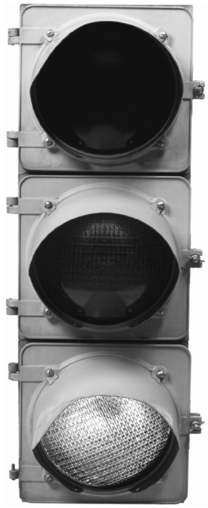

Introduction to Probability

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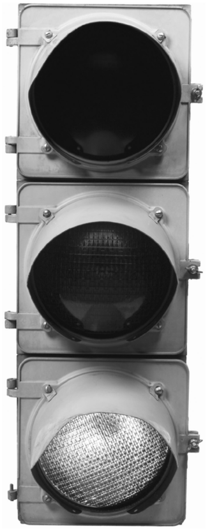
To David Blackwell
and in memory of
Lucien Le Cam



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Preface

Overview

This book is an introductory textbook in probability. No prior knowledge in probability is required; however, previous exposure to an elementary precalculus course in probability would prove beneficial in that the student would not see the basic concepts discussed here for the first time.

The mathematical prerequisite is a year of calculus. Familiarity with the basic concepts of linear algebra would also be helpful in certain instances. Often students are exposed to such basic concepts within the calculus framework. Elementary differential and integral calculus will suffice for the majority of the book. In some parts of Chapters 7 through 11, the concept of a multiple integral is used. In Chapter 11, the student is expected to be at least vaguely familiar with the basic techniques of changing variables in a single or a multiple integral.

Chapter Descriptions

The material discussed in this book is enough for a one-semester course in introductory probability. This would include a rather detailed discussion of Chapters 1 through 12, except, perhaps, for the derivations of the probability density functions following Definitions 1 and 2 in Chapter 11. It could also include a cursory discussion of Chapter 13.

Most of the material in Chapters 1 through 12—with a quick description of the basic concepts in Chapter 13—can also be covered in a one-quarter course in introductory probability. In such a case, the instructor would omit the derivations of the probability density functions mentioned above, as well as Sections 9.4, 10.3, 11.3, and 12.2.3.

A chapter-by-chapter description follows. Chapter 1 consists of 16 examples selected from a broad variety of applications. Their purpose is to impress upon the student the breadth of applications of probability, and draw attention to the wide range of situations in which probability questions are pertinent. At this stage, one could not possibly provide answers to the questions posed without the methodology developed in the subsequent chapters. Answers to most of these questions are given in the form of examples and exercises throughout the remaining chapters. In Chapter 2, the concept of a random experiment is introduced, along with related concepts and some fundamental results. The concept of a random variable is also introduced here, along with the basics in counting. Chapter 3 is devoted to the introduction of the concept of probability and the discussion of some basic properties and results, including the distribution of a random variable.

Conditional probability, related results, and independence are covered in Chapter 4. The quantities of expectation, variance, moment-generating function, median, and mode of a random variable are discussed in Chapter 5, along with some basic probability inequalities.

The next chapter, Chapter 6, is devoted to the discussion of some of the commonly used discrete and continuous distributions.

When two random variables are involved, one talks about their joint distribution, as well as marginal and conditional probability density functions and also conditional expectation and variance. The relevant material is discussed in Chapter 7. The discussion is pursued in Chapter 8 with the introduction of the concepts of covariance and correlation coefficient of two random variables.

The generalization of concepts in Chapter 7, when more than two random variables are involved, is taken up in Chapter 9, which concludes with the discussion of two popular multivariate distributions and the citation of a third such distribution. Independence of events is suitably carried over to random variables. This is done in Chapter 10, in which some consequences of independence are also discussed. In addition, this chapter includes a result, Theorem 6 in Section 10.3, of significant importance in statistics.

The next chapter, Chapter 11, concerns itself with the problem of determining the distribution of a random variable into which a given random variable is transformed. The same problem is considered when two or more random variables are transformed into a set of new random variables. The relevant results are mostly simply stated, as their justification is based on the change of variables in a single or a multiple integral, which is a calculus problem. The last three sections of the chapter are concerned with three classes of special but important transformations.

The book is essentially concluded with Chapter 12, in which two of the most important results in probability are studied, namely, the weak law of large numbers and the central limit theorem. Some applications of these theorems are presented, and the chapter is concluded with further results that are basically a combination of the weak law of large numbers and the central limit theorem. Not only are these additional results of probabilistic interest, they are also of substantial statistical importance.

As previously mentioned, the last chapter of the book provides an overview of statistical inference.

Features

This book has a number of features that may be summarized as follows. It starts with a brief chapter consisting exclusively of examples that are meant to provide motivation for studying probability.

It lays out a substantial amount of material—organized in twelve chapters—in a logical and consistent manner.

Before entering into the discussion of the concept of probability, it gathers together all needed fundamental concepts and results, including the basics in counting.

The concept of a random variable and its distribution, along with the usual numerical characteristics attached to it, are all introduced early on so that fragmentation in definitions is avoided. Thus, when discussing some special discrete and continuous random variables in Chapter 6, we are also in a position to present their usual numerical characteristics, such as expectation, variance, moment-generating function, etc.

Generalizations of certain concepts from one to more than one random variable and various extensions are split into two parts in order to minimize confusion and enhance understanding. We do these things for two random variables first, then for more than two random variables. Independence of random variables is studied systematically within the framework dictated by the level of the book. In particular, the reproductive property of certain distributions is fully exploited.

All necessary results pertaining to transformation of random variables are gathered together in one chapter, Chapter 11, rather than discussing them in a fragmented manner. This also allows for the justification of the distribution of order statistics as an application of a previously stated theorem. The study of linear transformations provides the tool of establishing Theorem 7 in Section 11.3, a result of great importance in statistical inference.

In Chapter 12, some important limit theorems are discussed, preeminently the weak law of large numbers and the central limit theorem. The strong law of large numbers is not touched upon, as not even an outline of its proof is feasible at the level of an introductory probability textbook.

The book concludes with an overview of the basics in statistical inference. This feature was selected over others, such as elements of Markov chains, of Poisson processes, and so on, in order to provide a window into the popular subject matter of statistics. At any rate, no justice could be done to the discussion of Markov chains, of Poisson processes, and so on, in an introductory textbook.

The book contains more than 150 examples discussed in great detail and more than 450 exercises suitably placed at the end of sections. Also, it contains at least 60 figures and diagrams that facilitate discussion and understanding. In the appendix, one can find a table of selected discrete and continuous distributions, along with some of their numerical characteristics, a table of some formulas used often in the book, a list of some notation and abbreviations, and often extensive answers to the even-numbered exercises.

Concluding Comments

An *Answers Manual*, with extensive discussion of the solutions of all exercises in the book, is available for the instructor.

A table of selected discrete and continuous distributions, along with some of their numerical characteristics, can also be found on the inside covers of the book. Finally, the appendix contains tables for the binomial, Poisson, normal, and chi-square distributions.

The expression $\log x$ (logarithm of x), whenever it occurs, always stands for the natural logarithm of x (the logarithm of x with base e).

The rule for the use of decimal numbers is that we retain three decimal digits, the last of which is rounded up to the next higher number (if the fourth decimal is greater or equal to 5). An exemption to this rule is made when the division is exact, and when the numbers are read out of tables.

On several occasions, the reader is referred to proofs for more comprehensive treatment of some topics in the book *A Course in Mathematical Statistics*, 2nd edition (1997), Academic Press, by G.G. Roussas.

Thanks are due to my project assistant, Carol Ramirez, for preparing a beautiful set of typed chapters out of a collection of messy manuscripts.