PRINCIPLES OF MATHEMATICAL MODELING

Second Edition

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This second edition of
Principles of Mathematical Modeling *is*
dedicated to my first grandchildren,

August Dym Noë and Courtney Makenna Anderson,

and to all of their future siblings
and first cousins!

Contents

Preface

My Goals For This Book

Science and engineering students depend heavily on concepts of mathematical modeling. In an age where almost everything is done on a computer, *it is my conviction that students of engineering and science are better served if they understand and "own" the underlying mathematics that the computers are doing on their behalf.* Mathematics is a *necessary* language for doing engineering and science. This will remain true no matter how good computation becomes. I repeatedly tell students that it is risky to accept computer calculations without having done some parallel closed-form modeling to benchmark the computer results. *Without such benchmarking and validation, how do we know that the computer isn't talking nonsense?* Finally, I find it satisfying and fun to do mathematical manipulations that explain how or why something happens, and to use mathematics to obtain corresponding numerical data or predictions.

Thus, as it was for the first edition, *my primary goal for this second edition remains to engage the reader in developing a foundation for mathematical modeling.* Further, knowing that mathematical models are built in a range of disciplines—including physics, biology, ecology, economics, sociology, military strategy, as well as all of the many branches of engineering—and knowing that mathematical modeling is comprised of a very diverse set of skills and tools, *I focused on techniques of particular interest to engineers, scientists, and others who model continuous systems.*

Features of This Edition

Aided by a variety of reviewers' comments and suggestions, this second edition features:

- A more formal statement of a principled approach to mathematical modeling (in Chapter 1). Ten principles are articulated and invoked as applications are developed, and each of them is identified by a key word (see below).
- Some 360 problems, many of which are designed to reinforce skills in mathematical manipulation. Many could be done with a computer algebra system (CAS), and there are others for which numerical programs could be used. However, given my goals for this book, I would ask students do the problems in "the old-fashioned way."
- A reordering and expansion of the applications chapters that reflects some sense of increasing complexity.
- Expanded figure captions that are intended to be more informative.

How This Book Is Organized

The book is organized into two parts: foundations and applications. The first part lays out the fundamental mathematical ideas of interest to the model builder: dimensional analysis, scaling, and elementary approximations of curves and functions. The applications part of the book develops a series of models and discusses their origins, their validity, and their meaning. These models include a host of exponential models, traffic flow models, free and forced vibration of linear (and occasionally nonlinear) oscillators, and optimization as done both with calculus and with elementary operations research techniques.

In the applications discussions, reference to the modeling principles is made by highlighting appropriate key words in the margin immediately adjacent to the appropriate text, as in:

Why? "Lanchester wanted to describe the attrition of opposing forces at war. This
Find? required modeling the changes of two army populations whose respective rates of attrition depend on the size of the opposing army."

The foundations and applications parts of the book are connected only loosely. The following matrix indicates roughly how the chapters in each part relate to each other. In fact, the reader—and the teacher—can easily start with Chapter 5 and work through the applications models, referring back to corresponding discussions of the foundations as needed.

The problems distributed throughout and at the end of each chapter (save Chapter 1) are an integral part of the book. Like bike riding and dancing and designing, mathematical modeling cannot be learned simply by reading. Skills are developed and honed by doing problems, both elementary and difficult. Thus, there are problems that provide drills in basic skills, and there are problems that either develop new models or expand on models developed earlier in the text. For example, in problems at the end of Chapter 3 we show how dimensional groups are used to interpret experimental results. The problems in Chapter 5 demonstrate how dimensional analysis interacts with other approaches to deriving the governing equations for the oscillating pendulum, and the problems in Chapter 7 include data on resonance and impedance for a variety of forced oscillators.

		Models				
		5	6	7	8	9
Tools		Exponential Growth and Decay	Traffic Flow Models	Modeling Free Vibration	Applying Vibration Models	What Is the Best?
2	Dimensional Analysis	•	•	•		•
3	Scaling	•	•	•		
4	Approximation	•	•	•	•	•

As noted earlier, many of the problems could be done with a computer, whether a symbolic manipulator, a spreadsheet, or an algorithmic number cruncher. However, in order to learn to do mathematical modeling, the problems should be done in "closed form," with pencil and paper, with access only to a simple electronic calculator. This will both reinforce skills and provide a basis for benchmarking future computer calculations.

Three appendices from the first edition have been moved closer to their use in the book. A brief review of elementary transcendental functions is now appended to Chapter 4; the mathematics of the first-order equation, $dN/dt - \lambda N = 0$, is outlined in Section 5.2.2; and the mathematics of the second-order oscillator equation, $md^2x/dt^2 + kx = F(t)$, is detailed in Sections 7.2.2. and 8.6.

Lastly, the book can be used in several ways. The first edition was developed for new courses in mathematical modeling that were offered to first-year *engineering students* at Carnegie Mellon University and at the University of Massachusetts at Amherst. The book could also serve as a first course in applied mathematics for *mathematics majors*, or as a "technical elective" for various science and engineering majors, or conceivably as a supplementary text in basic calculus courses. In hopes of extending its audience, I have tried to enhance both the book's accessibility and its flexibility.

I Presume That You, the Reader, Have. . .

. . . taken courses in elementary algebra, trigonometry, and first-year calculus. I further presume that you recognize what a differential equation is and what it means for $y(x)$ or $y(x, t)$ to be a solution of a differential equation. While you won't be asked to "solve" a differential equation, you will be asked to confirm and manipulate some of the solutions that are given. Finally, I do assume some basic understanding of first-year physics, mainly mechanics.

Acknowledgments

This book is the second edition of a text originally published in 1980 and written by me and Elizabeth S. Ivey; Betty was then both a professor of physics at Smith College and an adjunct professor of mechanical engineering at the University of Massachusetts at Amherst. When approached in the summer of 2000 by Academic Press to do a second edition, Betty and I decided that I would do the second edition alone, but I cannot view the second edition as complete without acknowledging the wonderful nature of our original collaboration and the long-standing friendship that resulted.

Many people deserve much credit for the good in this new version, although the responsibility for the bad (and the ugly) is entirely mine. Professors Robert L. Borrelli (Harvey Mudd College), Edward A. Connors (University of Massachusetts at Amherst), Ricardo Diaz (University of North Colorado), Michael Kirby (Colorado State University), Mark S. Korlie (Montclair State University), Thomas Seidman (University of Maryland Baltimore County), Caroline Smith (James Madison University), and William H. Wood (University of Maryland Baltimore County) were kind enough to provide pre-publication reviews of this second edition that were very helpful and supportive.

Professor Ewart Carson (University of London) provided a solid and helpful review of the first edition that prompted several changes in the second.

The artist Miriam Dym once again agreed to design the cover for one of her father's books, for which I am very grateful.

Drs. Ashkay Gupta and Ali Reza (Exponent FAA) provided a thoughtful review of the discussion of the vibration of a tall, slender building (Section 8.2).

Several Harvey Mudd colleagues provided helpful comments on various sections: Patrick Little (Engineering) of Chapter 9 on optimization; R. Erik Spjut (Engineering) of Section 9.5.1 on nucleation; Lisa M. Sullivan (Humanities and Social Sciences) of Section 5.5 on exponential modeling of money matters; and Harry E. Williams (Engineering), who commented on much of the early material in painstaking detail.

Dr. John H. McMasters (The Boeing Company) allowed me to use parts of an unpublished manuscript on geometric programming in constructing Sections 9.5.2 and 9.5.3, and graciously reviewed the final product.

Professor David Powers (Clarkson University) served once again as a reviewer of this book. In fact, Dr. Powers has also reviewed the prepublication prospectus of the first edition, provided a very useful re-review of that first edition that helped define the second edition, and then reviewed this manuscript during its preparation.

Professor Wilfred W. Recker (University of California, Irvine), a good friend for almost forty years, carefully reviewed the discussion of traffic flow modeling (Chapter 6) and provided some new materials.

Robert Ross, formerly of Academic Press, instigated this second edition, assisted ably by Mary Spencer. Subsequently, over the extended writing life of the project, Barbara Holland has been an encouraging and supportive editor, and she was assisted very ably by Tom Singer.

Professors Michael J. Scott (University of Illinois, Chicago) and William H. Wood (University of Maryland Baltimore County) commented insightfully on my adaptation (for Section 9.4) of our joint paper on choosing among alternatives.

Gautam Thatte, Harvey Mudd College ’03, did a great job of researching and rendering illustrations.

Finally, Joan Dym has provided her usual friendship, love, and oft-needed distraction while this second edition was being written. It is a true pleasure to acknowledge what this has meant to me.

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Claremont, California
May 2004

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