Chapter 3: Engine cycles and their efficiencies

The concept of internally reversible cycles was introduced in section 2.7. It was mentioned that a cycle could be internally reversible, while being externally irreversible. An internally reversible cycle is sometimes known as an endoreversible cycle. The advantage of considering endoreversible cycles is that the area of the cycle on a $T$ - $s$ diagram represents the work done by, or on, the fluid when it executes that cycle. If a cycle is not endoreversible then the area on the $T$-$s$ diagram of the cycle is not equivalent to the work done. There are a number of important endoreversible cycles and these are introduced below.

3.1 Heat Engines

3.1.1 Carnot cycle

The first thermodynamic cycle to be defined was that introduced by Carnot in 1824, and this led to the Second Law of Thermodynamics. The Carnot cycle, as originally proposed, is both internally and externally reversible. It consists of two isentropic and two isothermal processes, as shown in fig 3.1, for a cycle based on a perfect gas.

![Diagram of Carnot cycle](image)

The processes are defined on fig 3.1(b), and these definitions are repeated below:

- 1 - 2: reversible adiabatic (isentropic) compression;
- 2 - 3: reversible isothermal heat addition;
- 3 - 4: reversible adiabatic (isentropic) expansion;
- 4 - 1: reversible isothermal heat rejection.

Fig 3.1(b) has been based on processes for a perfect gas. The reversible, adiabatic processes obey the law $pV^\gamma = c$, and the reversible, isothermal processes obey the law $pV = c$. It can be seen from fig 3.1(b) that the $p$ - $V$ diagram for the cycle is very "thin". This means that if the processes are not reversible the effect on the area of the diagram, shown in fig 3.2, is dramatic. In fig 3.1(a) the temperatures of the hot and cold reservoirs are also shown: $T_H$ is only infinitesimally higher than $T_1$, while $T_C$ is only infinitesimally lower than $T_2$. This is necessary if the cycle is to be both internally and externally reversible. The consequence
of this is that the heat transfer rates associated with the Carnot cycle are very low, and the Carnot cycle produces no power! The effects of external irreversibilities on engine efficiency are discussed in Chapter 6.

Fig 3.2: "Carnot cycle" depicting processes using a perfect gas working fluid, with irreversibilities

One difficulty with using a perfect gas as the working fluid is that it can only absorb heat by changing temperature. For this reason, many of the cycles used in engines, and particularly heat engines, are based on a fluid that can change phase during the working cycle. The working fluid most commonly used in power generation is water, which can receive heat during the isothermal heat addition process (2 - 3) by evaporation, and can reject it during the isothermal heat rejection (4 - 1) by condensation. The diagrams of these cycles are shown in fig 3.3.

Fig 3.3: Carnot cycle for a working fluid which changes phase during the heat addition and rejection processes
(a) $T$-$s$ diagram for cycle; (b) $p$-$V$ diagram for cycle
It can be seen that in the Carnot cycle shown on the $T$-$s$ diagram the isothermal heating process now takes place between the saturated liquid and saturated vapour lines: in other words the liquid is evaporated at the high pressure state. This is depicted in the $p$-$V$ diagram shown in fig 3.3(b).

The fluid is then expanded from state 3, on the saturated vapour line, to state 4, which is in the two-phase liquid + vapour region. The heat rejection takes place between 4 and 1, and the fluid "condenses" isothermally at constant pressure. The cycle is completed by a process from 1 to 2, when the fluid is compressed reversibly and adiabatically to the saturated liquid line.

The energy transfers during the cycle can be evaluated for both cases by consideration of the $T$-$s$ diagram.

The heat addition is

$$Q_{23} = T_2(s_3 - s_2);$$  
(3.1)

the heat rejection is

$$Q_{41} = T_1(s_4 - s_1);$$  
(3.2)

and the work output is

$$W = Q_{23} + Q_{41} = (T_2 - T_1)(s_3 - s_2)$$  
(3.3)

The thermal efficiency of the cycle is

$$\eta = \frac{W}{Q_{23}} = \frac{(T_2 - T_1)(s_3 - s_2)}{T_2(s_3 - s_2)} = \frac{T_2 - T_1}{T_2} = \frac{T_H - T_C}{T_H},$$  
(3.4)

This is the same as the thermal efficiency of a reversible cycle derived by reference to the definition of the absolute scale of temperature in section 2.5, eqn 2.6. These energy transfers can be illustrated on a $T$-$s$ diagram, as shown on fig 3.4.

![T-s diagram](image.png)

**Fig 3.4:** Energy transfer processes depicted on $T$-$s$ diagram

Heat addition, $Q_{23} =$ area $6-2-3-5-6$

Heat rejection, $Q_{41} =$ area $6-1-4-5-6$

Work output, $W =$ area $1-2-3-4-1$

The thermal efficiency of the reversible cycle based on a perfect gas can also be evaluated. It should be realised that the thermodynamic scale of temperature and "absolute temperature" defined by the perfect gas
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law are the same by coincidence rather than definition. Considering the processes shown in fig 3.1(b), the work transfers are defined by the areas of the p - V diagram.

Work done,
\[ W_{12} = \frac{p_1V_1 - p_2V_2}{\kappa - 1}; \quad W_{23} = p_2V_2 \ln \frac{V_3}{V_2}; \quad W_{34} = \frac{p_3V_3 - p_4V_4}{\kappa - 1}; \quad W_{41} = p_4V_4 \ln \frac{V_1}{V_4} \]

which gives the work from the cycle as
\[ \int_{\text{cycle}} W = W_{12} + W_{23} + W_{34} + W_{41}. \]

Now
\[ W_{12} = \frac{p_1V_1 - p_2V_2}{\kappa - 1} = \frac{R(T_1 - T_2)}{\kappa - 1}, \quad \text{and} \quad W_{34} = \frac{R(T_3 - T_4)}{\kappa - 1}, \]

which means that \( W_{12} = -W_{34} \), giving \( W_{12} + W_{34} = 0 \).

Also
\[ W_{23} = p_2V_2 \ln \frac{V_3}{V_2} = RT_2 \ln \frac{V_3}{V_2}; \quad \text{and} \quad W_{41} = RT_1 \ln \frac{V_1}{V_4}, \]

giving
\[ \int_{\text{cycle}} W = R \ln \frac{V_3}{V_2} (T_2 - T_1). \] (3.5)

The energy addition to the cycle can be evaluated from process 2 - 3. Applying the First Law gives
\[ \delta Q_{23} = dU_{23} + \delta W_{23} = c_v (T_3 - T_2) + \delta W_{23} = \delta W_{23} = RT_2 \ln \frac{V_3}{V_2} \]

Hence, the thermal efficiency of the cycle is
\[ \eta = \frac{\int_{\text{cycle}} W}{\delta Q_{23}} = \frac{T_2 - T_1}{T_2} = 1 - \frac{T_1}{T_2} = 1 - \frac{T_C}{T_H}. \] (3.7)

This is the same as evaluated from the previous analysis, and given as eqn 3.4, which is not surprising because all reversible cycles operating between the same temperature limits have the same thermal efficiency. Hence, the efficiency of a reversible cycle is independent of the working fluid.

It can be seen that there are a number of benefits in using a working fluid which can change phase, including:

- constant pressure isothermal heat addition;
- a diagram that is less sensitive to inefficiencies than the perfect gas one;
- it is possible to replace the compression of a gaseous phase by the pumping of a liquid, in which case the compression work is significantly reduced.

The work output of a Carnot cycle is defined by the areas of either a p - V or a T - s diagram, and it can be seen that these are finite. However, while the Carnot cycle can produce a finite work output, the power output is zero because the rate of heat transfer to the engine across an infinitesimal temperature difference is zero. The temperature difference is zero to achieve external reversibility. This means that the Carnot efficiency defines the maximum efficiency of a heat engine between two temperature levels. This is never achieved in practice because all engines are, at least, significantly irreversible in the external heat transfer processes: this is discussed in Chapter 6.

### 3.1.2 Rankine cycle

It was shown above that the Carnot cycle for a fluid which changes phase during the working cycle requires an expansion device which operates with a fluid with a low quality (x) at the end of the expansion process. This tends to create problems in the design of the expansion device because of the amount of liquid in the working fluid. In the case of a steam turbine there can be very bad erosion of the low-pressure turbine blades due to water droplets in the steam, and this can reduce the reliability of the machine. Similarly, the
compression depicted in fig 3.3 goes from a low quality mixture of water and water vapour (at state 1) to saturated liquid water (at state 2). This requires careful control of state 1 to ensure that state 2 lies on the saturated liquid line, and also a compressor that can deal with a mixture that changes phase during the compression process. Both of these problems can be solved, at least partially, by use of the Rankine cycle.

The Rankine cycle is the basis of all steam turbine power plants, and a schematic diagram of a steam turbine power plant is shown in fig 3.5. It is an adaptation of the Carnot cycle to remove some of the latter's limitations. The basic Rankine cycle (see fig 3.6(a)) removes the problem of compressing a two-phase liquid from 1 to 2. In this case the fluid is condensed right up to the saturated liquid line (state 1). It is then compressed in the liquid phase from 1 to 2, where it is a compressed liquid. Heating, at constant pressure, commences at 2, and the fluid is raised in temperature from 2 to 3, with evaporation occurring from 3 to 4. At state 4 the liquid is fully evaporated, and state 4 is on the saturated vapour line. The fluid is expanded from 4 to 6 in the turbine, when it experiences the same change of state as in the Carnot cycle. the fluid at 6 is very wet and turbine blade erosion can be a problem. This situation can be improved by superheating the fluid from state 4 to state 5, as shown in fig 3.6(b). The effect of this is to raise the maximum temperature achieved by the fluid in executing the cycle, and the peak temperature is now $T_5$ rather than $T_2$, as it was in the other cycles up till now.

The work output of an endoreversible Rankine cycle is defined by the area of the $T$-$s$ diagram. This means that for the same pressures it is possible to increase the power output of the cycle by superheating, because the area of the diagram in fig 3.6(b) is greater than that in fig 3.6(a). Hence, superheating increases the work output, but what is the effect on the efficiency? First, it must be recognised that the energy input is greater in the superheat cycle than the standard one, and is defined by the enthalpy difference between 2 and 5, rather than 2 and 4. The real question is has the energy added between 4 and 5 in fig 3.6b been used more efficiently than that between 2 and 4. This can be answered by considering the superheat cycle to be made up of two cycles: 1-2-3-4-5-6-1, and 6'4-5-6-6'. The first cycle here is the basic Rankine cycle, while the second is a superheated cycle. Since the efficiency of a heat engine cycle is dependent on the temperature at which energy is received the efficiency of cycle 6'4-5-6-6' is greater than that of cycle 1-2-3-4-5-6-1 and, hence, the efficiency of the superheated cycle is greater than that of the basic one.
3.1.3 Comparison of efficiencies of Carnot and Rankine cycles

It is possible to use a simple analysis to compare the efficiencies of Carnot and Rankine cycles operating between the same temperature limits. Two cycles are shown superimposed in figs 3.7(a) and (b), which show the basic and superheated cycles respectively.

The Carnot cycle operating between the same temperature limits produces more work than the Rankine cycle, but this does not guarantee that the efficiency is higher. The efficiency can be considered by examining the work output and the heat rejected. By definition, the efficiency of a heat engine is

$$\eta = \frac{W}{Q_{in}} = \frac{W}{W + Q_{out}}.$$  

In these cases the values of $Q_{out}$ for the Carnot and Rankine cycles are the same, and, hence, the efficiency of the Carnot cycle must be greater than that of the Rankine cycle.

$$\eta_{\text{Carnot}} > \eta_{\text{Rankine}}$$  \hspace{1cm} (3.8)
Another way of looking at this problem is to consider the mean temperature of energy addition.

### 3.1.4 Mean temperature of energy addition and rejection

It was shown in figs 3.7(a) and (b) that the Carnot cycle has a greater thermal efficiency than a Rankine cycle operating between the same temperature limits. This is because the heat addition for the Carnot cycle takes place at the maximum temperature of the cycle, and the heat rejection occurs at the minimum temperature. Hence, the Carnot cycle takes maximum advantage of the temperature difference. The Carnot cycle in fig 3.8(a) has been broken down into an infinite number of infinitesimal cycles, and the efficiency of the cycle is given by

$$\eta = \frac{\sum \text{cycles} \ W}{\sum \text{cycles} \ Q}.$$  \hspace{1cm} (3.9)

Since all the cycles are identical in this case the efficiency of the whole cycle is equal to the individual efficiencies of the incremental cycles.

The situation changes with the superheated Rankine cycle in fig 3.8(b). Three incremental cycles have been depicted. Cycle a is in the region where the liquid water is being heated, and the efficiency is low because the peak temperature is low. The next cycle, b, is in the evaporation region, and the efficiency in this region will be the same as the Carnot cycle shown in fig 3.8(a). The final cycle, c, has been drawn in the superheat region, where the temperature is again rising. The efficiency of this cycle will be higher than during the evaporation region but lower than that of a Carnot cycle operating between $T_1$ and $T_5$. Hence, the efficiency of the Rankine cycle in fig 3.8(b) will be an “average” of the incremental cycles which make up the whole Rankine cycle. Since the temperature of heat rejection in this diagram is the same for all incremental cycles then the efficiency is governed by the temperature of energy addition. The efficiency of each incremental cycle is equivalent to that of the equivalent Carnot cycle, and the work output of each cycle is

$$\delta W = \delta Q \left(1 - \frac{T_r}{T_a} \right),$$  \hspace{1cm} (3.10)

where $T_a$ = temperature of heat addition

$T_r$ = temperature of heat rejection

Fig 3.8: Carnot and Rankine cycles broken down to incremental Carnot cycles

Now, the heat addition for each incremental cycle is $\delta Q = T \delta s$, and, hence, the work done over the cycle is
\[ W_{\text{cycle}} = \int_{\text{cycle}} \delta W = \int_{\text{cycle}} T_a (1-T_r/T_a) \, ds = \int_{s_2}^{s_5} T_a \, ds - T_r (s_5 - s_2) = \left( \frac{\int_{s_2}^{s_5} T_a \, ds}{s_5 - s_2} \right) (s_5 - s_2) \]

(3.11)

This equation is equivalent to eqn (3.3), but the value of the high temperature has been replaced by the term \( \int_{s_2}^{s_5} T_a \, ds \), which is the mean temperature of heat addition.

Mean temperature of energy addition, or rejection: \( \bar{T} = \frac{\int_{s_2}^{s_5} T \, ds}{s_5 - s_2} \)  

(3.12)

Hence, any cycle can be made equivalent to a Carnot cycle, and the efficiency of that cycle is the same as that of a Carnot cycle with the same mean temperatures of heat addition and rejection. This shows that any cycle in which the temperature of heat addition and rejection are not constant cannot achieve the same efficiency as a Carnot cycle with the same temperature limits.

3.1.5 Rankine cycle depicted on \( p-V-T \) surface

The diagrams given above show the Rankine cycle on \( T-s \) diagrams. This is the normal manner in which the cycle is shown. It is also possible to draw the Rankine cycle on the \( p-v-T \) surface for water, as shown in fig 3.9.
3.2 Air standard cycles

The Rankine cycle is based on a working fluid which changes phase during the cycle. This has the advantage of introducing regions of heat addition and rejection where the temperature is constant. However, the most readily available working fluid is air, which is a superheated gas at normal operating conditions. This results in a series of cycles in which the energy is received and rejected at variable temperature. These cycles can be used to examine the performance of internal combustion engines, e.g., petrol and diesel engines, and gas turbines. It should be realised that internal combustion engines and gas turbines are not heat engines - because mass flows across the boundaries as air and fuel to enter the engines, and exhaust gases leave. More realistic cycles for these engines are considered in Chapters 16 and 17, respectively. However, it is possible to define “engines” which can be analysed by endoreversible cycles: these “engines” replace the energy flows brought about by gas flows and combustion by heat transfer processes. Such cycles will be described below.

There are three air-standard cycles:

- constant volume ‘combustion’ (Otto)
- constant pressure ‘combustion’ (Diesel)
- dual ‘combustion’ – this is a combination of constant volume and constant pressure combustion, and results in a slightly more realistic cycle

These are the heat engine equivalent of the reciprocating engine and different from the actual engine cycle because:

- the cycle is a closed one with heat transfer;
- the working fluid does not change composition;
- the energy addition obeys closely defined rules e.g. constant volume energy addition;
- the rates of heat release (energy addition) are unrealistic;
- indicated work outputs are evaluated.

In a real cycle:

- the fluid changes as the gas passes through the engine;
- the properties of the fluid change;
- the rates of heat release are finite;
- the engine has frictional and heat transfer losses.

The effect of two of these differences will be examined in Chapter 14:

(i) frictional losses;
(ii) the finite rate of heat release.

3.2.1 Otto cycle

The Otto cycle is an air standard cycle which approximates the processes in petrol or diesel engines. It is based on constant volume heat addition (combustion) and heat rejection processes, and isentropic compression and expansion. The diagram is shown on fig 3.10, where it is superimposed on an actual p - V diagram for a diesel engine.

The actual p-V diagram for an engine has rounded corners because of the processes of combustion take place at a finite rate. The Otto cycle has sharp corners because the “combustion” is switched on and off instantaneously. It can be seen from fig 3.10 that the area of the Otto cycle is larger than that of the actual cycle, and this has to be taken into account when analysing engine cycles – the actual cycle will always produce less work output than the Otto cycle.
A typical engine "cycle" is defined in fig 3.11. It consists of a compression stroke (fig 3.11a), followed by a period of combustion close to top dead centre (tdc) (fig 3.11b), and then by expansion (fig 3.11c). These two strokes form the power producing processes, but afterwards the products of combustion have to be replaced by fresh air. This is symbolised in fig 3.11d, where the exhaust valve is open at the beginning of the exhaust stroke. In a four-stroke engine the piston executes two complete revolutions of the crankshaft, and uses two-strokes while the gas is pushed out by the piston on the up stroke, and then the intake valve is opened to enable air to be induced. In a two-stroke engine the intake and exhaust strokes occur at the end and the beginning of the expansion and compression strokes, respectively. These processes are called the gas exchange processes, and are one of the main reasons why real engines are not heat engines. The other reason is the combustion process, when the air is used to burn the fuel. This process of combustion means that the fluid in the engine cannot undergo a cycle.
The Otto, Diesel and dual-combustion cycles are air standard cycles that approximate the processes in a real engine. They can be achieved in the following way:

- the combustion process is replaced by a heat transfer process in which an amount of energy equivalent to the energy released by combustion is added to the air;
- the gas exchange process is replaced by a heat transfer process to a cold reservoir, so that the hot gases after expansion are returned to the state of the air after induction.

The resulting air standard cycle is defined in fig 3.12.

The basic Otto cycle is made up of four processes:

- isentropic compression;
- constant volume heat addition;
- isentropic expansion;
- constant volume heat rejection.

These can be depicted on $T$-$s$ and $p$-$V$ diagrams as shown in fig 3.13.

Since all the processes in fig 3.13 are reversible, the areas of the diagrams (a and b) are equal, and depict the work done in the cycle. The work done is

$$W = \int_{1}^{2} pdV = \int_{2}^{3} pdV + \int_{3}^{4} pdV$$

$$= \frac{p_1 V_1 - p_2 V_2}{\kappa - 1} + \frac{p_3 V_3 - p_4 V_4}{\kappa - 1} = \frac{mR}{\kappa - 1} \left( (T_1 - T_2) + (T_3 - T_4) \right)$$

(3.13)

The energy added to the cycle is that added at constant volume between 2 and 3, and is given by

$$Q_{23} = mc_v (T_3 - T_2) = \frac{mR}{\kappa - 1} (T_3 - T_2)$$

(3.14)

Hence the thermal efficiency is
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\[ \eta_{\text{Otto}} = \frac{\int p \, dV}{Q_{23}} = \frac{(T_1 - T_2) + (T_3 - T_4)}{T_3 - T_2} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} \]  

(3.15)

This equation can be rearranged into the more familiar one for the Otto cycle in the following way. First, it is necessary to define the compression ratio, \( r \), which is based on the ratio of the volume at ‘top dead centre’ (tdc) to that at bottom dead centre (bdc), i.e. \( r = V_2/V_1 \). It is then possible to write the temperatures around the cycle in terms of \( T_1 \) and \( T_3 \), and the compression ratio, \( r \). This gives

\[ T_4 = \frac{T_3}{r^{(k-1)}} \text{, and } T_2 = T_1 r^{(k-1)} \]  

which may be substituted into eqn (3.15) to give

\[ \eta_{\text{Otto}} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} = 1 - \frac{T_3}{T_1} \frac{r^{(k-1)} - T_1}{r^{(k-1)}} \]  

(3.16)

Now, the term \( \frac{1}{r^{(k-1)}} = \frac{T_1}{T_2} = \frac{T_4}{T_3} \), and hence \( \eta_{\text{Otto}} = 1 - \frac{T_1}{T_2} = 1 - \frac{T_4}{T_3} \).  

(3.17)

Thus, the efficiency of an Otto cycle is also related to a temperature ratio, but in this case it is the temperature ratio due to isentropic compression or expansion. Consideration of fig 3.13(b) shows that this is significantly less than the temperature ratio of the hot and cold reservoirs, and hence the efficiency of the Otto cycle is less than that of a Carnot cycle operating between the same two temperature limits which has an efficiency, \( \eta_{\text{Carnot}} = 1 - \frac{T_3}{T_1} = 1 - \frac{T_4}{T_3} \). The reason for this is simply that the heat is added and rejected over varying temperatures, and it can be shown that the efficiency of an engine operating on an Otto cycle

Fig 3.13: Otto cycle
(a) T-s diagram; (b) p-V diagram
is the same as a Carnot cycle operating between reservoirs at the mean temperatures of heat addition ($\bar{T}_a$) and rejection ($\bar{T}_r$). It is interesting to note that the efficiency of the Otto cycle approaches that of the Carnot cycle if $T_3 \equiv T_2$; such a cycle produces no output because lines 1-2 and 3-4 on fig 3.13 become coincident.

### 3.2.2 Diesel cycle

The diesel cycle is also a cycle applied to reciprocating engines, and is similar to the Otto cycle except that the heat is applied at constant pressure rather than constant volume. This removes the limitation of infinite rates of combustion implied by the Otto cycle, but still results in an unrealistic combustion pattern. The diesel cycle is shown in fig 3.14.

![Diagram of Diesel cycle](image)

Fig 3.14: Diesel cycle
(a) $T$-$s$ diagram; (b) $p$-$V$ diagram

The work done in a diesel cycle is

$$W = \int p \, dV = \frac{p_1 V_1 - p_2 V_2}{\kappa - 1} + \frac{p_1 V_1 - p_4 V_4}{\kappa - 1} + \frac{p_1 V_1 - p_3 V_3}{\kappa - 1}$$

$$= p_2 V_2 \left\{ (\beta - 1) + \frac{1}{\kappa - 1} \left[ \frac{p_1 V_1}{p_2 V_2} - 1 + \frac{p_3 V_3}{p_2 V_2} - \frac{p_4 V_4}{p_2 V_2} \right] \right\}$$

$$= \frac{mRT_2}{\kappa - 1} \left\{ (\beta - 1)(\kappa - 1) + \frac{1}{\kappa - 1} - 1 + \beta - \left( \frac{\beta}{r} \right)^{\frac{1}{\kappa - 1}} \right\}$$

(3.18)

where $\beta$ defines the size of the constant pressure heat addition region, $\beta = V_3/V_2$. The effect of the constant pressure heat addition region is to reduce the expansion ratio of the cycle, $r_e = V_4/V_3 = r/\beta$. This has a large effect on the efficiency of the cycle.

The heat addition is

$$Q_{23} = mc_p (T_3 - T_2) = \frac{m k R}{\kappa - 1} (T_3 - T_2) = \frac{m k R T_2}{\kappa - 1} (\beta - 1).$$

(3.19)

The efficiency of the cycle is

$$\eta = \frac{W}{Q_{23}} = \frac{\frac{m k R T_2}{\kappa - 1} (\beta - 1)}{\frac{m k R}{\kappa - 1} (T_3 - T_2)}.$$
\[ \eta_{\text{diesel}} = \frac{\int \rho dV}{Q_{23}} = \frac{\kappa (\beta - 1) - \left(\frac{\beta^\kappa - 1}{\kappa (\beta - 1)}\right)}{\kappa (\beta - 1)} = 1 - \frac{1}{r^{\kappa - 1}} \beta^\kappa - 1 \]  

(3.20)

This efficiency is less than that of the Otto cycle because the term \( \frac{\beta^\kappa - 1}{\kappa (\beta - 1)} > 1 \)

### 3.2.3 Dual combustion cycle

The dual-combustion cycle is shown in Fig 3.15. The cycle gets its name because a proportion of the ‘combustion’ (heat addition) takes place at constant volume, from 2 to 3, and then the remainder occurs at constant pressure, from 3 to 4. This cycle is the most representative of real engine cycles, in which the initial combustion takes place rapidly, and then slows down later in the process (although the dual-combustion cycle requires the heat release to increase as the volume increases). It can be shown (this is left to the reader) that the efficiency of a dual-combustion cycle is

\[ \eta_{\text{h}} = 1 - \frac{1}{r^{(\kappa - 1)}} \left[ \frac{\alpha \beta^\kappa - 1}{(\alpha - 1) + \alpha \kappa (\beta - 1)} \right] \]  

(3.21)

where \( \alpha = \frac{p_3}{p_2} \), the pressure ratio caused by constant volume combustion.

![Fig 3.15 The dual-combustion cycle](image)

### 3.2.4 The most efficient internal combustion engine cycle - based on various constraints

In the case of ideal cycles it is possible to define the relationship between the thermal efficiency (indicated) and the compression ratio. The indicated thermal efficiency for an Otto cycle (constant volume combustion), shown in Fig.3.13, is given by eqn (3.16)

\[ \eta_{\text{ih}} = 1 - \frac{1}{r^{(\kappa - 1)}} \]
where \( r \) = compression ratio; 
\( \kappa \) = ratio of specific heats \((c_p/c_v)\).

The indicated thermal efficiencies of the diesel and dual combustion cycles are lower than that of the Otto cycle for the same compression ratio. The relationships for these cycles are:

Constant pressure cycle (diesel cycle), shown in Fig. 3.14 – efficiency given by eqn (3.20):

\[
\eta_{th} = 1 - \frac{1}{r^{(\kappa-1)}} \left[ \frac{\beta - 1}{\kappa (\beta - 1)} \right] 
\]

(3.20)

where \( \beta \) = cut-off ratio.

Dual combustion cycle, shown in Fig. 3.15 – efficiency given by eqn (3.21),

\[
\eta_{th} = 1 - \frac{1}{r^{(\kappa-1)}} \left[ \frac{\alpha \beta^{\kappa - 1}}{(\alpha - 1) + \alpha \kappa (\beta - 1)} \right] 
\]

(3.21)

where \( \alpha \) = pressure ratio caused by constant volume combustion.

Equations 3.16, 3.20, 3.21 define the efficiency of air-standard cycles. These can be generalized as:

\[
\eta_{th} = 1 - \frac{C}{r^{(\kappa-1)}} 
\]

(3.22)

where \( C = 1 \) for Otto cycle 
\( C = \left[ \frac{\alpha \beta^{\kappa - 1}}{(\alpha - 1) + \alpha \kappa (\beta - 1)} \right] > 1 \) for a dual combustion cycle; 
\( C = \left[ \frac{\beta^{\kappa - 1}}{\kappa (\beta - 1)} \right] > 1 \), for a diesel (constant pressure) cycle. (3.25)

Hence, for a given compression ratio \( r \) the thermal efficiencies are related by

\[
\left( \eta_{th} \right)_{\text{Otto}} > \left( \eta_{th} \right)_{\text{dual comb}} > \left( \eta_{th} \right)_{\text{diesel}} 
\]

(3.26)

However, the situation changes if the maximum pressure is limited: in fact, if all three cycles are compared for the same peak pressure and same work output then

\[
\left( \eta_{th} \right)_{\text{diesel}} > \left( \eta_{th} \right)_{\text{dual comb}} > \left( \eta_{th} \right)_{\text{Otto}} 
\]

(3.27)

Why is this the case? Considering first the cycles with the same compression ratio, then it is apparent that the average expansion ratio of each of the cycles is different. The average expansion ratio of the Otto cycle is equal to the compression ratio, \( r \). In Fig. 3.15, \( V_e \) represents a typical cycle and the typical expansion ratio \( r_e = V_1/V_e \) and, since \( V_e > V_2 \), then \( r_e < r \). So if there is any constant pressure combustion then there must be a lower mean expansion ratio than in the case of the Otto cycle.
Fig. 3.16 Comparison of Otto and Diesel cycles with the same maximum pressure and work outputs

Fig. 3.16 shows a comparison of two cycles (an Otto and a Diesel cycle). These have the same peak pressure and the same work output. It can be seen that the compression ratio of the diesel engine is higher than that of the Otto cycle engine; in fact, the expansion ratio of the diesel cycle throughout the cycle is higher than the compression ratio of the Otto cycle. This means that the thermal efficiency of each element of the diesel cycle is more efficient than the Otto cycle, and hence this diesel cycle is more efficient than the Otto cycle. A similar argument applies for the dual combustion cycle, which lies between the diesel and Otto cycles.

It is useful to consider these idealised cycles because they enable general concepts to be understood. They have enabled us to see why a diesel engine cycle can be more efficient than an Otto one if the peak pressure is limited. They have shown that expansion ratio is a more relevant parameter than compression ratio when considering the efficiency of a cycle. However, they have not allowed us to see how the performance of real engines, with finite rates of combustion, varies when parameters are changed. These are considered in Chapter 16, by means of an engine simulation program. The results of the simulation are presented in a form that should be intelligible to the reader of this chapter, without having assimilated all necessary theory to understand the detailed workings of the program.

In the previous sections engines have been compared based on their efficiencies. Another way of assessing the output of reciprocating engines is to compare them on the basis of mean effective pressure (mep). The indicated mean effective pressure of an engine (imep, $\bar{p}_i$) is defined as the average (mean) pressure that would have to operate over the whole stroke ($V_s = V_1 - V_2$) to give the same work output as the actual cycle, i.e.

$$\bar{p}_i = \frac{\int p \, dV}{V_s}$$  \hspace{1cm} (3.28)

The concept of mean effective pressures will be returned to in Chapter 16.

### 3.2.5 Joule (or Brayton) cycle

The Joule cycle is the air standard cycle which describes the processes in the gas turbine. The Joule cycle has constant pressure combustion and constant pressure heat rejection. It is depicted in fig 3.17.

This cycle, which is examined in exactly the same manner as the Otto and diesel cycles in Chapter 16.2, results in the following expression for efficiency eqn (17.8))
\[ \eta_{\text{Joule}} = 1 - \frac{1}{r_p^{\frac{\kappa-1}{\kappa}}} , \]  
where \( r_p = p_2 / p_1 \), the pressure ratio for the gas turbine.

The term \( \frac{1}{r_p^{\frac{\kappa-1}{\kappa}}} \) is equivalent to the temperature ratio, \( \frac{T_1}{T_2} \), and the significance of this is discussed below.

The efficiency of the Carnot cycle is directly related to the temperature ratio of the hot and cold reservoirs. Examination of the other efficiencies will show that they are also related to temperature ratios of the cycle. The efficiency of the Otto cycle is (eqn (3.17))

\[ \eta_{\text{Otto}} = 1 - \frac{1}{r^{(\kappa-1)}} = 1 - \frac{T_1}{T_2} = 1 - \frac{T_4}{T_3} . \]

The equation for the diesel engine is more complex and will not be considered here. However, the efficiency of the Joule cycle can also be related to the temperature ratio because \( \frac{\kappa-1}{\kappa} \), \( r_p^{\frac{\kappa-1}{\kappa}} = \frac{T_2}{T_1} \), and hence

\[ \eta_{\text{Joule}} = 1 - \frac{1}{r_p^{\frac{\kappa-1}{\kappa}}} = 1 - \frac{T_1}{T_2} = 1 - \frac{T_4}{T_3} , \text{ on fig 3.17.} \]

Hence, all heat engines have efficiencies of the form

\[ \eta_{\text{th}} = 1 - \frac{T_n}{T_m} , \]
where $T_n$ and $T_m$ are particular temperatures relating to the individual cycle.

### 3.4 Reversed heat engines

Reversed heat engines were introduced in section 2.7, and comprise refrigerators and heat pumps. The objectives of these devices are to transfer energy from a low temperature reservoir to a higher temperature one. In the case of the refrigerator the aim is to cool the ice box, and reject the energy extracted from it into ambient conditions at a higher temperature. The purpose of the heat pump is to warm a building by "pumping" low grade energy up to a higher temperature. Both of these devices work by using power input to drive the processes: it is not possible, by the Second Law, for energy to pass from a low to a higher temperature reservoir. The cycles used to analyse reversed heat engines are similar to those introduced above for heat engines producing power - the processes are simply executed in reverse order.

#### 3.4.1 Reversed Carnot cycle

If the Carnot cycle shown in fig 3.1 is operated in reverse, i.e. the arrows on the diagram are turned round, as in fig 3.16, then the directions of the work and heat transfer terms are also reversed. This means that net work is provided to the cycle during processes 1-2 and 3-4, and that energy $Q_{\text{in}}$ is "pumped" from the low temperature reservoir ($T_C$), and energy $Q_{\text{out}}$ is delivered to the high temperature reservoir $T_H$. Hence, the addition of work to the cycle is able to raise the "quality" of the energy in the low temperature reservoir.

![Fig 3.18: $T$-$s$ diagram for reversed heat engine (refrigerator or heat pump) operating on reversed Carnot cycle](image)

**Fig 3.18:** $T$-$s$ diagram for reversed heat engine (refrigerator or heat pump) operating on reversed Carnot cycle

#### 3.4.2 Actual refrigerator and heat pump cycles

A device which would execute this cycle is shown schematically in fig 3.19(a): in this case, unlike that of the steam turbine, the pumping work is significantly larger than the work obtained from the expander turbine. In most small, domestic, refrigeration plant the expander turbine is replaced with throttle, as shown in fig 3.19(b). This modifies the $T$-$s$ diagram for the refrigerator to that given in fig 3.20(a). Many refrigeration plants operate with sub-cooling of the working fluid, which further modifies the cycle to that in fig 3.20(b).

As described in Section 2.7.1, thermal efficiency is not the correct parameter to use to define the "efficiency" of a reversed cycle, and the parameter used in this case is the coefficient of performance, which is defined as
\[
\beta = \frac{\text{Heat transferred from cold reservoir}}{\text{Net work done}} = \frac{Q_C}{W_{net}} = \frac{Q_C}{Q_H - Q_C}, \text{ for a refrigerator}
\]

\[
\beta' = \frac{\text{Heat transferred to hot reservoir}}{\text{Net work done}} = \frac{Q_H}{W_{net}} = \frac{Q_H}{Q_H - Q_C}, \text{ for heat pump}
\]

It was also shown in Section 2.7.1 that \(\beta = \beta + 1\). The coefficients of performance of all reversible heat engines operating between the same two temperature reservoirs will be equal, irrespective of the working fluid. This can be defined in terms of the reservoir temperatures if the devices are internally and externally reversible. Substituting for temperatures gives

\[
\beta = \frac{T_C}{T_H - T_C}, \text{ and } \beta' = \frac{T_H}{T_H - T_C}.
\]

**Fig 3.19** Schematic diagrams of refrigerator (or heat pump)

(a) with expander turbine    (b) with throttle

### 3.4.3 Example:

Calculate the coefficients of performance of the refrigerator and heat pump working with ammonia as the refrigerant. The evaporation takes place at a pressure of 0.7177 bar, after which the ammonia is compressed isentropically to the saturated vapour line at a pressure of 15.54 bar. After constant pressure condensation to a temperature of 28°C, the fluid is expanded irreversibly through a throttle to the evaporator pressure.

**Solution:**

The \(T\)-\(s\) diagram for the cycle is shown in fig 3.20.

The values of properties at the salient points can be obtained from tables of properties.

Conditions at 4:
\[ p_3 = p_2 = 0.7177 \text{bar}; \]
\[ h_l = 0 \text{kJ/kg}; \quad h_g = 1390 \text{kJ/kg}; \]
\[ s_l = 0 \text{kJ/kgK}; \quad s_g = 5.962 \text{kJ/kgK}; \]
\[ p_1 = 15.54 \text{bar}; \quad h_l = 371.9 \text{kJ/kg}; \quad h_g = 1473.3 \text{kJ/kg}; \]
\[ s_l = 1.360 \text{kJ/kgK}; \quad s_g = 4.877 \text{kJ/kgK}; \]

The values of enthalpy and entropy on the saturated liquid line at 40°C have been arbitrarily set at zero in the tables.

Conditions at 3:

\[ p_3 = p_2 = 15.54 \text{bar}; \]
\[ h_l = 371.9 \text{kJ/kg}; \quad h_g = 1473.3 \text{kJ/kg}; \]
\[ s_l = 1.360 \text{kJ/kgK}; \quad s_g = 4.877 \text{kJ/kgK}; \]

---

**Fig 3.20:** Irreversible refrigeration cycles
(a) with simple throttle (see fig 3.19b)
(b) with throttle and sub-cooling

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**Fig 3.21:** T - s diagram for reversed heat pump cycles.

Conditions at 2:
This is a sub-cooled state where the liquid is compressed, and it will be assumed that the enthalpy is equal to the enthalpy on the saturated liquid line at 28°C. Hence

\[ h_l = 313.4 \text{kJ/kg} \]

Now, since the compression from 4 to 3 is isentropic \( s_3 = s_4 \). Hence,

\[ s_3 = 4.877 = xx' + (1-x)s_l = 5.962x \]

giving \( x = 4.877/5.962 = 0.8180 \)

and \( h_4 = xh_g + (1-x)h_l = 0.8180 \times 1390 = 1137 \text{kJ/kg} \)

Work done in compressor

\[ W_C = W_{43} = h_4 - h_3 = 1137 - 1473.3 = -336.3 \text{kJ/kg} \]

Heat extracted from the evaporator (cold reservoir)

\[ Q_C = Q_{41} = h_4 - h_1 = h_4 - h_2 = 1137 - 313.4 = 823.6 \text{kJ/kg} \]

Hence, \( \text{coefficient of performance of refrigerator} \quad \beta = \frac{h_4 - h_1}{h_3 - h_4} = \frac{823.6}{336.3} = 2.449 \)

Heat transferred from the condenser (hot reservoir)

\[ Q_H = Q_{32} = h_2 - h_3 = 313.4 - 1473.3 = -1159.9 \text{kJ/kg} \]

Hence, \( \text{coefficient of performance of heat pump} \quad \beta' = \frac{h_2 - h_1}{h_4 - h_3} = \frac{1159.9}{336.3} = 3.449 \)

Thus, both the coefficients of performance are greater than unity, and the coefficient of performance of the heat pump is one greater than that of the refrigerator - even though the devices are not reversible. If the devices had been reversible, i.e. had followed the reversed Carnot cycle, then the values would have been

\[ \beta = \frac{T_C}{T_H - T_C} = \frac{T_1}{T_3 - T_1} = \frac{233}{313 - 233} = 2.913 \]

\[ \beta' = \frac{T_H}{T_H - T_C} = \frac{T_3}{T_3 - T_1} = \frac{313}{313 - 233} = 3.913 \]

It can be seen that the reversible reversed heat engines, operating on the Carnot cycle, have higher coefficients of performance than those undergoing cycles with irreversibilities.

### 3.5 Concluding remarks

This chapter has introduced a range of different cycles, from the fundamental Carnot cycle through more realistic cycles for simulating actual powerplant. It is possible to evaluate the thermal efficiency of these cycles, and these efficiencies can be compared to that of the Carnot cycle. The Carnot cycle is shown to be the most efficient cycle operating between two temperature levels, simply because it is able to receive and reject energy at the upper and lower temperatures. None of the other cycles can achieve this, although the basic Rankine cycle can come close.
Air standard cycles have been introduced, and cycles that can be used to analyse reciprocating engines, e.g. spark ignition and diesel engines, and gas turbines have been described. It has been explained that engines following these cycles are usually not heat engines because working fluid flows across their boundaries. It has also been demonstrated that they can never achieve the Carnot efficiency because the energy addition and rejection occurs at varying temperature, and the efficiency of the cycles is related to the *mean* temperatures of energy addition and rejection.

Finally, reversed heat engine cycles, i.e. refrigerators and heat pumps, have been introduced and their ‘efficiency’ has been defined as the coefficient of performance. Reciprocating engines and gas turbines will be discussed further in chapters 16 and 17 respectively.

All these cycles have been internally and externally reversible, and while they provide useful information on the effect of important parameters they are not directly applicable in the ‘real world’.

### 3.6 Problems

**P3.1** A steam turbine operates on a Carnot cycle, with a maximum pressure of 20 bar and a condenser pressure of 0.5 bar. Calculate the salient points of the cycle, the energy addition and work output per unit mass, and hence the thermal efficiency of the cycle.

Compare this value to the Carnot efficiency based on the temperatures of energy addition and rejection.  
\[ 26.98\%; 27.0\% \]

**P3.2** A steam powerplant operating on a basic Rankine cycle has the following parameters: maximum (boiler) pressure 20 bar; minimum (condenser) pressure 0.5 bar. Calculate the thermal efficiency of the cycle, and compare it to that of a Carnot cycle operating between the same temperature limits (see P3.1). Calculate the specific power output and the back work ratio (defined as \( \dot{w}_p/\dot{w}_T \)) for the cycle in this question and that in P3.1. Comment on the results obtained.

(*Assume the pump and turbine operate isentropically*)

\[ 24.26\%; 27.0\%; 598.5\text{ kW/(kg/s)}; 0.34\%; 14.7\% \]

**P3.3** Recalculate P3.1 assuming that the pump efficiency, \( \eta_p \), and the turbine efficiency, \( \eta_T = 0.9 \). Comment on the effect on the thermal efficiency of the plant, and also the back work ratio.

\[ 22.64\%; 20.48\% \]

**P3.4** Recalculate P3.2 assuming that the pump efficiency, \( \eta_p = 0.8 \), and the turbine efficiency, \( \eta_T = 0.9 \). Comment on the effect on the thermal efficiency of the plant, and also the back work ratio.

\[ 21.81\%; 0.47\% \]

**P3.5** The engine designed by Lenoir was essentially an atmospheric engine based on the early steam engines. In this, a combustible mixture was contained in a cylinder: it was ignited and the pressure increased isochorically to the maximum level. After this the gas expanded isentropically through an expansion ratio, \( r_e \), during which it produced work output. The air standard cycle returned the gas to state 1 through an isochoric expansion to \( p_1 \) and an isobaric compression to \( V_1 \).

Assume \( p_1 = 1 \text{ bar}, T_i = 15^\circ\text{C}, p_2 = 10\text{ bar} \) and the expansion ratio, \( r_e = 5 \). Calculate the specific work output and thermal efficiency of this cycle. How does this compare with the efficiency of an equivalent Carnot cycle?

\[ 620.5\text{ kJ}; 33.4\%; 90.0\% \]

**P3.6** A Lenoir engine (described in P3.5) operates with inlet conditions of \( p_1 = 1 \text{ bar}, \text{ and } T_i = 27^\circ\text{C} \). The energy added to the charge is 1000 kJ/kg. Calculate the maximum pressure and temperature achieved in the cycle, and its thermal efficiency.

\[ 5.65\text{ bar}; 1422^\circ\text{C}; 21.7\% \]
P3.7 A cycle is proposed as a development of the Lenoir cycle, in which the working fluid is expanded isentropically from its peak pressure down to a point where its temperature is equal to $T_1$, the initial temperature. The gas is then compressed isothermally back to the initial pressure. Prove that the thermal efficiency of the cycle is given by

$$\eta_{th} = 1 - \frac{T_1}{T_2 - T_\text{1}} \ln \frac{T_2}{T_1}$$

where $T_2$ is the maximum temperature achieved in the cycle.

Calculate the thermal efficiency of the cycle if the initial pressure is 10 bar, and the maximum pressure is 35 bar. Compare this to the Carnot efficiency achievable between the temperature limits, and explain why this cycle would not be used in practice.

[49.9%; 71.4%]

P3.8 Yet another cycle is proposed in which energy is added at constant volume until the fluid achieves state 2, the gas is then expanded to its initial pressure (state 3) before being compressed isobarically back to its initial conditions (state 1). Show that the thermal efficiency of this cycle is

$$\eta_{th} = 1 - \frac{\kappa(T_3 - T_\text{1})}{(T_2 - T_\text{1})}$$

If the initial conditions are 27°C and 1.0 bar, and the energy added is 2000 kJ/kg, calculate the thermal efficiency of the cycle. What is the specific work output of the cycle?

[35.4%; 708 kJ/kg]

Examples P3.2 and P3.9 to P3.13 follow the development of a basic Rankine cycle to demonstrate how the efficiency of such cycles can be improved.

P3.9 The condenser pressure of the turbine in P3.2 is reduced to 0.15 bar. Calculate the same parameters for this cycle as in the previous example. Why have the parameters improved so much?

[28.95%; 32.63%; 743.7 kW/(kg/s)]

P3.10 Both cycles in P3.2 and P3.9 resulted in extremely ‘wet’ steam (low quality) at the exit to the turbine. This would cause erosion of the blades, and should be avoided. One way of achieving this is to superheat the steam before it leaves the boiler: assuming that the temperature of the steam leaving the superheater is 400°C, calculate the same parameters for this cycle using the basic data in P3.9. What is the quality of the steam leaving the turbine?

Also calculate the mean temperatures of energy addition and rejection, and show that a Carnot cycle with these temperatures would have the same efficiency as this Rankine cycle.

[30.97%; 51.40%; 935kW/(kg/s); 0.879; 474.1K; 326.98K]

P3.11 Problems P3.2, P3.9 and P3.10 have shown how the efficiency of a basic Rankine cycle can be improved, but even after superheating the steam leaving the turbine is still wet. This situation could be alleviated by using two turbine stages and reheating the steam between them. Calculate the basic parameters for the cycle if the steam is withdrawn from the HP turbine at 10 bar and reheated to 400°C. What are the specific power outputs of each turbine?

[32.04%; 51.40%; 1035 kW/(kg/s); 0.925; 842.8 kW; 194 kW]

P3.12 Recalculate P3.11 with the pressure at which steam is reheated reduced to 5 bar. What have been the benefits of using this lower pressure?

[32.35%; 51.40%; 1101 kW/(kg/s); 0.971; 743.2 kW; 359.7 kW]

P3.13 Problem P3.12 seems to demonstrate that the efficiency of the reheated Rankine cycle gets better as the work distribution between the HP and LP turbines becomes more equal. Do some calculations to see if...
this proposition is true. At what reheat pressure are the turbine power outputs approximately equal, and what are the salient parameters of the cycle?

\[ 1.5 \text{ bar; 32.12%; 51.40%; 1169 kW/(kg/s); 0.983; 578 kW; 591 kW} \]

P3.14 What has the development of the basic Rankine cycle carried out in Problems P3.9 to P3.14 shown you about the effect of the salient parameters on the efficiency of the cycle? Evaluate the mean temperature of energy addition and rejection for the cycles.

P3.15 The previous examples, P3.9 to P3.13, have all been based on a boiler pressure of 20 bar. What is the effect of raising the boiler pressure to (a) 40 bar and (b) 80 bar on the steam plant described in P3.10?

\[(a) \ 34.06%; \ 1016\text{kW/(kg/s)}; \ (b) \ 36.82%; \ 1069\text{kW/(kg/s)}\]

P3.16 Recalculate P3.15(a) with the condenser pressure lowered to 0.07 bar.

\[36.36%; \ 1108\text{kW/(kg/s)}\]

P3.17 Considering P3.15 and P3.16, which is the most effective method for increasing the thermal efficiency of the plant – raising the boiler pressure, or decreasing the condenser pressure? Explain your answer.

P3.18 An air standard Otto cycle operates with a compression ratio, \( r = 10:1 \). If the initial conditions at bottom dead centre (bdc) are 1 bar and 27°C, and the energy addition is 2000kJ/kg of air, calculate the salient points around the cycle, and the thermal efficiency. Show that the efficiency calculated from the cycle calculation is equal to that from eqn (3.16). Assume the compression and expansion are isentropic, and \( \kappa = 1.4 \). How does this compare with the efficiency of a Carnot cycle between the two temperature limits?

\[60.2%; \ 91.5\%\]

P3.19 Recalculate P3.18 by assuming that the energy addition results from the combustion of fuel in the cylinder – this increases the mass of gas after ignition. The combustion occurs when an air-fuel with a strength, \( \varepsilon \), of 20:1 is burned at top dead centre (tdc): the calorific value (\( Q_p \)) of the fuel is 40,000kJ/kg. How does this compare with the efficiency of a Carnot cycle between the two temperature limits?

\[61.0%; \ 91.2\%\]

P3.20 An air standard Diesel cycle operates with a compression ratio, \( r = 10:1 \). If the initial conditions at bottom dead centre (bdc) are 1 bar and 27°C, and the energy addition is 2000kJ/kg of air, calculate the salient points around the cycle, and the thermal efficiency. Show that the efficiency calculated from the cycle calculation is equal to that from eqn (3.20). Assume the compression and expansion are isentropic, and \( \kappa = 1.4 \). How does this compare with the efficiency of a Carnot cycle between the two temperature limits, and what is the value of \( \beta \)?

\[45.02%; \ 89.07%; \ 3.6453\]

P3.21 The Otto cycle in P3.18 achieved a peak pressure of 118.1 bar, whilst the diesel cycle in P3.20 only reached 25.12 bar. If the compression ratio of the diesel cycle was increased to reach 118.1 bar at the end of compression what would be the cycle efficiency? How does this fit in with the analysis in this chapter?

\[67.57\%\]

P3.22 The final air standard cycle associated with reciprocating engines is the dual combustion cycle. Assume the Otto cycle in P3.18 is modified so that half the energy is added at constant volume, and the other half at constant pressure. What is the efficiency of this cycle based on the ratio of work output to energy addition? Evaluate \( \alpha \) and \( \beta \), defined in the text, and calculate the efficiency using eqn (3.21)?

\[58.49%; \ 2.850; \ 1.465\]

*There are more problems relating to reciprocating engines in Chapter 16, and gas turbines in Chapter 17.*