9.4.1 SINUSOIDAL INPUTS*

Sinusoidal signals are an important class of inputs to electronic circuits. So, as a first example of specific inputs to the circuits shown in Figures 9.31 through 9.34, consider the special cases of

\[
I(t) = \begin{cases} 
0 & t \leq 0 \\
I_o \sin(\omega t) & t > 0 
\end{cases}
\] (9.67)

\[
V(t) = \begin{cases} 
0 & t \leq 0 \\
V_o \sin(\omega t) & t > 0 
\end{cases}
\] (9.68)

Note that both sources are zero for \( t \leq 0 \), but nonzero for \( t > 0 \), so that they effectively turn on at \( t = 0 \). A sketch of \( I(t) \) is shown in Figure 9.35a.

To complete the analysis of the circuits, we substitute the corresponding source function from either Equation 9.67 or 9.68 into Equations 9.63 through 9.66 and carry out the indicated integration or differentiation. This results in

\[
v(t) = \begin{cases} 
0 & t \leq 0 \\
\frac{I_o}{\omega C} (1 - \cos(\omega t)) & t > 0 
\end{cases}
\] (9.69)

FIGURE 9.35 The current \( I \), the voltage \( V \), the power \( vI \), and the energy \( \frac{1}{2}C v^2 \) stored in the capacitor, for the circuit shown in Figure 9.31 given the sinusoidal source current from Equation 9.67.
for the capacitor circuit shown in Figure 9.31,

\[ i(t) = \begin{cases} 
0 & t \leq 0 \\
\omega CV \cos(\omega t) & t > 0 
\end{cases} \quad (9.70) \]

for the capacitor circuit shown in Figure 9.32,

\[ i(t) = \begin{cases} 
0 & t \leq 0 \\
\frac{V_0}{\omega L} (1 - \cos(\omega t)) & t > 0 
\end{cases} \quad (9.71) \]

for the inductor circuit shown in Figure 9.33, and

\[ v(t) = \begin{cases} 
0 & t \leq 0 \\
\omega LI \cos(\omega t) & t > 0 
\end{cases} \quad (9.72) \]

for the inductor circuit shown in Figure 9.34. Note that for these equations to make sense, the units of \( \omega C \) must be conductance and the units of \( \omega L \) must be resistance; they are. We will encounter these products again in future chapters.

A comparison of the circuit inputs given in Equations 9.67 and 9.68 to the circuit responses given in Equations 9.69 through 9.72 shows that the sinusoidal components of the current and voltage in each circuit are \( \pi/2 \) radians out of phase with each other. This is in keeping with the observation that the circuits perform integration or differentiation from current to voltage or voltage to current. In the case of the capacitor circuits, the current leads the voltage because the current must be present first to build up the charge to which voltage is proportional. In the case of the inductor circuits, the voltage leads the current because the voltage must be present first to build up the flux linkage to which the current is proportional.

The operation of the circuits in Figures 9.31 through 9.34 demonstrates that inductors and capacitors are capable of reversible energy storage. To see this, let us examine the circuit shown in Figure 9.31 in detail; an examination of the three remaining circuits would yield identical observations. For this circuit, the power delivered by the source to the capacitor is given by

\[ v(t)i(t) = \begin{cases} 
0 & t \leq 0 \\
\frac{P_0}{\omega C} \sin(\omega t) (1 - \cos(\omega t)) & t > 0. 
\end{cases} \quad (9.73) \]
Integration of this power, or rate of energy delivery to the capacitor, yields

\[
\nu_E(t) = \begin{cases} 
0 & t \leq 0 \\
\frac{L_2^2}{\omega^2 C} \left( \frac{3}{4} - \cos(\omega t) + \frac{1}{4} \cos(2\omega t) \right) & t > 0
\end{cases}
\]

(9.74)
as the energy stored in the capacitor. The current \( I \), the voltage \( v \), the power \( vI \) into the capacitor, and the energy \( \nu_E \) stored in the capacitor are all shown in Figure 9.35. From the figure we see that the power can be both positive and negative indicating that energy can be delivered to and retrieved from the capacitor. In fact, during odd intervals of \( \pi/\omega \) in time, energy is delivered to the capacitor. It is then retrieved without loss during the following even interval of \( \pi/\omega \) in time. Thus, ideal capacitors are lossless energy reservoirs. The same is true for inductors.