INTRODUCTION AND TERMINOLOGY

1.1 INTRODUCTION

Historically, a signal meant any set of signs, symbols, or physical gesticulations that transmitted information or messages. The first electronic transmission of information was in the form of Morse code. In the most general sense a signal can be embodied in two forms: (1) some measured or observed behavior or physical property of a phenomenon that contains information about that phenomenon or (2) a signal that can be generated by a manufactured system and have the information encoded. Signals can vary over time or space. Our daily existence is replete with the presence of signals, and they occur not only in man-made systems but also in human and natural systems. A simple natural signal is the measurement of air temperature over time, as shown in Figure 1.1. Study of the fluctuations in temperature informs us about some characteristics of our environment. A much more complex phenomenon is speech. Speech is intelligence transmitted through a variation over time in the intensity of sound waves. Figure 1.2 shows an example of the intensity of a waveform associated with a typical sentence. Each sound has a different characteristic waveshape that conveys different information to the listener. In television systems the signal is the variation in electromagnetic wave intensity that encodes the picture information. In human systems, measurements of heart and skeletal muscle activity in the form of electrocardiographic and electromyographic voltages are signals. With respect to these last three examples, the objective of signal analysis is to process these signals in order to extract information concerning the characteristics of the picture, cardiac function, and muscular function. Signal processing has been implemented for a wide variety of applications. Many of them will be mentioned throughout this textbook. Good sources for other applications are Chen (1988) and Cohen (1986).
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FIGURE 1.1 The average monthly air temperature at Recife, Brazil. [Adapted from Chatfield, fig. 1.2, with permission]

FIGURE 1.2 An example of a speech waveform illustrating different sounds. The utterance is “should we chase . . . .” [Adapted from Oppenheim, fig. 3.3, with permission]
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A time-dependent signal measured at particular points in time is synonymously called a *time series*. The latter term arose within the field of applied mathematics and initially pertained to the application of probability and statistics to data varying over time. Some of the analyses were performed on economic or astronomic data such as the Beveridge wheat price index or Wolfer’s sunspot numbers (Anderson, 1971). Many of the techniques that are used currently were devised by mathematicians decades ago. The invention of the computer and now the development of powerful and inexpensive computers have made the application of these techniques very feasible. In addition, the availability of inexpensive computing environments of good quality has made their implementation widespread. All of the examples and exercises in and related to this textbook were implemented using subroutine libraries or computing environments that are good for a broad variety of engineering and scientific applications (Ebel, 1995; Foster, 1992). These libraries are also good from a pedagogical perspective because the algorithms are explained in books such as those written by Blahut, 1985; Ingle and Proakis, 2000; Press et al., 2002; and Stearns, 2003. Before beginning a detailed study of the techniques and capabilities of signal or time series analysis, an overview of terminology and basic properties of signal waveforms is necessary. As with any field, symbols and acronyms are a major component of the terminology, and standard definitions have been utilized as much as possible (Granger, 1982; Jenkins and Watts, 1968). Other textbooks that will provide complementary information of either an introductory or an advanced level are listed in the reference section.

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1.2.1 Domain Types

The *domain* of a signal is the independent variable over which the phenomenon is considered. The domain encountered most often is the time domain. Figure 1.3 shows the electrocardiogram (ECG) measured from the abdomen of a pregnant woman. The ECG exists at every instant of time, so the ECG evolves in the *continuous time domain*. Signals that have values at a finite set of time instants exist in the *discrete time domain*. The temperature plot in Figure 1.1 shows a discrete time signal with average temperatures given each month. There are two types of discrete time signals. If the dependent variable is processed in some way, it is an *aggregate* signal. Processing can be averaging, such as the temperature plot, or summing, such as a plot of daily rainfall. If the dependent variable is not processed but represents only

**FIGURE 1.3** The abdominal ECG from a pregnant woman showing the maternal ECG waveform (m) and the fetal ECG waveform (f). [Adapted from Inbar, p. 73, fig. 8, with permission]
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an instantaneous measurement, it is simply called instantaneous. The time interval between points, called the sampling interval, is very important. The time intervals between successive points in a time series are usually equal. However, there are several applications that require this interval to change. The importance of the sampling interval will be discussed in Chapter 3.

Another domain is the spatial domain and usually this has two or three dimensions in the sense of having two or three independent variables. Images and moving objects have spatial components with these dimensions. Image analysis has become extremely important within the last two decades. Applications are quite diverse and include medical imaging of body organs, robotic vision, remote sensing, and inspection of products on an assembly line. The signal is the amount of whiteness, called gray level, or color in the image. Figure 1.4 illustrates the task of locating a tumor in a tomographic scan. In an assembly line the task may be to inspect objects for defects as in Figure 1.5. The spatial domain can also be discretized for computerized analyses. Image analysis is an extensive topic of study and will not be treated in this textbook.

![Figure 1.4](image)

FIGURE 1.4  (a) A tomographic scan of a cross section of the abdomen. (b) A digitized model of the scan showing the calculated outlines of the major anatomical regions. [Adapted from Strohbehn and Douple, fig. 4, with permission]
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FIGURE 1.5 (a) Test image of a steel slab with imperfections. (b) Processed image with defects located. [Adapted from Suresh et al., figs. 12 & 13, with permission]

1.2.2 Amplitude Types

The amplitude variable, like the time variable, also can have different forms. Most amplitude variables, such as temperature, are continuous in magnitude. The most pertinent discrete-amplitude variables involve counting. An example is the presentation of the number of monthly sales in Figure 1.6. Other phenomena that involve counting are radioactive decay, routing processes such as in telephone exchanges, or other queueing processes. Another type of process exists that has no amplitude value. These are called point processes and occur when one is only interested in the time or place of occurrence. The study of neural coding of information involves the mathematics of point processes. Figure 1.7 illustrates the reduction of a measured signal into a point process. The information is encoded in the time interval between occurrences or in the interaction between different channels (neurons).
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1.2.3 Basic Signal Forms

There are different general types of forms for signals. One concerns periodicity. A signal, $x(t)$, is periodic if; it exists for all time, $t$, and

$$x(t) = x(t + P)$$

(1.1)

where $P$ is the duration of the period. These signals can be constituted as a summation of periodic waveforms that are harmonically related. The triangular waveform in Figure 1.8 is periodic. Some signals can be constituted as a summation of periodic waveforms that are not harmonically related. The signal itself is not periodic and is called quasi-periodic. Most signals are neither periodic nor quasi-periodic and are called aperiodic. Aperiodic signals can have very different waveforms as shown in the next two figures. Figure 1.9 shows a biomedical signal, an electroencephalogram, with some spike features indicated by dots. Figure 1.10 shows the output voltage of an electrical generator.
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The time span over which a signal is defined is also important. If a signal has zero value or is nonexistent during negative time, $t < 0$, then the signal is called causal. The unit step function is a causal waveform. It is defined as

$$U(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$  \hspace{1cm} (1.2)$$

Any signal can be made causal by multiplying it by $U(t)$. If a signal’s magnitude approaches zero after a relatively short time, it is transient. An example of a transient waveform is a decaying exponential function which is defined during positive time; that is,
FIGURE 1.10 Output voltage signal from an electrical generator used for process control. [Adapted from Jenkins and Watts, fig. 1.1, with permission]

\[ x(t) = e^{-at} U(t), \quad a > 0 \]  

(1.3)

The wind gust velocity measurement shown in Figure 1.11 is a transient signal.

1.2.4 The Transformed Domain—The Frequency Domain

Other domains for studying signals involve mathematical transformations of the signal. A very important domain over which the information in signals is considered is the frequency domain. Knowledge of the distribution of signal strength or power over different frequency components is an essential part of many engineering endeavors. At this time it is best understood in the form of the Fourier series. Recall that any

FIGURE 1.11 Record of a wind gust velocity measurement. [Adapted from Bendat and Piersol, fig. 1.3, with permission]
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A periodic function, with a period of \( P \) units, as plotted in Figure 1.8, can be mathematically modeled as an infinite sum of trigonometric functions. The frequency terms in these functions are harmonics, integer multiples, of the fundamental frequency, \( f_0 \). The form is

\[
x(t) = C_0 + \sum_{m=1}^{\infty} C_m \cos(2\pi mf_0 t + \theta_m)
\]

where \( x(t) \) is the function, \( f_0 = 1/P \), \( C_m \) are the harmonic magnitudes, \( f_m = mf_0 \) are the harmonic frequencies, and \( \theta_m \) are the phase angles. The signal can now be studied with the harmonic frequencies assuming the role of the independent variable. Information can be gleaned from the plots of \( C_m \) versus \( f_m \), called the magnitude spectrum, and \( \theta_m \) versus \( f_m \), called the phase spectrum. The magnitude spectrum for the periodic waveform in Figure 1.8 is shown in Figure 1.12. Different signals have different magnitude and phase spectra. Signals that are aperiodic also have a frequency domain representation and are much more prevalent than periodic signals. This entire topic will be studied in great detail under the titles of frequency and spectral analysis.

1.2.5 General Amplitude Properties

There are two important general classes of signals that can be distinguished by waveform structure. These are deterministic and random. Many signals exist whose future values can be determined with certainty.

![Magnitude spectrum of the periodic triangular waveform in Figure 1.8.](image)
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FIGURE 1.13 Random signal (---) with exponential trend (---).

if their present value and some parameters are known. These are called deterministic and include all waveforms that can be represented with mathematical formulas, such as cosine functions, exponential functions, and square waves. Random or stochastic signals are those whose future values cannot be determined with absolute certainty based upon knowing present and past values. Figure 1.13 is an example. Notice there is a decaying exponential trend that is the same as in the deterministic waveform; however, there are no methods to predict the exact amplitude values. Figures 1.9, 1.10, and 1.11 are examples of actual random signals. The concepts of probability must be used to describe the properties of these signals.

A property associated with random signals is whether or not their characteristics change with time. Consider again the temperature signal in Figure 1.1. If one calculates average and maximum and minimum values over short periods of time and they do not change then the signal is stationary. Contrast this with the trends in the sales quantities in Figure 1.6. Similar calculations show that these parameters change over time; this signal is nonstationary. In general, stationary signals have average properties and characteristics that do not change with time, whereas the average properties and characteristics of nonstationary signals do change with time. These concepts will be considered in detail in subsequent chapters.

1.3 ANALOG TO DIGITAL CONVERSION

As mentioned previously, most signals that are encountered in man-made or naturally occurring systems are continuous in time and amplitude. However, since it is desired to perform computerized signal analysis, the signal values must be acquired by the computer. The input procedure is called analog to digital (A/D) conversion. This procedure is schematically diagrammed in Figure 1.14. The signal \( g(t) \) is measured continuously by a sensor with a transducer. The transducer converts \( g(t) \) into an electrical
signal \( f(t) \). Usually the transduction process produces a linear relationship between these two signals. The sampler measures \( f(t) \) every \( T \) time units and converts it into a *discrete time sequence*, \( f(nT) \). Typical A/D converters are capable of taking from 0 to 100,000 samples per second. The technology of telecommunications and radar utilizes A/D converters with sampling rates up to 100 MHz. Also inherent in the process is the *quantization* of the magnitude of the signal. Computer memory is composed of words with a finite bit length. Within the hardware of the A/D converter, each sampling (measurement) of the analog signal’s magnitude is converted into a digital word of a finite bit length. These integer words are then stored in memory. The word length varies from 4 to 32 bits. For many applications a 12-bit quantization has sufficient accuracy to ignore the quantization error. For applications requiring extreme precision or analyzing signals with a large *dynamic range*, defined as a range of magnitudes, converters with the longer word lengths are utilized.

For mathematical operations that are implemented in software, the set of numbers is transformed into a floating point representation that has proper units such as voltage, force, degrees, and so on. For mathematical operations that are implemented in hardware, such as in mathematical coprocessors or in special purpose digital signal processing chips, the set of numbers remains in integer form but word lengths are increased up to 80 bits.

### 1.4 MEASURES OF SIGNAL PROPERTIES

There are many measures of signal properties that are used to extract information from or to study the characteristics of signals. A few of the useful simple ones will be defined in this section. Many others will be defined as the analytic techniques are studied throughout the text. Initially both the continuous time and discrete time versions will be defined.

#### 1.4.1 Time Domain

The predominant number of measures quantizes some property of signal magnitude as it varies over time or space. The simplest measures are the maximum and minimum values. Another measure that everyone uses intuitively is the average value. The magnitude of the *time average* is defined as

\[
x_{av} = \frac{1}{P} \int_0^P x(t) dt
\]

(1.5)

in continuous time and

\[
x_{av} = \frac{1}{N} \sum_{n=1}^N x(nT)
\]

(1.6)

in discrete time, where \( P \) is the time duration, \( N \) the number of data points, and \( T \) is the sampling interval.
Signal energy and power are also important parameters. They provide a major classification of signals and sometimes determine the types of analyses that can be applied (Cooper and McGillem, 1967). Energy is defined as

$$ E = \int_{-\infty}^{\infty} x^2(t)dt $$  \hspace{1cm} (1.7)

or in discrete time

$$ E = T \sum_{n=-\infty}^{\infty} x^2(nT) $$  \hspace{1cm} (1.8)

An energy signal is one in which the energy is finite. Examples are pulse signals and transient signals, such as the wind gust velocity measurement in Figure 1.11. Sometimes signal energy is infinite as in periodic waveforms, such as the triangular waveform in Figure 1.8. However, for many of these signals the power can be finite. Power is energy averaged over time and is defined as

$$ PW = \lim_{p \to \infty} \frac{1}{2p} \int_{-p}^{p} x^2(t)dt $$  \hspace{1cm} (1.9)

or in discrete time

$$ PW = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x^2(nT) $$  \hspace{1cm} (1.10)

Signals with nonzero and finite power are called power signals. The class of periodic functions always has finite power.

### 1.4.2 Frequency Domain

Power and energy as they are distributed over frequency are also important measures. Again periodic signals will be used to exemplify these measures. From elementary calculus, the power in constant and sinusoidal waveforms is known. The power in the average component with a magnitude \( C_0 \) is \( C_0^2 \). For the sinusoidal components with amplitude \( C_1 \) the power is \( C_1^2/2 \). Thus for a periodic signal, the power, \( PW_M \), within the first \( M \) harmonics is

$$ PW_M = \frac{C_0^2}{T} + 0.5 \sum_{m=1}^{M} C_m^2 $$  \hspace{1cm} (1.11)

This is called the integrated power. A plot of \( PW_M \) versus harmonic frequency is called the integrated power spectrum. More will be studied about frequency domain measures in subsequent chapters.
REFERENCES


