CHAPTER 7 PROBLEMS AND EXERCISES

Problem 1: Consider a PWAS transducer with length $l = 7\ \text{mm}$, width $b = 1.65\ \text{mm}$, thickness $t = 0.2\ \text{mm}$, and material properties as given in Table 7.1.

(i) Find the free capacitance, C, of the PWAS.

(ii) Calculate the voltage, V, measured at PWAS terminals by an instrument with input capacitance $C_e = 1\ \text{pF}$ when the applied strain is $S_1 = -1000\ \mu\varepsilon$.

(iii) Plot the variation of measured voltage with instrument capacitance $C_e = 0.1\ ...\ 10\ \text{nF}$ when $S_1 = -1000\ \mu\varepsilon$ (use log scale for C_e axis).

(iv) Plot the variation of measured voltage with strain in the range $S_1 = 0 ... -1000\ \mu\varepsilon$; make a carpet plot of V vs. S_1 for various values of C_e (0.1 nF, 1 nF, 10 nF).

(v) Extend to dynamic strain: calculate the voltage V for dynamic strain of amplitude $\hat{S}_1 = 1000\ \mu\varepsilon$ with frequency f = 100 kHz (take $C_e = 1\ \text{pF}$)

Solution

(i) Recall the free capacitance C of the PWAS, Eq. (7.7) of the textbook, i.e.,

$$C = \varepsilon_{33}^{T} \frac{A}{t} \tag{1}$$

Note that the A is the PWAS surface area, i.e., $A = bl = 11.55\ \text{mm}^2$. Upon calculation, Eq. (1) yields $C = 0.894\ \text{nF}$.

(ii) To calculate the voltage V measured at PWAS terminals by an instrument with a given input capacitance C_e, consider the measuring circuit depicted in Figure 7.5 of the textbook Chapter7 in which the PWAS transducer is connected in to the instrument input capacitance C_e ., Elementary analysis yields the formula of Eq. (7.46) of Section 7.3.2, i.e.,

$$V(S_1, C_e) = \frac{1}{C_e + C\left(1 - k_{31}^2\right)} \frac{Ad_{31}}{s_{11}^{T}} S_1 \tag{2}$$

It is important to notice that the output voltage depends on both the applied strain S_1 and the instrument input capacitance C_e. For a given strain, the maximum output voltage is obtained when the instrument input capacitance approaches zero (i.e., infinite capacitive impedance at the instrument input). For the numerical case $S_1 = -1000\ \mu\varepsilon$ and $C_e = 1\ \text{pF}$ one gets $V = 169\ \text{V}$.

(iii)To plot the variation of measured voltage with instrument capacitance $C_e = 0.1\ ...\ 10\ \text{nF}$ when $S_1 = -1000\ \mu\varepsilon$, use Eq. (2) with the value of S_1 fixed at $S_1 = -1000\ \mu\varepsilon$, as shown in Figure 1 below.

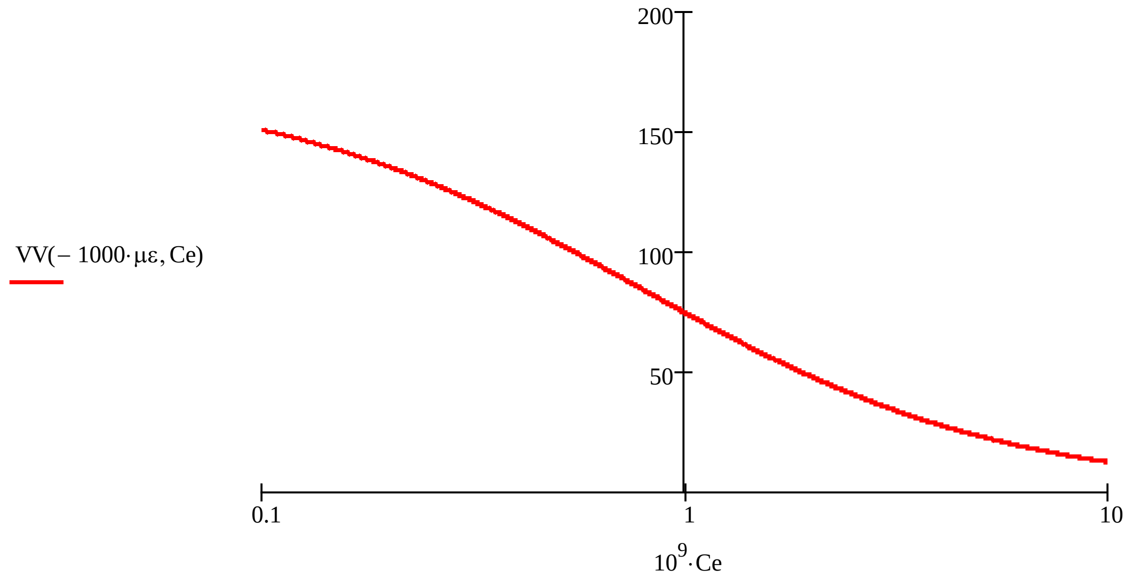


Figure 1 Variation of measured voltage with instrument capacitance C_e for a constant applied strain $S_1 = -1000\ \mu\varepsilon$.

(iv) To plot the variation of measured voltage with strain and to make a carpet plot of V vs. S_1 for various values of C_e, use Eq. (2), give to C_e the values 0.1 nF, 1 nF, 10 nF, and plot them overlapped. One obtains the plot in Figure 2 below.

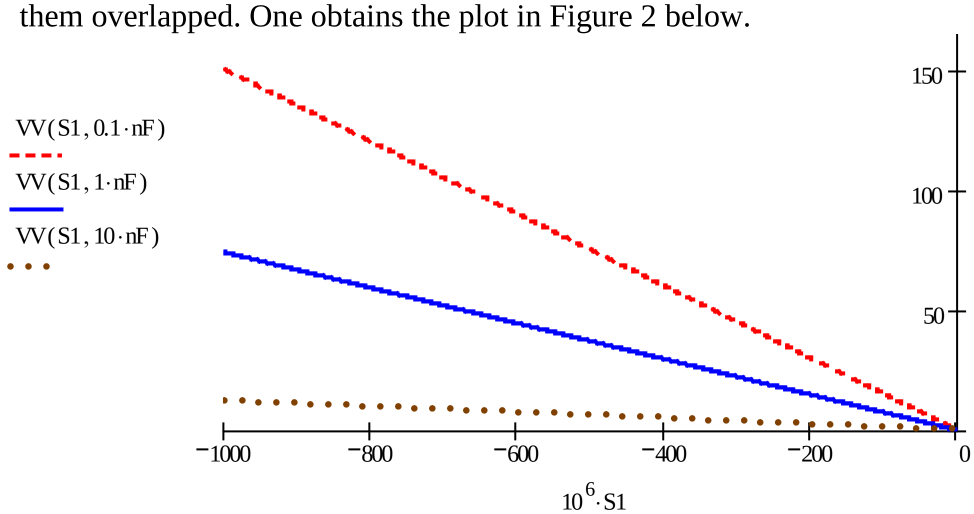


Figure 2 Variation of measured voltage with strain for various values of C_e: 0.1 nF, 1 nF, 10 nF

(v) To extend the analysis to dynamic strain $S_1(t) = \hat{S}_1 e^{i\omega t}$, recall the analysis of Chapter 7, Section 7.3.4 and Eq. (7.71), i.e.,

$$V(t) = A\frac{d_{31}}{s_{11}^T}\frac{1}{Y_e + Y_0\left(1-k_{31}^2\right)}\dot{S}_1(t) \tag{3}$$

where $Y(\omega) = i\omega C$ is the PWAS admittance and $Y_e(\omega) = i\omega C_e$ is the instrument admittance. The strain rate $\dot{S}_1(t)$ is calculate as

$$\dot{S}_1(t) = i\omega \hat{S}_1 e^{i\omega t} = i\omega S_1(t) \tag{4}$$

Substitution of Eq. (4) into Eq. (3) yields

$$V(t) = A\frac{d_{31}}{s_{11}^T}\frac{1}{Y_e + Y_0\left(1-k_{31}^2\right)} i\omega S_1(t) \tag{5}$$

or, in terms of amplitudes,

$$\hat{V} = i\omega A\frac{d_{31}}{s_{11}^T}\frac{1}{Y_e + Y_0\left(1-k_{31}^2\right)}\hat{S}_1 \tag{6}$$

At a frequency $f = 100\ \text{kHz}$, the PWAS admittance is $Y = 562\ \mu\text{S}$. The admittance of an instrument of capacitance $C_e = 1\ \text{pF}$ is $Y_e = 0.628\ \mu\text{S}$. For dynamic strain of amplitude $\hat{S}_1 = 1000\ \mu\varepsilon$, Eq. (6) yields the voltage amplitude $\hat{V} = 169\ \text{V}$.

PROBLEM 7.1 SOLUTION

$kHz := 1000 \cdot Hz \quad pF := 10^{-12} \cdot F \quad mm = 1 \times 10^{-3}\,m \quad nF := 10^{-9} \cdot F \quad \mu\varepsilon := 10^{-6} \quad ..$

Given: $L := 7 \cdot mm \quad b := 1.65 \cdot mm \quad t := 0.2 \cdot mm$

$\varepsilon 0 := 8.85 \cdot 10^{-12} \cdot \frac{F}{m} \quad \varepsilon 33 := 1750 \cdot \varepsilon 0 \quad s11 := 15.3 \cdot \frac{10^{-12}}{Pa} \quad d31 := -175 \cdot 10^{-12} \cdot \frac{m}{V} \quad k31 := 0.36$

Solution: $\varepsilon 33 = 15.487 \frac{nF}{m}$

1. Free capacitance $A := b \cdot L \quad A = 11.55\,mm^2 \quad C := \varepsilon 33 \cdot \frac{A}{t} \quad C = 0.894\,nF$

2. General expression of voltage at PWAS terminals:

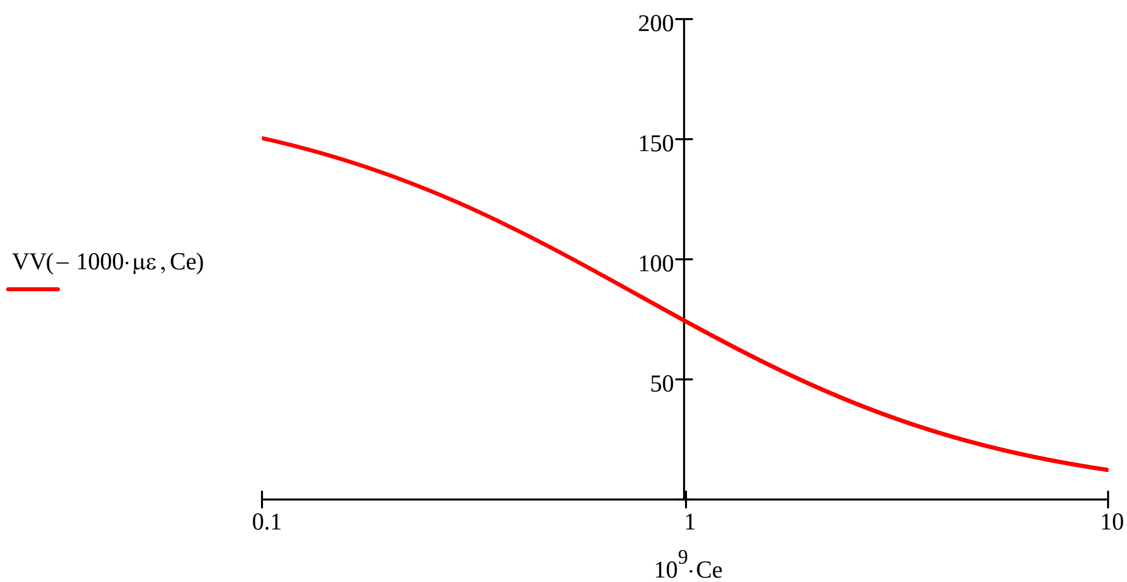
$$VV(S1, Ce) := \frac{A \cdot \frac{d31}{s11}}{Ce + C \cdot \left(1 - k31^2\right)} \cdot S1 \qquad d31 = -1.75 \times 10^{-10} \frac{m}{V}$$

3. Voltage for given Ce and S1

$$VV(-1000 \cdot \mu\varepsilon, 1 \cdot pF) = 169\,V$$

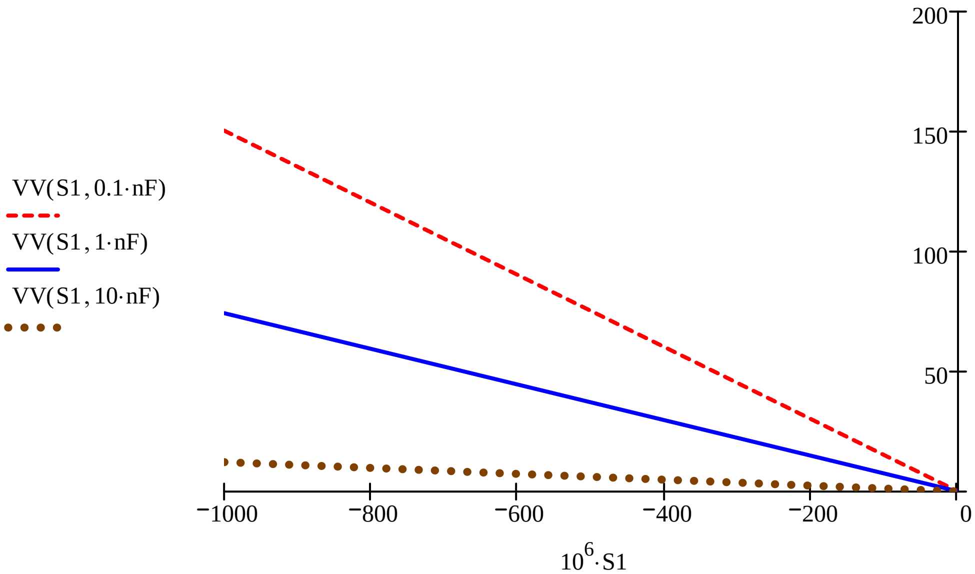
4. Voltage vs. instrument capacitance:

$Ce\_min := 0.1 \cdot nF \quad Ce\_max := 10 \cdot nF \quad \delta Ce := 1 \cdot pF \quad Ce := Ce\_min, Ce\_min + \delta Ce .. Ce\_max$



5. Voltage vs. strain for various instrument capacitance

$\mathrm{S1\_min} := -1000\cdot\mu\varepsilon \qquad \mathrm{S1\_max} := 0 \qquad \delta\mathrm{S1} := 1\cdot\mu\varepsilon \qquad \mathrm{S1} := \mathrm{S1\_min}, \mathrm{S1\_min} + \delta\mathrm{S1} \,..\, \mathrm{S1\_max}$



- What happens if the strain is dynamic? Calculate the voltage V for $S_1 = 1000$ microstrain amplitude with frequency f = 100 kHz? (take $C_e = 1$ pF)

$\mathrm{Ce} := 1\cdot\mathrm{pF}$

$f := 100\mathrm{kHz} \qquad \omega := 2\cdot\pi\cdot f \qquad \omega = 628\,10^{3}\cdot\frac{\mathrm{rad}}{\mathrm{s}}$

$\mathrm{S1} := -1000\cdot\mu\varepsilon$

$\mathrm{Ye} := i\cdot\omega\cdot\mathrm{Ce} \qquad |\mathrm{Ye}| = 0.628\,10^{-6}\cdot\mathrm{S}$

$\mathrm{Y} := i\cdot\omega\cdot\mathrm{C} \qquad |\mathrm{Y}| = 562\,10^{-6}\cdot\mathrm{S}$

$\mathrm{S1dot} := i\cdot\omega\cdot\mathrm{S1} \qquad \mathrm{S1dot} = -628.319i\,\mathrm{Hz}$

$\mathrm{d31} = -1.75\times10^{-10}\,\frac{\mathrm{m}}{\mathrm{V}} \qquad \mathrm{s11} = 15.3\,10^{-12}\cdot\frac{1}{\mathrm{Pa}} \qquad \mathrm{k31}^2 = 0.13$

$$\mathrm{VV} := \mathrm{S1dot}\cdot\mathrm{d31}\cdot\frac{A}{\mathrm{s11}}\cdot\left[\frac{1}{\mathrm{Ye} + \mathrm{Y}\cdot\left(1 - \mathrm{k31}^2\right)}\right]$$

$|\mathrm{VV}| = 169\,\mathrm{V}$

Problem 2: Consider a PWAS transducer with length $l = 7$ mm, width $b = 1.65$ mm, thickness $t = 0.2$ mm, and material properties as given in Table 7.1.

(vi) Calculate the approximate value of Young's modulus, Y^E, in GPa. What common material is this value close to?

(vii) Calculate the electric field, E_3, for an applied voltage $V = 100$ V. Express the electric field E_3 in kV/mm.

(viii) Calculate the strain S_1 for the simultaneous application of stress $T_1 = -1$ MPa and voltage $V = -100$ V.

(ix) Plot on the same chart the variation of strain S_1 with voltage V for $T_1 = 0; -2.5; -5; -7.5; -10$ MPa

(x) Plot on the same chart the variation of strain S_1 with stress T_1 for $V = 0, 25, 50, 75, 100$ V

Solution

(i) To calculate the approximate value of Young's modulus, Y^E use the approximate formula

$$Y_{11}^E \approx \frac{1}{s_{11}^E}, \quad \text{i.e.,} \quad Y_{11}^E = 65.4 \text{ GPa} \tag{1}$$

The value given in Eq. (1) is closed to the elastic modulus of aluminum (~ 70 GPa)

(ii) To calculate the electric field, E_3, between the plates of a conventional capacitor, one divides the voltage by the distance between the plates. In the case of a PWAS, we note that the electrodes are placed on the top and bottom surfaces of the PWAS; hence the electric field is obtained by dividing the voltage by the PWAS thickness, as given by Eq. (7.13), i.e.,

$$E = -\frac{V}{t} \tag{2}$$

For an applied voltage $V = 100$ V and a PWAS thickness $t = 0.2$ mm, the corresponding electric field is $E_3 = -0.5$ kV/mm.

(iii) To calculate the strain S_1 for the simultaneous application of stress $T_1 = -5$ MPa and voltage $V = 100$ V use the textbook Eqs. (2.22) of Chapter 2 and (7.13) of Chapter 7, i.e.,

$$S_1(T_1, E_3) = s_{11}^E T_1 + d_{31} E_3 \tag{3}$$

Using Eq. (2) into Eq. (3) yields

$$S_1(T_1, V) = s_{11}^E T_1 + d_{31}\left(-\frac{V}{t}\right) \tag{4}$$

For the for the simultaneous application of stress $T_1 = -1$ MPa and voltage $V = -100$ V the strain is $S_1 = 72\ \mu\varepsilon$.

(iv) The plot on the same chart the variation of strain S_1 with voltage V for $T_1 = 0; -2.5; -5; -7.5; -10$ MPa is show in Figure 3 below.

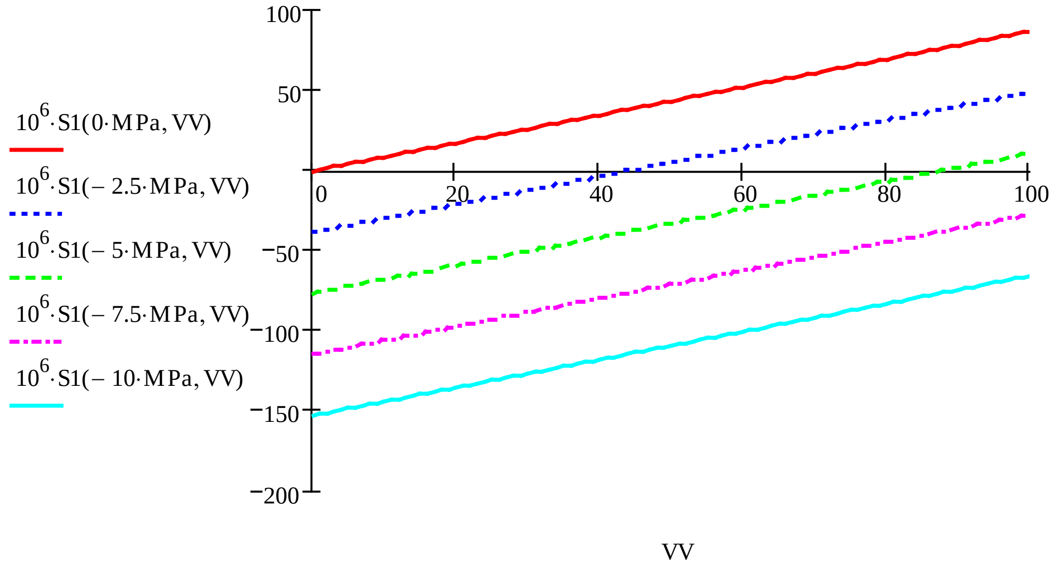


Figure 3   Variation of strain S_1 with voltage V for various values of stress T_1

(v) The plot on the same chart the variation of strain S_1 with stress T_1 for $V = 0, 25, 50, 75, 100$ V is given in Figure 4 below

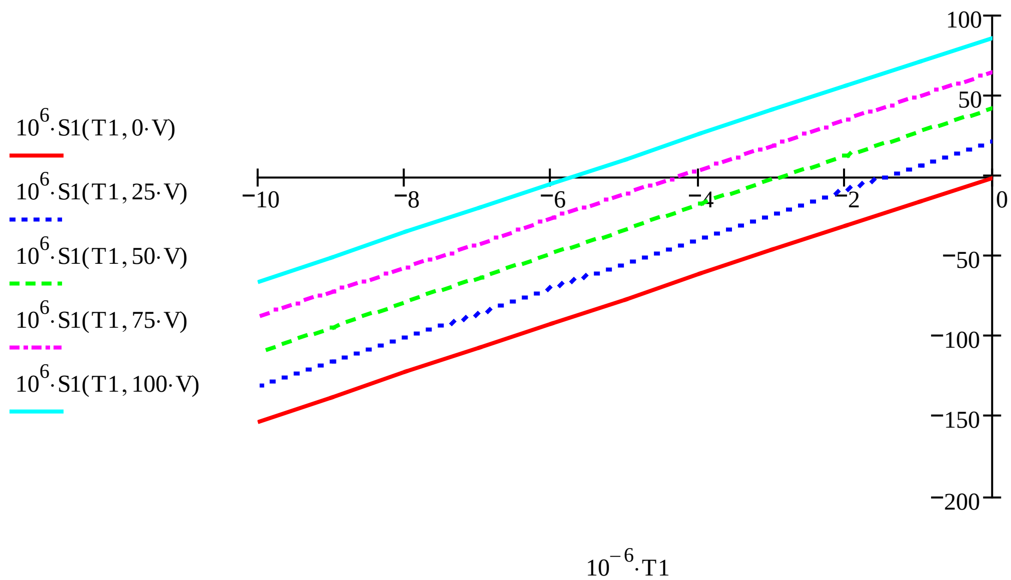


Figure 4   Variation of strain S_1 with stress T_1 for various values of voltage V

PROBLEM 7.2 SOLUTION

Units $\quad kHz := 1000\cdot Hz \quad pF := 10^{-12}\cdot F \quad nF := 10^{-9}\cdot F \quad \mu\varepsilon := 10^{-6} \quad MPa := 10^{6}\cdot Pa \quad GPa := 10^{9}\cdot Pa$

Given: $\quad L := 7\cdot mm \quad b := 1.65\cdot mm \quad t := 0.2\cdot mm$

$$\varepsilon 0 := 8.85\cdot 10^{-12}\cdot\frac{F}{m} \quad \varepsilon 33 := 1750\cdot\varepsilon 0 \quad s11 := 15.3\cdot 10^{-12}\cdot\frac{1}{Pa} \quad d31 := -175\cdot 10^{-12}\cdot\frac{m}{V} \quad k31 := 0.36$$

Solution:

$$Vmax := 100\cdot V \qquad VV := 0\cdot V, 1\cdot V .. Vmax \qquad \varepsilon 33 = 15.487\frac{nF}{m}$$

$$Tmax := -10\cdot MPa \quad \delta T := -1\cdot MPa \qquad T1 := 0, \delta T .. Tmax$$

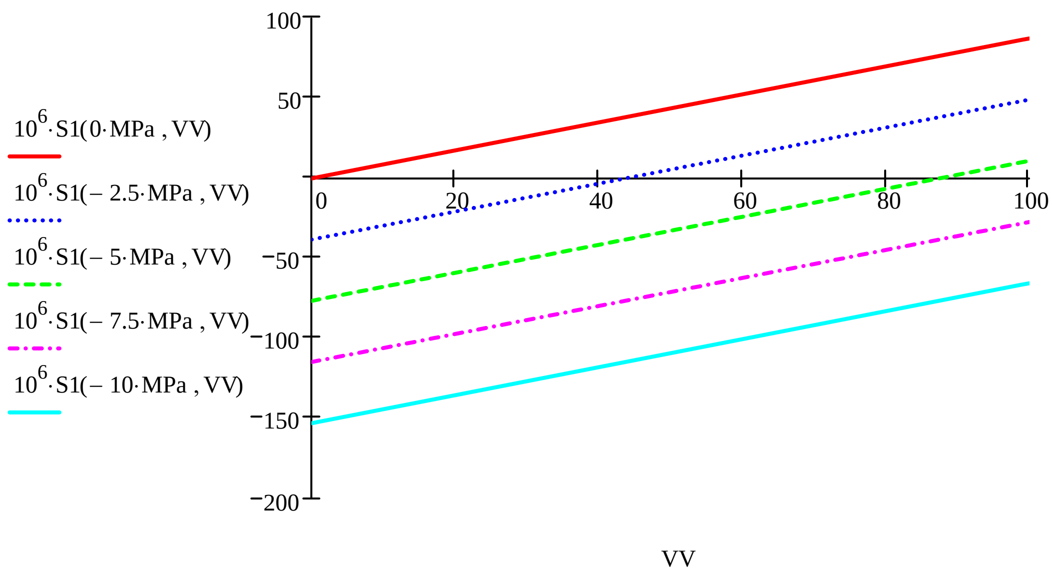
$$YE11 := \frac{1}{s11} \qquad YE11 = 65.4\,GPa$$

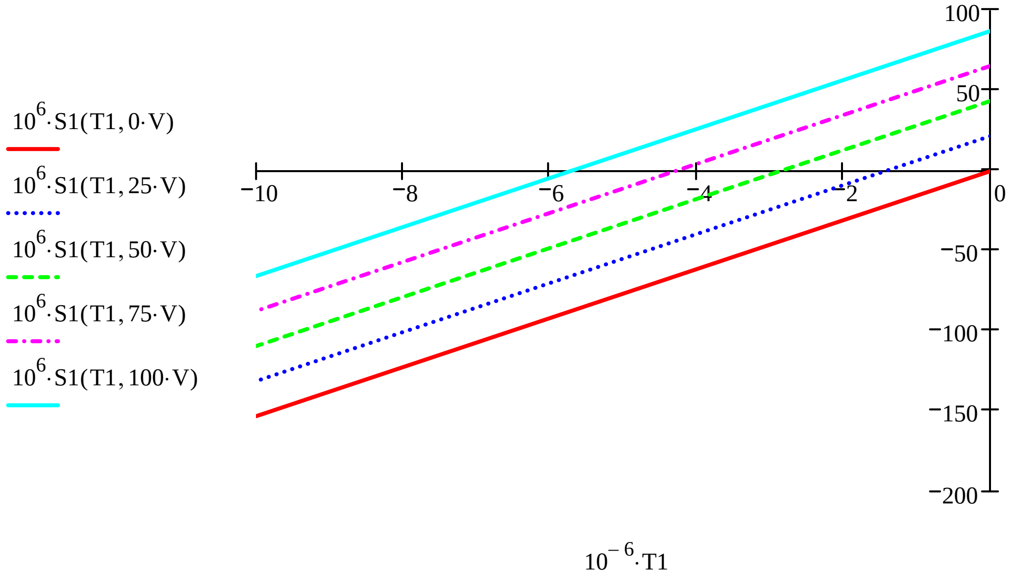
Close to aluminum Young's modulus = 70 GPa

$$E3(VV) := -\frac{VV}{t} \qquad E3(100\cdot V) = -0.500\frac{kV}{mm}$$

$$S1(T1, VV) := s11\cdot T1 + d31\cdot\left(-\frac{VV}{t}\right)$$

$$S1(-1\cdot MPa, 100\cdot V) = 72.2\mu\varepsilon$$




$10^6 \cdot S1(T1, 0 \cdot V)$
$10^6 \cdot S1(T1, 25 \cdot V)$
$10^6 \cdot S1(T1, 50 \cdot V)$
$10^6 \cdot S1(T1, 75 \cdot V)$
$10^6 \cdot S1(T1, 100 \cdot V)$
100
50
−10
−8
−6
−4
−2
0
−50
−100
−150
−200
$10^{-6} \cdot T1$