2.7 A FORMULATION SUITABLE FOR A COMPUTER SOLUTION *

Thus far we have seen several circuit examples that we solved by writing a set of equations based on the constituent relations for the elements, KVL, and KCL. There were as many independent equations as unknown variables, which allowed us to solve for any variable by simple algebra. The same set of equations can be written in matrix form so that they are amenable to a computer solution. For example, the circuit in Figure 2.1 analyzed using the basic method in Section 2.3.5 resulted in ten equations and ten unknowns. These ten equations are summarized as follows:

\[ v_1 = i_1 R_1 \]  
\[ v_2 = i_2 R_2 \]  
\[ v_3 = i_3 R_3 \]  
\[ v_4 = i_4 R_4 \]  
\[ v_5 = V \]  
\[ -v_5 + v_1 - v_2 = 0 \]  
\[ +v_2 + v_3 + v_4 = 0 \]  
\[ -i_5 - i_1 = 0 \]  
\[ +i_1 + i_2 - i_3 = 0 \]  
\[ i_3 - i_4 = 0. \]  

The ten unknowns are \( v_1, v_2, v_3, v_4, v_5, i_1, i_2, i_3, i_4, \) and \( i_5 \). The equations can be rewritten so that constant voltages and currents appear on the left-hand side of the equation.

\[ 0 = v_1 - i_1 R_1 \]  
\[ 0 = v_2 - i_2 R_2 \]  
\[ 0 = v_3 - i_3 R_3 \]  
\[ 0 = v_4 - i_4 R_4 \]  
\[ V = v_5 \]  
\[ 0 = v_1 - v_2 - v_5 \]  
\[ 0 = v_2 + v_3 + v_4 \]  
\[ 0 = -i_5 - i_1 \]  
\[ 0 = i_1 + i_2 - i_3 \]  
\[ 0 = i_3 - i_4. \]
This set of equations can be written in matrix form as follows:

\[
\begin{bmatrix}
0 \\
0 \\
0 \\
V \\
0
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 & -R_1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & -R_2 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & -R_3 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & -R_4 & 0 \\
1 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\
\end{bmatrix}
\begin{bmatrix}
v_1 \\
v_2 \\
v_3 \\
v_4 \\
v_5 \\
i_1 \\
i_2 \\
i_3 \\
i_4 \\
i_5 \\
\end{bmatrix}
\]

(2.222)

This matrix equation is in the form

\[b = Ax\]

where \(x\) is a column vector of unknowns and \(b\) is the column vector of drive voltages and currents. This vector of unknowns can be solved by a computer using standard linear algebraic techniques such as Cramer’s rule. In fact, the well known SPICE software package uses methods such as these to solve circuits.\(^5\)

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\(^5\) The examples in this chapter focused on linear circuits, which result in a set of linear simultaneous equations. However, the fundamental method of solving circuits based on KVL, KCL, and constituent relations applies equally well to nonlinear circuits. A nonlinear circuit might contain nonlinear circuit elements with constituent relations such as \(v = i^2 R\), or, \(i = k(e^{v/V_T} - 1)\). The resulting set of equations that arise will be nonlinear. Computer solution of such circuits makes use of another technique called linearization, which is discussed in Chapter 4. Further discussions of linearity are in Chapter 3, and a further treatment of nonlinear circuits is in Chapter 4.