), one gets $f_1 = 294.2\text{ kHz}$, $f_2 = 763.8\text{ kHz}$, $f_3 = 1213.5\text{ kHz}$, $f_4 = 1660.3\text{ kHz}$, $f_5 = 2106.1\text{ kHz}$

Of these frequencies, only three reside in the interval 100 kHz to 2MHz = 2,000 kHz, i.e., $f_1 = 294.2\text{ kHz}$, $f_2 = 763.8\text{ kHz}$, $f_3 = 1213.5\text{ kHz}$,.